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SPECIAL ISSUE

Controlled Random Search and Likelihood Ratio in Boolean Programming Problems

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Abstract—Based on the sequential probability ratio test (likelihood ratio) method, a controlled random search algorithm is proposed for the approximate solution of large-scale discrete programming problems. The reduction in the exhaustive search of feasible sets of the decision variables of the problem is achieved by introducing non-zero probabilities of false recognition of the optimal solution. As a practical application of the algorithm, the problem of forming optimal well placement patterns in oil and gas reservoirs is considered. The results of computational experiments are presented, the purpose of which was to study the accuracy of the problem's solution depending on its dimension (the number of blocks where well placement is possible and the number of wells to be placed were varied). The optimal solution of the problem obtained by one of the exact methods of discrete programming was used as a reference solution, against which the accuracy of the approximate solution generated by the proposed algorithm was evaluated.

Keywords: algorithm, Boolean programming, probability, statistical hypotheses, maximum likelihood method, probability density, optimization, transportation problem

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1. INTRODUCTION

One of the main approaches to solving large-scale discrete programming problems is the use of approximate algorithms that allow estimating the error in determining the exact (optimal) solution [1–3]. This approach is quite justified when solving applied problems, for which a typical situation is not only their large dimensionality but also a significant error in the values of the initial parameters. In this case, the value of the exact solution decreases, and it becomes possible to reduce the number of analyzed feasible solutions of the problem by abandoning the guarantee of obtaining an exact solution.

In accordance with [2], a large-scale problem will be understood as a problem for which the search for an optimal solution on a specific computer takes a time exceeding a specified value, or requires an amount of memory exceeding the allocated volume.

A significant class of approximate methods is composed of controlled random search methods [1, 2]. Unlike "blind" random search, these methods allow accumulating information obtained at previous stages of the search for an optimal solution to correct subsequent stages.

The algorithm proposed in this paper belongs to the controlled random search methods; it represents an iterative procedure, at each step of which, given the probabilities of erroneous inference, the truth of one of two statistical hypotheses is tested: can the feasible solution, which is the most preferable relative to all previously generated feasible solutions in terms of the objective function, be considered optimal, or should the search be continued?

2. PROBLEM STATEMENT AND ALGORITHM DESCRIPTION

Consider the linear Boolean programming problem:

$$F(X) \to \min \tag{1}$$

$$X = (x_1, \dots, x_k) \in A \tag{2}$$

$$x_i \in \{0, 1\}, \quad i = 1, \dots, k, \tag{3}$$

where A is a subset of k-dimensional Euclidean space represented by linear constraints, and $F(X)$ is a linear function defined on the subset A.

For the large-scale problem (1)–(3), solving it would require a computationally prohibitive exhaustive search of all feasible solutions (in the general case, such an exhaustive search is impossible due to the practically unlimited volume of required memory and, accordingly, computation time). Let us introduce the set B:

$$B \equiv \{X = (x_1, \dots, x_k) : X \in A, 0 \le x_i \le 1,\ i = 1, \dots, k\}. \tag{4}$$

Assuming that an exhaustive search of all feasible solutions is impossible due to enormous time costs, let us consider the main steps of the algorithm.

1. Two problems are solved by any available linear programming method:

$$F(X) \to \min_{X \in B}, \tag{5}$$

$$F(X) \to \max_{X \in B}. \tag{6}$$

Let $F_{\min}$ and $F_{\max}$ be the objective function values in the optimal solutions of problems (5) and (6), respectively. Then, taking into account (4), for the values of the objective function (1) under constraints (2) and (3), it will follow that:

$$F_{\min} \le F(X) \le F_{\max}. \tag{7}$$

2. A set of n feasible solutions to problem (1)–(3) is generated randomly (according to some probability distribution): $X_1, \dots, X_n$, and a set $F_1 \equiv F(X_1), \dots, F_n \equiv F(X_n)$ is formed, which represents a realization of a random variable—the value of the objective function of problem (1)–(3).

If for $r, j \in \{1, \dots, n\}$ it turns out that the conditions $F_r \le F_j$ and $F_r = F_{\min}$ are satisfied, then it follows from inequalities (7) that X_r is the optimal solution to problem (1)–(3), and all calculations are terminated. Otherwise, the following testing stages are required, where the case $F_r > F_{\min}$ is considered. If over the next $n+1, n+2, \dots$ "trials" the condition $F_r \le F_j$ is satisfied, where $j \in \{n+1, n+2, \dots\}$, a suspicion arises: is X_r the optimal solution to problem (1)–(3)?

The main goal of the subsequent stages of the proposed modification of the random search method is to answer the question: how many additional trials, in each of which $F_{\min} < F_r \le F_j$ is satisfied, where $j \in \{n+1, n+2, \dots\}$, should be conducted so that, given the probabilities of false inference, one could assert that X_r is the optimal solution to problem (1)? The answer to these questions is achieved as follows.

3. Let us introduce the notations $X^* \equiv X_r$ and $F^* \equiv F(X_r)$. Based on the obtained values of the objective function (1), a histogram is constructed. From the shape of the histogram, one of the possible probability distributions for $F(X)$—the values of the objective function (1)—is selected. This selected probability distribution is applied to the values of the random variable $F(X)$ belonging both to the interval $[F^*, F_{\max}]$ and to the interval $[F_{\min}, F_{\max}]$. In this regard, two hypotheses H_1 and H_2 are put forward:

$$\mathrm{H}_1 : F(X) \in [F^*, F_{\max}]; \tag{8}$$

$$\mathrm{H}_2 : F(X) \in [F_{\min}, F_{\max}]. \tag{9}$$

To test the proposed hypotheses, it is suggested to use the sequential probability ratio test (or likelihood ratio) [4]. One of its advantages is the ability not to specify the required number of trials (measurements, iterations) in advance, since after each trial, either the validity of hypothesis H_1 is established or a conclusion is made about the need to continue the trials. Thus, unnecessary trials are not conducted.

4. The implementation of the sequential probability ratio test (SPRT) when applied to problem (1)–(3) consists of the following steps.

4.1. The probabilities of false recognition are introduced: γ_{lt} is the probability of accepting H_l when H_t is actually true, where $l, t \in \{1, 2\}$. Two thresholds are determined: α and β, $\alpha > \beta$ [4]:

$$\alpha \equiv (1 - \gamma_{21})/\gamma_{12}, \quad \beta \equiv \gamma_{21}/(1 - \gamma_{12}). \tag{10}$$

4.2. Based on the selected type of probability distribution of the random variable $F(X)$ and its realizations $F_1, \ldots, F_n$ obtained during the trials, the vectors of parameters Y_1 and Y_2 of two conditional probability density functions of the objective function values (1) are determined using the maximum likelihood method [5]. The first conditional probability density function $p_{1n}(f/Y_1, F^*)$ corresponds to hypothesis H_1 (condition (8) is valid), i.e., $F^* \le F(X) \le F_{\max}$. The second conditional probability density function $p_{2n}(f/Y_2, F_{\min})$ corresponds to hypothesis H_2 (condition (9) is valid), i.e., $F_{\min} \le F(X) \le F_{\max}$. In other words, two probability density functions $p_{1n}(f/Y_1, F^*)$ and $p_{2n}(f/Y_2, F_{\min})$ are reconstructed, which is reduced to solving two problems:

$$\prod_{j=1}^{n} p_{1n}(F_j/Y_1, F^*) \to \max_{Y_1}, \tag{11}$$

$$\prod_{j=1}^{n} p_{2n}(F_j/Y_2, F_{\min}) \to \max_{Y_2}. \tag{12}$$

After determining Y_1 and Y_2, one can proceed to testing hypotheses H_1 and H_2, i.e., to answering the question: is the random variable $F(X)$ distributed according to the first law $p_{1n}(f/Y_1, F^*)$ or the second law $p_{2n}(f/Y_2, F_{\min})$?

4.3. A new feasible solution X_{n+1} is randomly generated, and F_{n+1}—the value of the objective function (1) corresponding to this solution—is calculated.

4.4. The likelihood ratio δ_{n+1} [4] is introduced:

$$\delta_{n+1} = \frac{p_{1n}(F_{n+1}/Y_1, F^*)}{p_{2n}(F_{n+1}/Y_2, F_{\min})}.$$

4.5. The conditions for stopping or continuing the trials [4] are checked:

a) if $\delta_{n+1} \ge \alpha$, then H_1 is considered true, i.e., the hypothesis is accepted: X^* is the optimal solution to problem (1)–(3);

b) if $\beta < \delta_{n+1} < \alpha$, then it is necessary to generate a new feasible solution X_{n+2}, calculate F_{n+2}, determine δ_{n+2} using the formula

$$\delta_{n+2} = \frac{p_{1n}(F_{n+1}/Y_1, F^*)}{p_{2n}(F_{n+1}/Y_2, F_{\min})} \cdot \frac{p_{1n}(F_{n+2}/Y_1, F^*)}{p_{2n}(F_{n+2}/Y_2, F_{\min})},$$

again compare δ_{n+2} with the thresholds α and β, etc. (with each iteration, the number of factors in the likelihood ratio increases by one);

c) if the condition $\delta_{n+1} \le \beta$ is satisfied (H_2 is true), i.e., $\delta_{n+1} = 0$ because $F_{n+1} < F^*$ and $p_{1n}(F_{n+1}/Y_1, F^*) = 0$, then it is necessary to set $X^* \equiv X_{n+1}$, $F^* \equiv F(X_{n+1})$, find new values of Y_1 and Y_2 by solving problems (11) and (12) where $j = 1, 2, \ldots, n+1$, and repeat steps 4.2–4.5, setting $n \equiv n + 1$.

Let $(m-1)$ be the number of iterations of the recognition process in each of which the condition of item b) was satisfied, and realizations of the random variable $F(X)$ were obtained: $F_{n+1}, F_{n+2}, \ldots, F_{n+m-1}$. Then, at the mth iteration after calculating F_{n+m}, the likelihood ratio takes the form:

$$\delta_{n+m} = \prod_{j=n+1}^{n+m} \frac{p_{1n}(F_j/Y_1, F^*)}{p_{2n}(F_j/Y_2, F_{\min})}. \tag{13}$$

3. APPLICATION OF THE ALGORITHM TO FORM OPTIMAL WELL PLACEMENT PATTERNS IN OIL AND GAS RESERVOIRS

Consider the application of the algorithm to solve the problem of placing a given number of wells in oil and gas reservoirs, which is a key task in the list of problems for designing the development of oil and gas fields. Numerous studies are devoted to its solution (see, for example, reviews [6, 7]), most of which use placement models where the objective functions represent final performance indicators of the reservoir development processes, such as the hydrocarbon recovery factor from the subsurface. This necessitates reservoir simulation of oil and gas field development processes [8], the implementation of which is associated with significant time costs.

Consider one of the alternative approaches to designing gas (oil) well placement patterns, in which the well placement problem is reduced to a linear Boolean programming model [9]. In this model, the objective function is a formalization of heuristic rules for rational well placement, verified by years of practice in oil and gas field development. In this case, calculating the objective function values will not require involving reservoir simulation. According to these rules, the placement of a given number of wells (more precisely, bottom-holes) should ensure [9]:

a) the smallest possible distance from the wells to any point of the productive formation;

b) equality of the drainage areas of the wells;

c) proximity of the wells to the reservoir areas with higher reserve values.

Fulfilling these rules aims to achieve the maximum possible reservoir coverage by a given number of wells and to increase the hydrocarbon recovery factor from the reservoir.

The substantive formulation of the problem is as follows: let the reservoir be divided into blocks, in each of which a bottom-hole can be placed; the number of wells to be placed is given; it is required to determine the set of blocks containing bottom-holes for which the heuristic rules of rational well placement (rules *a, b, c*) are violated to a minimum extent.

Let us proceed to constructing the optimization model, focusing on the results of paper [9]. Preliminarily, the productive area is approximated by a two-dimensional domain consisting of equal-area square blocks. The gas (oil) reserves of each block are known. It is assumed that when placing a well in any block, the coordinates of the bottom-hole coincide with the center of this block.

Let us introduce the initial parameters. Let S be the number of production wells, K be the number of blocks per well, N be the total number of blocks, $N = KS$. In order for the problem to admit non-trivial solutions, we set $N > S \geq 1$, i.e., $K > 1$.

Let V_j be the geological reserves of the jth block, $V_j > 0$, and $V \equiv \max\{V_j\}, j = 1, \ldots, N$. We introduce the parameter λ_j, characterizing the productivity ("importance," "usefulness") of the jth block or the potential efficiency of a well placed in this block. Then the ratio $\lambda_j = V_j/V$ can be chosen as λ_j [9]. Let us introduce the parameter R_{ij}—the distance between the centers of the ith and jth blocks, $R_{ij} > 0$, $j \neq i$, $R_{ii} = 0$, $R \equiv \max\{R_{ij}\}$, $i = 1, \ldots, N$, $j = 1, \ldots, N$. Let $r_{ij} = R_{ij}/R$.

Let us define the parameter c_{ij}—the "weighted distance" between the ith and jth blocks:

$$c_{ij} = \begin{cases} (\lambda_j)^{1-\gamma}(r_{ij})^{\gamma}, & i \neq j, \\ 0, & i = j, \end{cases} \tag{14}$$

where γ is an expert assessment of the importance of the "distance" indicator relative to the "reserves" indicator, $0 \leq \gamma \leq 1$. The choice of γ for oil and gas wells may vary. Taking into account the higher mobility of gas compared to oil, γ for gas reservoirs will have lower values. The parameter c_{ij} can be interpreted as a penalty for violating rules *a, b, c* when including the jth block in the area of influence of a well located in the ith block. The area of influence is understood as the set of blocks around the well from which the well receives the main influx of formation fluids.

To increase the adequacy of well placement models, instead of the Euclidean distance expressed in units of length, one can use parameters characterizing the resistance to the flow of formation fluids from one block to another as the distance between any pair of blocks (a detailed description of such a substitution can be found in [10]).

Let us introduce the decision variables x_{ij}: $x_{ij} = 1$ if the jth block is included in the area of influence (drainage area) of a well located in the ith block, and $x_{ij} = 0$ otherwise. It follows from the definition of x_{ij} that if $x_{ii} = 1$, then there is a well in the ith block, otherwise $x_{ii} = 0$.

Taking into account the above-formulated concept of rationality (rules *a, b, c*), the formation of the best well placement pattern is reduced to finding such decision variables x_{ij} that

$$\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}x_{ij} \to \min_{x}, \tag{15}$$

$$\sum_{i=1}^{N} x_{ii} = S, \tag{16}$$

$$\sum_{j=1}^{N} x_{ij} = Kx_{ii}, \quad i = 1, \ldots, N, \tag{17}$$

$$\sum_{i=1}^{N} x_{ij} = 1, \quad j = 1, \ldots, N, \tag{18}$$

$$x_{ij} \in \{0, 1\}, \quad i = 1, \ldots, N, \quad j = 1, \ldots, N. \tag{19}$$

In model (15)–(19): the objective function in criterion (15) is the total penalty for violating rules *a, b, c*; constraint (16) is the limit on the number of wells; constraint (17) is the condition that each area of influence consists of an equal number of blocks; constraint (18) is the condition that any block can be included in only one area of influence. Since it follows from (14) that $c_{ij} = 0$ for $j = i$, the variables x_{ii} for $j = i$ are absent in the objective function (15). Considering (14) and $V_j > 0$, $R_{ij} > 0$, $j \neq i$, $i = 1, \ldots, N$, $j = 1, \ldots, N$, the objective function (15) will be strictly greater than zero for any feasible solution.

The algorithm considered in the previous section utilizes the specificity of problem (15)–(19), which lies in the following. First, the system of equations (16)–(19) is linearly dependent due to the condition $N = KS$. Second, with a certain fixed well placement, i.e., with known feasible values of the variables x_{ii}, problem (15), (17)–(19) becomes a classical balanced transportation problem with a cost objective (T-problem) [11]. We will demonstrate the transformation of problem (15), (17)–(19) into a T-problem for one of the feasible well placements, according to which wells are placed in the first S blocks:

$$x_{ii} = 1, \quad i = 1, \ldots, S, \quad x_{ii} = 0, \quad i = S + 1, \ldots, N. \tag{20}$$

Taking into account relations (17), (18), (20), problem (15)–(19) takes the form:

$$\sum_{i=1}^{S}\sum_{j=S+1}^{N} c_{ij}x_{ij} \to \min_{x}, \tag{21}$$

$$\sum_{j=S+1}^{N} x_{ij} = K-1, \quad i=1,\ldots,S, \tag{22}$$

$$\sum_{i=1}^{S} x_{ij} = 1, \quad j=S+1,\ldots,N, \tag{23}$$

$$x_{ij} \geq 0, \quad i=1,\ldots,S, \quad j=S+1,\ldots,N. \tag{24}$$

In problem (21)–(24), considering that $N = KS$, the sum of the right-hand sides of equations (22) and the sum of the right-hand sides of equations (23) coincide and equal $(N-S)$. Due to constraint (23) and conditions (24), the values of the decision variables cannot be greater than 1. Since the right-hand sides of constraints (22) and (23) are strictly positive integers and condition (24) is introduced, condition (19) will be satisfied "automatically" [11]. Therefore, when condition (20) is satisfied, the optimal values of x_{ij} (where $i=1,\ldots,S$, $j=S+1,\ldots,N, j\neq i$) from the point of view of criterion (21) can be obtained using linear programming algorithms, which are more efficient compared to discrete programming algorithms when solving large-scale problems. The remaining decision variables of the original problem (15)–(19) are set to zero to satisfy constraints (16)–(19). Thus, a value of the objective function (15) will be obtained that is equal to the value of the objective function (21) in the optimal solution of problem (21)–(24).

By assigning values equal to one to some S elements of the main diagonal of the square matrix $\{x_{ij}\}_{N\times N}$ using a random number generator and setting the remaining elements of the main diagonal to zero (to satisfy constraint (16)), it is possible to generate optimal solutions to problem (21)–(24), which are feasible solutions to problem (15)–(19). Each feasible solution corresponds to a value of the objective function (15), which makes it possible to apply the proposed controlled random search algorithm to solve the original problem (15)–(19).

Below are the results of testing the proposed algorithm with various sets of initial data for problem (15)–(19). Natural and technological parameters of the development of a hypothetical gas field were used as initial data. Computational experiments were conducted by varying the key parameters of the problem: the total number of blocks (N), the number of placed wells (S), and γ—the parameter of the objective function (15), $0 \leq \gamma \leq 1$. The parameter γ determines the relative "weights" of rules a and c, namely: at $\gamma = 0$, only the block reserves are taken into account (complete dominance of rule c); at $\gamma = 1$, only the distances between wells are considered (complete dominance of rule a). Note that at $\gamma = 0$, the solution of problem (15)–(19) is trivial: wells are placed in the S blocks that exceed the other $(N-S)$ blocks in terms of reserves.

The proposed algorithm (PA) was compared with the Branch and Bound Method (BBM) [11], which allows obtaining an exact solution to problem (15)–(19). A computation time limit of 1200 seconds (20 minutes) was set for the BBM. The analysis is based on a comparison of the obtained objective function values (15): F^* corresponds to the best solution found by PA; F_{opt} to the optimal solution found by BBM (if available); $F_{\min}$ is the theoretical lower bound of the objective function (15), when problem (15)–(18), (24) is solved instead of problem (15)–(19). In addition, the time spent obtaining the final solutions was compared: T^* for PA, T_{opt} for BBM. The comparison results are presented in Table. Besides the parameters mentioned above, Table shows the error of the best (approximate) solution obtained using PA relative to the exact solution obtained using BBM: $\varepsilon = (F^* - F_{\text{opt}})/F_{\text{opt}}, \%$. The symbol ">" means that the allowed solution time was exceeded. In the PA implementation, the limit on the number of iterations was 5000.

Comparison of the approximate algorithm (PA) and the exact algorithm (BBM)

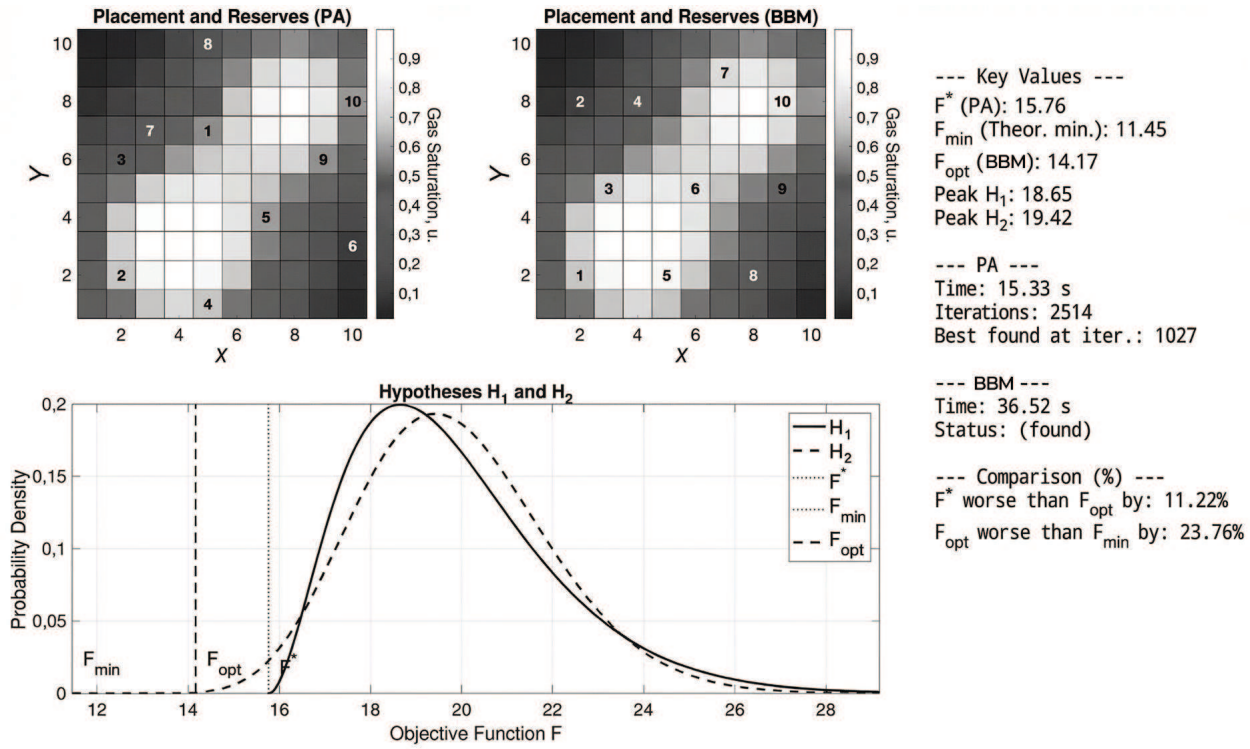
N	S	γ	F^*	F_{opt}	$F_{\min}$	T^*, s	T_{opt}, s	Number of PA iterations	ε, %
16	4	0.0	11.06	11.06	11.06	13.63	0.02	5000	0.00%
16	4	0.3	7.45	7.45	7.34	9.36	0.03	3562	0.00%
16	4	0.7	4.28	4.28	4.26	13.19	0.02	5000	0.00%
16	4	1.0	2.83	2.83	2.83	13.08	0.03	5000	0.00%
100	5	0.0	45.94	45.45	45.45	26.71	0.13	5000	1.08%
100	5	0.3	30.36	29.93	25.43	15.05	5.62	3203	1.44%
100	5	0.7	19.61	18.65	12.26	14.43	28.09	3102	5.15%
100	5	1.0	14.65	13.95	7.46	11.70	28.31	2743	5.02%
100	10	0.0	42.56	41.10	41.10	32.74	0.16	5000	3.55%
100	10	0.3	26.94	26.01	23.32	19.56	14.67	3151	3.58%
100	10	0.7	15.76	14.17	11.45	15.33	36.52	2514	11.22%
100	10	1.0	10.78	9.53	7.07	15.31	51.19	2728	13.12%
100	25	0.0	33.32	29.46	29.46	49.85	0.20	5000	13.10%
100	25	0.3	20.70	19.50	17.46	26.94	2.88	2943	6.15%
100	25	0.7	10.80	9.86	9.11	29.30	147.14	3217	9.53%
100	25	1.0	7.03	>	5.89	36.29	> 1200	3608	>
400	10	0.0	184.80	182.74	182.74	218.29	13.15	5000	1.13%
400	10	0.3	110.43	>	81.97	95.08	> 1200	2792	>
400	10	0.7	60.47	>	29.53	91.40	> 1200	2852	>
400	10	1.0	41.83	>	14.51	85.58	> 1200	2712	>
400	40	0.0	168.14	156.33	156.33	462.45	10.43	5000	7.55%
400	40	0.3	88.06	>	71.75	337.64	> 1200	3435	>
400	40	0.7	39.15	>	26.65	285.09	> 1200	3279	>
400	40	1.0	22.66	>	13.40	247.32	> 1200	2749	>
400	100	0.0	137.21	110.68	110.68	959.47	10.12	5000	23.97%
400	100	0.3	67.65	>	53.29	592.15	> 1200	2835	>
400	100	0.7	26.91	>	21.13	639.56	> 1200	3273	>
400	100	1.0	14.14	>	11.16	577.67	> 1200	2921	>

From the results presented in Table, the following conclusions can be drawn.

1. Impact of problem dimensionality on computation time. For the problem with $N = 16$, both methods find the solution almost instantly (BBM is faster). Already at $N = 100$, T_{opt}—the computation time for BBM—increases noticeably and becomes comparable to T^*—the computation time for PA (or $T_{\text{opt}} > T^*$), for example, at $N = 100$, $S = 5$, $\gamma = 0.70$: $T_{\text{opt}} = 28$ s, $T^* = 14$ s; $N = 100$, $S = 10$, $\gamma = 1.00$: $T_{\text{opt}} = 51$ s, $T^* = 15$ s; $N = 100$, $S = 25$, $\gamma = 0.70$: $T_{\text{opt}} = 147$ s, $T^* = 29$ s. At $N = 400$, the BBM consistently fails to meet the 1200-second limit. In contrast, the PA completed its work significantly faster than this time threshold in all experiments without exception, finding a solution in a time ranging from one and a half minutes to ∼16 minutes (90–960 s), which demonstrates its applicability to large-scale problems.

2. Deviation from the exact solution. For small N ($N = 16$), the PA finds the exact solution ($F^* = F_{\text{opt}}$). For $N = 100$, the PA finds solutions that are only slightly inferior to the exact ones. The deviation of F^* from F_{opt} varies within reasonable limits (1–13%), as it does not exceed the error in the initial data for real production facilities. However, for extreme values of γ or a large number of wells, the deviation can reach 24%.

3. Influence of the parameter γ. The extreme values $\gamma = 0$ and $\gamma = 1$ simplify the problem for BBM, which finds the solution quickly (for $N \le 100$). In these cases, the PA shows poorer relative performance: it often reaches the iteration limit, and the error of the approximate solution may be higher than at intermediate γ values (0.3; 0.7). At $\gamma = 0$ or $\gamma = 1$, solutions with similar quality



Graphs of the probability density functions of the objective function values (15) for hypotheses H_1 and H_2 at $N = 100$, $S = 10$, $\gamma = 0.7$.

can arise, which hinders the operation of the PA. As noted above, at $\gamma = 0$ the solution is trivial: wells are concentrated in zones with maximum reserves; at $\gamma = 1$, wells are distributed as evenly as possible. Intermediate γ values (0.3; 0.7) create a more complex objective function landscape, slowing down BBM, but allowing the PA to demonstrate its speed advantages while maintaining high solution quality.

4. Estimation of the number of iterations required to obtain the final solution using PA. The PA stops either when the iteration limit (5000) is reached or when hypothesis H_1 is accepted ($\delta_{n+1} \geq \alpha$). The termination of the algorithm in many cases by the condition $\delta_{n+1} \geq \alpha$ (hypothesis H_1 is true) confirms its operability.

The figure shows the probability density functions of the random variable—the objective function (15) at $N = 100$, $S = 10$, $\gamma = 0.7$, corresponding to hypotheses H_1 (solid curve) and H_2 (dashed curve). Based on the histogram constructed using the realizations of this random variable, the β-distribution was chosen as the probability distribution.

4. CONCLUSION

It is advisable to use the proposed algorithm in integer programming problems where the process of generating feasible solutions is not computationally demanding. An example of such a problem is the model of optimal well placement in oil and gas reservoirs considered in this article.

The formation of oil and gas well placement patterns belongs to the main tasks of designing hydrocarbon field development processes. The application of the algorithm to solve this problem allows reducing the time for generating and selecting acceptable field development scenarios, from which the final option to be implemented is selected. It is important to note that the positive qualities of the algorithm manifest themselves specifically when solving large-scale problems, which is typical when designing the development of real oil and gas production facilities. The significant uncertainty in the values of the initial parameters perfectly justifies the use of approximate opti-

mization methods. The algorithm consistently finds near-optimal solutions, spending significantly less time on this than exact methods. This difference in performance is especially noticeable on large-scale problems, where the Branch and Bound Method regularly reached the 1200-second limit, while the proposed algorithm always remained far from this threshold.

The results of computational experiments showed the operability of the proposed controlled random search algorithm, which is based on the likelihood ratio implemented in the form of a sequential probability ratio test.

The reduction in the exhaustive search of feasible solutions is achieved not only by introducing non-zero probabilities of false inference but also by using all the information contained in the probability distributions of the random variable, whose realizations are the values of the optimization problem's objective function.

Since any linear discrete programming problem in which each decision variable takes integer values from some finite set, e.g., $\{0, 1, 2, \ldots, l\}$, where $l < \infty$, can be reduced to an equivalent linear Boolean programming model, the proposed controlled random search algorithm can be extended to solve discrete optimization problems of a more general form.

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REFERENCES

1. Finkelstein, Yu.Yu., *Priblizhennye metody i prikladnye zadachi diskretnogo programmirovaniya* (Approximate Methods and Applied Problems of Discrete Programming), Moscow: Nauka, 1976.
2. Sigal, I.Kh. and Ivanova, A.P., *Vvedenie v prikladnoe diskretnoe programmirovanie: modeli i vychislitel'nye algoritmy* (Introduction to Applied Discrete Programming: Models and Computational Algorithms), Moscow: Fizmatlit, 2003.
3. Khachaturov, V.R., Veselovskii, V.E., Zlotov, A.V., et al., *Kombinatornye metody i algoritmy resheniya zadach diskretnoi optimizatsii bol'shoi razmernosti* (Combinatorial Methods and Algorithms for Solving Large-Scale Discrete Optimization Problems), Moscow: Nauka, 2000.
4. Fu, K.S., *Sequential Methods in Pattern Recognition and Machine Learning*, Academic Press, 1968. Translated under the title *Posledovatel'nye metody v raspoznavanii obrazov i obuchenii mashin*, Moscow: Nauka, 1971.
5. Cramér, H., *Mathematical Methods of Statistics*, Princeton University Press, 1946. Translated under the title *Matematicheskie metody statistiki*, Moscow: Mir, 1975.
6. AlQahtani, G.D., Vadapalli, R., Siddiqui, S., and Bhattacharya, S., Well Optimization Strategies in Conventional Reservoirs. *SPE 160861*, 2012.
7. Latipov, A.R., Analysis of Models and Methods for Optimal Well Placement in Oil and Gas Reservoirs, *Avtom. Inform. TEK*, 2025, no. 6 (623), pp. 50–57.
8. Kanevskaya, R.D., *Matematicheskoe modelirovanie gidrodinamicheskikh protsessov razrabotki mestorozhdenii uglevodorodov* (Mathematical Modeling of Hydrodynamic Processes of Hydrocarbon Field Development), Moscow-Izhevsk: Institut komp'yuternykh issledovanii, 2003.
9. Ermolaev, A.I. and Ibragimov, I.I., Modeli ratsional'nogo razmeshcheniya skvazhin pri razrabotke gazovykh i gazokondensatnykh mestorozhdenii (Models of Rational Well Placement in the Development of Gas and Gas Condensate Fields), *Trudy Instituta problem upravleniya im. V.A. Trapeznikova RAN*, 2006, vol. XXVII, pp. 118–123.
10. Ermolaev, A.I. and Kuvichko, A.M., HPC-Based Optimal Well Placement, *Proc. of the EAGE Conference (ECMOR XIII)*, Biarritz, France, 10–13 September 2012.
11. Korbut, A.A. and Finkelstein, Yu.Yu., *Diskretnoe programmirovanie* (Discrete Programming), Moscow: Nauka, 1969.

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