Design of Generalized H_∞-suboptimal Controllers Based on Experimental and A Priori Data

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Abstract—This paper considers a linear continuous- or discrete-time dynamic object in the absence of its mathematical model. ...

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1. INTRODUCTION

As was established in [3], a single trajectory can be used to fully characterize a linear time-invariant dynamic system under the so-called persistency of excitation. In view of this fundamental result, different direct control design schemes based on experimental data were proposed in [4] for objects with unknown state dynamics matrices and given target output matrices under the persistency of excitation. According to [5], it suffices to fulfill the data informativity condition in order to construct control laws from experimental data, which is less restrictive than the persistency of excitation. For a fully uncertain object, H_2- and H_∞-optimal control laws were constructed based on input and output measurements using a matrix version of S-lemma [6] in the publication [7] and using Petersen's lemma [8] in the publication [5–7, 9]. In [10, 11], the state feedback parameters were calculated from a priori data and open-loop measurements of the input and output of a discrete-time uncertain object subjected to an unmeasured disturbance from a definite class.

2. PROBLEM STATEMENT

2.1. Cccccccc Vvvvvvv

Consider an uncertain system described by

$$\begin{aligned} \partial x(t) &= Ax(t) + Bu(t) + w(t), \quad x(0) = x_0, \\ z(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

with the following notations: ∂ is the differentiation operator in the continuous-time case or the shift operator in the discrete-time case; $x(t) \in \mathrm{R}^{n_x}$ is the state vector, $u(t) \in \mathrm{R}^{n_u}$ is the control vector (input), $w(t) \in \mathrm{R}^{n_w}$ is an exogenous disturbance, and $z(t) \in \mathrm{R}^{n_z}$ is the target output. By assumption, the disturbance $w(t) \in L_2(l_2)$ and the system matrices A, B, C, and D are unknown. In general, it is required to design linear state-feedback control laws based on a priori and experimental data so that the damping level of the disturbances in the closed loop system does not exceed a specified value.

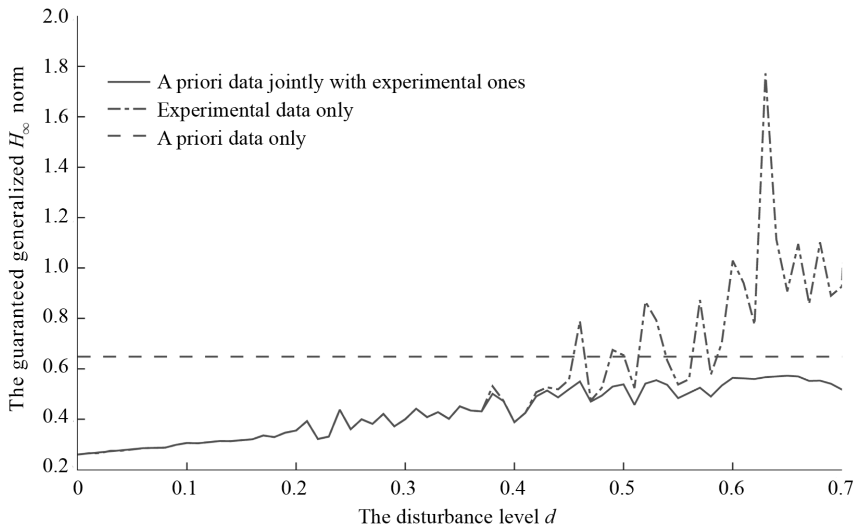

Fig. 1. The guaranteed estimates of the H_∞ norm as functions of the disturbance level in experimental data for different types of available information..

Proposition 1. *For any $m \in \mathbb{Z}^+ \setminus 0$, the optimal projection estimator $\widehat{f}^{\,0}_m(x) \in \mathbb{M}_{2,m}(\widetilde{\mathsf{P}})$ exists if and only if there are $\{\widehat{c}^{\,0}_m\} \in \widetilde{\mathbb{M}}_{2,m}(\mathsf{P})$ such that*

$$\inf_{\{\widehat{c}_{j,m}\}_{j\geqslant 0} \in \widetilde{\mathbb{M}}_{2,m}(\mathsf{P})} \mathsf{E}\sum_{j=0}^{\infty}\left[c_j - \widehat{c}_{j,m}\right]^2 = \mathsf{E}\sum_{j=0}^{\infty}\left[c_j - \widehat{c}^{\,0}_{j,m}\right]^2. \tag{2}$$

The proof of Proposition 1 is given in Appendix A.

The information about the unknown parameters of system (1) is extracted from a finite set of measurements of its trajectory. For the discrete-time system, there are available measurements of its state and target output, $x_0, x_1, \ldots, x_N$ and $z_0, \ldots, z_{N-1}$, respectively, under chosen controls $u_0, \ldots, u_{N-1}$ and some unknown disturbance $w_0, \ldots, w_{N-1}$. We compile the matrices

$$\Phi = (x_0 \cdots x_{N-1}), \quad \Phi_+ = (x_1 \cdots x_N),$$
$$U = (u_0 \cdots u_{N-1}), \quad W = (w_0 \cdots w_{N-1}), \quad Z = (z_0 \cdots z_{N-1}).$$

In the continuous-time case, there are measurements of the system state, its derivative, and the target output, $x(t_0), \ldots, x(t_{N-1})$, $\dot{x}(t_0), \ldots, \dot{x}(t_{N-1})$, and $z(t_0), \ldots, z(t_{N-1})$, respectively, under chosen controls $u(t_0), \ldots, u(t_{N-1})$ and some unknown disturbances $w(t_0), \ldots, w(t_{N-1})$ at time instants $t_0, \ldots, t_{N-1}$.

Lemma 2.1. *If the information matrix $\widehat{\Phi}\widehat{\Phi}^{\mathrm{T}}$ is nonsingular, then the set $\mathbf{\Delta_p}$ is a nondegenerate "matrix ellipsoid" centered at Δ_{LS} given by*

$$(\Delta - \Delta_{LS})(\widehat{\Phi}\widehat{\Phi}^{\mathrm{T}})(\Delta - \Delta_{LS})^{\mathrm{T}} \leq \Gamma, \tag{3}$$

where

$$\Gamma = \widehat{\Omega} + \widetilde{\Phi}[\widehat{\Phi}^{\mathrm{T}}(\widehat{\Phi}\widehat{\Phi}^{\mathrm{T}})^{-1}\widehat{\Phi} - I]\widetilde{\Phi}^{\mathrm{T}} \geq 0, \tag{4}$$

and $\Delta_{LS} = \widetilde{\Phi}\widehat{\Phi}^{\mathrm{T}}(\widehat{\Phi}\widehat{\Phi}^{\mathrm{T}})^{-1}$ is the optimal least-squares estimate of the unknown matrix Δ_{real} ....

Corollary 2.1. *For $v(t) \equiv 0$, the γ_0 norm of system satisfies the condition $\gamma_0 < \gamma$ iff there exists a quadratic form $V_a(x_a) = x_a^{\mathrm{T}} P x_a$ with $P > R$ such that the corresponding inequality is valid for $z_a(t) \equiv 0$ along the trajectories of the dual system*

$$\begin{aligned} \partial x_a(t) &= \mathcal{A}^{\mathrm{T}} x_a(t) + \mathcal{C}^{\mathrm{T}} v_a(t), \\ z_a(t) &= \mathcal{B}^{\mathrm{T}} x_a(t). \end{aligned} \tag{2.5}$$

Remark 1. Formally, the dual system is described by the equations

$$\begin{aligned} \dot{\widehat{x}}_a &= -\mathcal{A}^{\mathrm{T}} \widehat{x}_a - \mathcal{C}^{\mathrm{T}} \widehat{v}_a, \\ \widehat{z}_a &= \mathcal{B}^{\mathrm{T}} \widehat{x}_a \end{aligned} \tag{2.6}$$

in the continuous-time case.

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APPENDIX A

A.1. PROOF OF LEMMA 2.1

Let $f(x) \in L_2(\mathrm{K}, \Lambda)$ and $\widehat{f}_m(x) \in \mathbb{M}_{2,m}(\widetilde{\mathsf{P}})$ be some projection estimator. ...

A.1.1. Proof of Lemma 2.1

Let $f(x) \in L_2(\mathrm{K}, \Lambda)$ and $\widehat{f}_m(x) \in \mathbb{M}_{2,m}(\widetilde{\mathsf{P}})$ be some projection estimator. ...

APPENDIX B

EXAMPLE

Hence, for any ...

$$V_m(N) = \sum_{j=N+1}^{\infty} c_j^2 + \mathsf{E} \sum_{j=0}^{N} \left(c_j - \widehat{c}_{j,m}^{\,0}\right)^2. \tag{B.1}$$

Since ... we have

$$\widehat{c}_{j,m}^{\,0} = \frac{1}{m} \sum_{k=1}^{m} \left(c_j + n_k^j\right) = c_j + \frac{1}{m} \sum_{k=1}^{m} n_k^j. \tag{B.2}$$

In view of...

B.1. PROOF OF PROPOSITION 1

Hence, for any ...

$$V_m(N) = \sum_{j=N+1}^{\infty} c_j^2 + \mathsf{E} \sum_{j=0}^{N} \left(c_j - \widehat{c}_{j,m}^{\,0}\right)^2. \tag{B.1.1}$$

Since ... we have

$$\widehat{c}_{j,m}^{\,0} = \frac{1}{m} \sum_{k=1}^{m} \left(c_j + n_k^j\right) = c_j + \frac{1}{m} \sum_{k=1}^{m} n_k^j. \tag{B.1.2}$$

In view of...

B.1.1. Proof of Corollary 2.1

Hence, for any ...

$$V_m(N) = \sum_{j=N+1}^{\infty} c_j^2 + \mathsf{E} \sum_{j=0}^{N} \left(c_j - \widehat{c}_{j,m}^{\,0}\right)^2. \tag{B.1.3}$$

Since ... we have

$$\widehat{c}_{j,m}^{\,0} = \frac{1}{m} \sum_{k=1}^{m} \left(c_j + n_k^j\right) = c_j + \frac{1}{m} \sum_{k=1}^{m} n_k^j. \tag{B.1.4}$$

In view of...

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