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SPECIAL ISSUE

Robust Approach to Dynamic Compensation of Disturbances

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Abstract—In this paper, the classical problem statement of external disturbances compensation in linear systems is extended to the case of nonlinear systems in a robust formulation using the dynamic compensation method. The robust properties of closed-loop systems are provided through using of deep feedback loops and discontinuous control inputs. For the output variable regulation, it is proposed to consider part of the controlled plant as a generator of model uncertainties. In this case, unlike a disturbance generator, it is possible to correct the dynamics of the model generator and convergence rate. As an example of the application of the proposed approach, the problem of altitude control of an unmanned aerial vehicle with a spring-loaded suspension is solved.

Keywords: dynamic compensation, sliding modes, external disturbances, decomposition

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1. INTRODUCTION

In the control theory, the method of dynamic disturbance compensation is widely used in the following problem formulation. If it is possible to describe the behavior of external disturbances using a known mathematical model, then invariance can be ensured using the dynamic compensation method [1–3], or using the theory of asymptotic observers [4–7]. By using the first approach, asymptotic observers calculate estimates of unknown disturbances, which are then used for feedback generation. By utilizing the second one, the dynamic feedback is synthesized with the help of dynamic compensators in accordance with the disturbance model, thereby compensating for the influence of disturbances on output variables through control channels. When using both approaches, the synthesis of combined control law is based on an extended model of the original plant by introducing a dynamic subsystem corresponding to the disturbance subspace.

Some of the disadvantages of the dynamic compensation method are, firstly, the excessive expansion of the dynamic order of the control plant model due to the inclusion of a reference generator and, secondly, the sensitivity of the closed-loop system to parametric uncertainties in both the control plant model itself and the disturbance generator. The latter circumstance makes the use of the dynamic compensation method ineffective in practice.

To solve the problem of dynamic disturbance compensation for high-dimensional mathematical models of control plants, decomposition methods based on back-stepping control, decentralized control methods, and the block approach have proven effective. These methods split the solution to the original problem into independent stages, each of which synthesizes local feedback [8] in a reduced-dimensional subsystem. On the other hand, methods for systems with discontinuous sliding mode controls [9] and deep feedback [9, 10], which have already become classic in the synthesis of robust and invariant systems, make it possible to ensure the robustness of closed-loop systems to parametric uncertainties.

Section 2 presents a well-known result of the dynamic disturbance compensation method applied to linear systems using the motion separation method, which allows the synthesis problem to be broken down into independent, lower-dimensional subproblems for synthesizing the control plant itself and independently selecting the disturbance compensator. Within the framework of the approaches described above, Section 3 extends the problem of dynamic disturbance compensation to nonlinear systems. Section 4 considers the problem of dynamic compensation in the case of a parametrically uncertain disturbance generator.

Section 5 is devoted the problem of horizontal attitude control for an unmanned aerial vehicle (UAV) carrying a payload suspended by a tether. A unique feature of the problem formulation in this example is that the disturbance generator is a model of the behavior of a load suspended on a weightless string, which, generally speaking, is an integral part of the control object model. Unlike a classical disturbance generator, the model of a load suspended on a weightless string considered in this article is a non-autonomous, yet controllable, subsystem. Feedback allows for influencing the dynamics of the load's behavior and compensating for external disturbances. The authors believe the proposed approach has significant potential in control theory.

2. PRELIMINARY DISCUSSION. FORMULATION OF THE PROBLEM

For the preliminary discussion let us consider the linear system

$$\dot{x} = Ax + Bu + Q\eta, \quad y = Dx, \tag{1}$$

combined with the model of the external perturbations η

$$\dot{\omega} = W\omega, \;\; \eta = H\omega, \;\; \omega \in R^r, \;\; \dim W = (r \times r), \;\; \dim H = (l \times r), \tag{2}$$

where $x \in R^n$ is the system state space vector, $u \in R^m$ is the vector of control inputs, $y \in R^p$ is the output vector, $\eta \in R^l$ is the disturbances vector, $A,\ B,\ Q,\ D,\ W,\ H$ are constant matrices of the appropriate dimensions.

Under condition, that the disturbances vector is not measured, the problem of its attenuation with respect to the output vector is stated in the paper.

Also, it is assumed, that for the plant (1)–(2) the matching condition is satisfied, under which the channels of action of the disturbances and control vectors coincide:

$$\mathrm{Im}Q \subset \mathrm{Im}B \Rightarrow \exists \Lambda : Q = B\Lambda, \;\dim \Lambda = (m \times l). \tag{3}$$

In this case, the equations (1) can be rewritten as

$$\dot{x} = Ax + B(u + \Lambda\eta), \;\; y = Dx. \tag{4}$$

It should be noted, that for the object (4), the invariance conditions for the whole state space vector are satisfied (full disturbances suppression). In [11], the decomposition approach is proposed, which provides the solution of the stated problem of the invariance in the plant (2), (4). According to this methodology, the synthesis task for the system with $n + r$ dimension to be broken down into independently solvable subproblems of $n - m$, m, r dimensions.

Using a non-singular change of variables, the system (2)–(4) is reduced to the regular controllability form [12]:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2, \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2u + Q_2\omega, \\ \dot{\omega} &= W\omega, \end{aligned} \tag{5}$$

where $x_1 \in R^{n-m}$, $x_2 \in R^m$, $\mathrm{rank}B_2 = m$.

The dynamic compensator is formed like the following

$$\dot{z} = Wz - v,\ \ v, z \in R^r. \tag{6}$$

The relation for the control input is

$$u = -B_2^{-1}Q_2 z + \overline{u}, \tag{7}$$

After combining (5)–(7) one can get the system for the variables $x_1,\ x_2,\ \varepsilon = \omega - z$:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2, \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2\overline{u} + Q_2\varepsilon, \\ \dot{\varepsilon} &= W\varepsilon + v. \end{aligned} \tag{8}$$

After the coordinate transformation $\overline{x}_2 = x_2 + F_1x_1$ and $\overline{\varepsilon} = \varepsilon + Lx_2$ the last plant (8) is written in the form

$$\begin{aligned} \dot{x}_1 &= (A_{11} - A_{12}F)x_1 + \overline{x}_2, \\ \dot{\overline{x}}_2 &= \overline{A}_{21}x_1 + \overline{A}_{22}\overline{x}_2 + B_2\overline{u} + Q_2\overline{\varepsilon}, \\ \dot{\overline{\varepsilon}} &= (W + LQ_2)\overline{\varepsilon} + G_{11}x_1 + G_{12}\overline{x}_2 + LB_2\overline{u} + v. \end{aligned}$$

The control inputs of the dynamic compensator and the system $v,\ \overline{u}$ are chosen according to the formulas

$$\begin{aligned} v &= -G_{11}x_1 - G_{12}\overline{x}_2 - LB_2\overline{u}, \\ \overline{u} &= -B_2^{-1}(\overline{A}_{21}x_1 + (\overline{A}_{22} - Q_2L)\overline{x}_2 - F_2\overline{x}_2), \end{aligned} \tag{9}$$

The equations of the closed-loop system are

$$\begin{aligned} \dot{x}_1 &= (A_{11}x_1 - A_{12}F_1)x_1 + \overline{x}_2, \\ \dot{\overline{x}}_2 &= -F_2\overline{x}_2 + Q_2\overline{\varepsilon}, \\ \dot{\overline{\varepsilon}} &= (W + LQ_2)\overline{\varepsilon}. \end{aligned} \tag{10}$$

Assuming, that the original system is controllable and observable, the desired spectrum can be assigned by choosing the matrices $L,\ F_1,\ F_2$ in the system (10). The solution of this problem can be broken down into independently solvable subproblems of smaller dimensions $n-m$, m, r, respectively [11].

3. METHOD OF DYNAMIC COMPENSATION IN NONLINEAR SYSTEMS

In nonlinear case the equations of the control object and disturbances are

$$\begin{gathered} \dot{x} = f(x) + B(x)u + Q(x)\eta,\ \ \dot{\omega} = W\omega,\ \ \eta = H\omega, \\ x \in R^n,\ \ \ u \in R^m,\ \ \ \eta \in R^r. \end{gathered} \tag{11}$$

For the system (11) additionally requirement is introduced, that is

$$f(0) = 0. \tag{12}$$

The matching condition (3) is transformed to the representation

$$\mathrm{Im}Q(x) \subset \mathrm{Im}B(x), \forall x \Rightarrow \exists\Lambda(x) : Q(x) = B(x)\Lambda(x),$$

According to this matching condition, system (11) can be represented in regular controllability form [12] with the help of some nonsingular nonlinear coordinate transformation

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2), \\ \dot{x}_2 &= f_2(x_1, x_2) + B_2(x_1, x_2)[u + \Lambda_2(x_1, x_2)\omega], \end{aligned} \tag{13}$$

where $x_1 \in R^{n-m},\ x_2 \in R^m,\ u \in R^m,\ \det B_2 \neq 0$.

Following the block approach [8], a step-by-step procedure is proposed below to solve the system stabilization problem (13).

Step 1. Introducing new variable $\overline{x}_2 = x_2 - \varphi(x_1)$ one can write from (13)

$$\begin{aligned} &\dot{x}_1 = f_1(x_1, \overline{x}_2 + \varphi(x_1)), \\ &\dot{\overline{x}}_2 = f_2(x_1, \overline{x}_2 + \varphi(x_1)) + B_2(x_1, \overline{x}_2 + \varphi(x_1))[u + \Lambda_2(x_1, \overline{x}_2 + \varphi(x_1))\omega] - \frac{\partial \varphi(x_1)}{\partial x_1}\dot{x}_1, \end{aligned} \tag{14}$$

Further, the notations for the functions $f_2(x_1, \overline{x}_2 + \varphi(x_1))$, $B_2(x_1, \overline{x}_2 + \varphi(x_1))$, $\Lambda_2(x_1, \overline{x}_2 + \varphi(x_1))$ are simplified to $f_2(\cdot)$, $B_2(\cdot)$, $\Lambda_2(\cdot)$.

In the first subsystem, $\varphi(x_1)$ is chosen to ensure its stabilization $x_1 \to 0$ under the assumption $\overline{x}_2 = 0$.

Step 2. The stability of the second subsystem (13) can be provided by choosing the control input u in the form

$$u = B_2^{-1}(\cdot)\left[F_2\overline{x}_2 - f_2(\cdot) + \frac{\partial \varphi(x_1)}{\partial x_1}\dot{x}_1\right] + \Lambda_2(\cdot)\left[-z_2 + L_2\overline{x}_2\right], \tag{15}$$

where

$$\dot{z}_2 = Wz_2 + v_2, \ v_2, z_2 \in R^r. \tag{16}$$

The second subsystem (14), closed by feedback (15), taking into account (16), with the variables $\overline{x}_2$, $\varepsilon_2 = \omega - z_2 + L_2\overline{x}_2$ takes the representation

$$\begin{aligned} &\dot{\overline{x}}_2 = F_2\overline{x}_2 + B_2(\cdot)\Lambda_2(\cdot)\varepsilon_2, \\ &\dot{\varepsilon}_2 = W\varepsilon_2 + L_2\left(F_2\overline{x}_2 + B_2(\cdot)\Lambda_2(\cdot)\varepsilon_2\right) + v_2. \end{aligned} \tag{17}$$

Step 3. By using the feedback $v_2 = -L_2F_2\overline{x}_2$, one can rewrite (17)

$$\begin{aligned} &\dot{\overline{x}}_2 = F_2\overline{x}_2 + B_2(\cdot)\Lambda_2(\cdot)\varepsilon_2, \\ &\dot{\varepsilon}_2 = [W + L_2B_2(\cdot)\Lambda_2(\cdot)]\varepsilon_2. \end{aligned} \tag{18}$$

The stabilization problem of the system (18) is solved by appropriate choice of the matrix F_2, L_2. The separation principle may be used. The elements of the matrix F_2 is chosen to provide negative real parts of the matrix F_2 eigen values. Then with $L_2 = L_{20}B_2^{-1}(0,0)$ $(L_{20} \in R^{r\times m})$ the stability of the second subsystem (18) may be provided

$$\dot{\varepsilon}_2 = [W + L_{20}\Lambda_2(0,0)]\varepsilon_2,$$

where the matrix $W + L_{20}\Lambda_2(0,0)$ is Hurwitz, if the pair $(\Lambda_2(0,0), W)$ is observable.

Thus, as in the linear case, the stabilization problem of the nonlinear system (11) can be solved based on the choice of local feedbacks in subsystems of lower dimension.

By applying the theory of the systems with separable motions based on deep feedback and/or sliding modes, it is also possible to decompose the solution of the object (11) stabilization problem, while the stability of the subsystem (16) is provided by using the methods of the modal control theory.

Let us consider a solution of the stabilization problem of the plant (13) by using sliding mode theory [9]. In the second subsystem (14), let us introduce the control input like the following

$$u = v - \Lambda_2(\cdot)\overline{z}_2 + B_2^{-1}(\cdot)\frac{\partial \varphi(x_1)}{\partial x_1}\dot{x}_1, \ v = -M(x)\operatorname{sign}\overline{x}_2,$$

where the variable $\overline{z}_2$ is described by the dynamic system

$$\dot{\overline{z}}_2 = W\overline{z}_2 + Lv, \quad \overline{z}_2 \in R^r. \tag{19}$$

For coordinates $\overline{x}_2$, $\overline{\varepsilon}_2 = \omega - \overline{z}_2$ from (14), taking into account (19), the following relations can be obtained

$$\begin{aligned} \dot{\overline{x}}_2 &= f_2(\cdot) + B_2(\cdot)[v + \Lambda_2(\cdot)\overline{\varepsilon}_2], \\ \dot{\overline{\varepsilon}}_2 &= W\overline{\varepsilon}_2 + Lv. \end{aligned} \tag{20}$$

Let us choose the matrix

$$M(x) = B_2^{-1}(x)\mathrm{diag}\{M_i\}, \quad M_i = \mathrm{const} > 0 \ \ (i = \overline{1, m}),$$

where $\mathrm{diag}M_i$ is diagonal matrix.

With the sufficiently large M_i, the sliding mode arises along the manifold

$$\overline{x}_2 = x_2 - \varphi(x_1) = 0.$$

By choosing a vector function $\varphi(x_1)$, that stabilizes the first equation (14), the problem of the stability of the first subsystem (14) is solved.

In the sliding mode, after convergence of the variables $x_1, \ x_2$ to the origin ($f_2(x_1, x_2) = 0$), one can use the equivalent control method [9] and write

$$v_{eq} = -B_2^{-1}(0,0)\Lambda_2(0,0)\overline{\varepsilon}_2.$$

Given that the pair of matrices $(\Lambda_2(0,0), W)$ is observable, by assigning the coefficients of the matrix $L = L_0 B_2^{-1}(0,0)$, a spectrum with negative real parts of the eigenvalues is provided for the matrix of the internal dynamic for the variable $\overline{\varepsilon}_2$ from (20):

$$\dot{\overline{\varepsilon}}_2 = [W - L_0\Lambda(0,0)]\overline{\varepsilon}_2.$$

4. DISTURBANCES GENERATOR WITH UNKNOWN PARAMETERS

This section discusses one of the variants of the plant (1) with a control vector of the unit dimension. It is assumed, that the behavior of disturbances is described by a linear non-stationary dynamic system with a certain interval parameters:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2, \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2(u + \eta), \end{aligned} \tag{21}$$

where $x_1 \in R^{n-1}, \ x_2 \in R, \ u, \ \eta \in R^1$ and the disturbances are generated by a dynamic model of the form

$$\dot{\omega} = W\omega, \quad \eta = q^{\mathrm{T}}\omega, \ \ \omega \in R^r, \ \ \dim W = (r \times r). \tag{22}$$

The parameters of the matrix W are assumed to be unknown, but belonging to some intervals $|w_{ij}| \le W_{ij} = \mathrm{const}$.

In the previous sections, it was indicated, that the controllability of the pair (A_{11}, A_{12}) implies the controllability of the original system. Using a non-singular change of variables $\overline{x}_1 = x_2 - f_1^{\mathrm{T}}x_1$, the equations of the system (22) can be rewritten like the following

$$\begin{aligned} \dot{x}_1 &= (A_{11} + a_{12}f_1^{\mathrm{T}})x_1 + a_{12}\overline{x}_1, \\ \dot{\overline{x}}_1 &= \overline{a}_{21}^{\mathrm{T}}x_1 + \overline{a}_{22}\overline{x}_1 + b_2u + q^{\mathrm{T}}\omega, \end{aligned} \tag{23}$$

By choosing the vector f_1^{T}, the eigenvalues of the matrix $A_{11}+a_{12}f_1^{\mathrm{T}}$ are assigned, and the problem of stabilizing the (23) object is reduced to ensuring the stability of the second subsystem.

Since the parameters of the disturbances generator (22) are unknown, a dynamic compensator in the form of (6) cannot be selected to solve stated problem. Without loss of generality, under the assumption of observability of the pair (q^{T}, W), the disturbances generator model (22) can be transformed to the canonical representation

$$\dot{\omega}_{\mathrm{i}} = \omega_{i+1},\ \ i = \overline{1, r-1},\ \ \dot{\omega}_r = \sum_{\mathrm{i}=1}^{r} g_i\omega_i,\ \ |g_i| \le G_i = \mathrm{const},\ \ \omega_1 = q^{\mathrm{T}}\omega. \tag{24}$$

The second subsystem (23) with the control input $b_2u = -\overline{a}_{21}^{\mathrm{T}}x_1 - \overline{a}_{22}\overline{x}_1 + \overline{u}$ is written in the form

$$\dot{\overline{x}}_1 = \overline{u} + \omega_1. \tag{25}$$

Therefore, the original problem has been transformed into the problem of the dynamic compensation of the disturbances in the plant (25) with the perturbations described by the dynamic model (24).

This problem can be solved by choosing $\overline{u} = -z_1$, where z_1 is the output of the dynamic system

$$\dot{z}_i = z_{i+1},\ \ i = \overline{1, r-1},\ \ \dot{z}_r = \sum_{\mathrm{i}=1}^{r} d_iz_i + v,\ \ d_i = \mathrm{const}, \tag{26}$$

where v is the new control input.

Introducing the notation $e_i = \omega_i - z_i$, $i = \overline{1, r}$, one can rewrite the system (25) as

$$\dot{\overline{x}}_1 = e_1. \tag{27}$$

Differentiating the variable (27) r times, the expanded system can be obtained in the canonical form [13]

$$\dot{\overline{x}}_i = \overline{x}_{i+1},\ \ i = \overline{1, r-1},\ \ \dot{\overline{x}}_r = \sum_{i=1}^{r} [g_ie_i + (g_i - d_i)z_i] + v, \tag{28}$$

where $\overline{x}_{i+1} = e_i,\ i = \overline{1, r-1}$.

By choosing a sufficiently large amplitude of discontinuous control inputs [9], taking into account the constraints $|g_i| \le k_i$, the system (28) can be stabilized by the following control action:

$$v = -M\mathrm{sign}(s), \tag{29}$$

due to which a sliding mode is organized on the manifold

$$s = \overline{x}_r + \sum_{i=1}^{r-1} C_i\overline{x}_i = 0,$$

with motion along it described by the equivalent control method

$$\dot{\overline{x}}_i = \overline{x}_{i-1},\ \ i = \overline{1, r-1},\ \ \dot{\overline{x}}_r = \sum_{i=1}^{r} C_i\overline{x}_i.$$

If C_i are chosen as the coefficients of the Hurwitz polynomial, then sliding mode motion ensures asymptotic convergence of the system.

The proposed scheme for constructing invariant systems, using a canonical representation and sliding mode theory methods, provides the robustness properties of whole system, which includes plant dynamics and the disturbances model. Note, that the perturbation generator (24) allows the assumption of variable model parameters, that are bounded in absolute value: $|g_i(t)| \le G_i = \text{const}$.

To obtain the estimates of the system state space vector (28), necessary for the control input (29) synthesis, a state space observer must be constructed.

For order r, further a procedure for synthesizing an observer-differentiator of the variable (27) is synthesized.

1. Introduce a filter with stable eigen motions

$$\begin{aligned} &\dot{\xi}_i(t) = \xi_{i+1}(t) + \overline{x}_1,\ i = \overline{1, r-1}, \\ &\dot{\xi}_r(t) = c^{\mathrm{T}}\xi(t) + \overline{x}_1, \end{aligned} \tag{30}$$

where $\xi^{\mathrm{T}}(t) = (\xi_1(t), \ldots, \xi_r(t)) \in R^r$, $c \in R^r$ is the coefficient vector of Hurwitz polynomial.

2. Represent the system (30) in the canonical form

$$\begin{aligned} &\dot{\overline{\xi}}_i(t) = \overline{\xi}_{i+1}(t),\ i = \overline{1, r-1}, \\ &\dot{\overline{\xi}}_r(t) = c^{\mathrm{T}}\xi(t) + \overline{\xi}_{r+1}(\mathrm{t}), \end{aligned} \tag{31}$$

where

$$\begin{aligned} &\overline{\xi}_1(t) = \xi_1(t), \\ &\overline{\xi}_2(t) = \xi_2(t) + \overline{x}_1, \\ &\overline{\xi}_3(t) = \xi_3(t) + \overline{x}_1 + \overline{x}_2, \\ &\ldots \\ &\overline{\xi}_r(t) = \xi_r(t) + \sum_{i=1}^{r-1} \overline{x}_i, \\ &\overline{\xi}_{r+1}(t) = \sum_{i=1}^{r} \overline{x}_i. \end{aligned} \tag{32}$$

3. Let us construct an observer of the state vector $\overline{\xi}^{\mathrm{T}}(t) = (\overline{\xi}_1(t), \ldots, \overline{\xi}_3(t))$ and the signal $\overline{\xi}_{r+1}(t)$ as applied to the system (31):

$$\begin{aligned} &\dot{\hat{\xi}}_i(t) = \hat{\xi}_{i+1}(t) + \nu_i,\ i = \overline{1, r-1}, \\ &\dot{\hat{\xi}}_r(t) = c^{\mathrm{T}}\hat{\xi}(t) + \nu_r, \quad \hat{\xi}^{\mathrm{T}}(t) = (\hat{\xi}_1(t), \ldots, \hat{\xi}_r(t)). \end{aligned} \tag{33}$$

4. The equations (31) and (33) in terms of residuals $\overline{\varepsilon}_{\mathrm{i}}(t) = \overline{\xi}_i(t) - \hat{\xi}_i(t)$ are:

$$\begin{aligned} &\dot{\overline{\varepsilon}}_{\mathrm{i}}(t) = \overline{\varepsilon}_{i+1}(t) - \nu_i,\ i = \overline{1, r-1}, \\ &\dot{\overline{\varepsilon}}_r(t) = c^{\mathrm{T}}\overline{\varepsilon}(t) + \overline{\xi}_{r+1}(t) - \nu_r, \end{aligned} \tag{34}$$

where $\overline{\varepsilon}^{\mathrm{T}}(t) = (\overline{\varepsilon}_1(t), \ldots, \overline{\varepsilon}_r(t))$.

In papers [14–16], it was shown that the choice of the corrective input actions of the observer (34) in the class of linear functions with the saturation according to the hierarchy

$$\nu_1 = \mathrm{sat}_{L_1}(l_1\overline{\varepsilon}_1(t)),\ \ \nu_i = \mathrm{sat}_{L_i}(l_i v_{i-1}),\ \ i = \overline{2, r},\ L_1,\ \ L_i, l_i = \text{const}$$

solves the stabilization problem of the system (34) with a given accuracy.

Definition 1. For $M = \text{const} > 0$ and an arbitrary time function $s(t)$, by definition

$$\mathrm{sat}_M(s) = \min(|s|, M)\mathrm{sign}(s(t)).$$

Indeed, under the assumptions

$$|\overline{\varepsilon}_i(t)| \le E_i,\ |\dot{\overline{\varepsilon}}_i(t)| \le \overline{E}_i;\quad \mathrm{E_i}, \overline{E}_i = \text{const},$$
$$\left|\overline{\xi}_{r+1}(t)\right| \le E_{r+1} = \text{const}$$

We schematically describe the stabilization procedure for the system (34) .

1. In the first subsystem

$$\dot{\overline{\varepsilon}}_1(\mathrm{t}) = \overline{\varepsilon}_2(t) - \mathrm{sat}_{L_1}(l_1\overline{\varepsilon}_1(t)),$$

with the amplitude

$$L_1 > |\overline{\varepsilon}_2(t)|$$

the variable $\overline{\varepsilon}_1(t)$ converges to the linear zone after a finite time, and the equality holds:

$$\dot{\overline{\varepsilon}}_1(\mathrm{t}) = \overline{\varepsilon}_2(t) - l_1\overline{\varepsilon}_1(t).$$

In this case, the relations $|\overline{\varepsilon}_1(t)| \le E_2/l_1,\ |\overline{\varepsilon}_2(t) - l_1\overline{\varepsilon}_1(t)| \le \overline{E}_2/l_1$ are satisfied, and by increasing the coefficient l_1 for the variable $\overline{\varepsilon}_1(t) \approx 0$, a given degree of stabilization can be ensured. As a result, the estimation with a given accuracy of the variable $\overline{\varepsilon}_2$ is $\overline{\varepsilon}_2(t) \approx l_1\overline{\varepsilon}_1(t)$.

Let us describe the first point with the help of logical diagram:

$$\begin{aligned}
&\dot{\overline{\varepsilon}}_1(t) = \overline{\varepsilon}_2(t) - \mathrm{sat}_{L_1}(l_1\overline{\varepsilon}_1(t)),\ L_1 > |\overline{\varepsilon}_2(t)| \Rightarrow \dot{\overline{\varepsilon}}_1(\mathrm{t}) = \overline{\varepsilon}_2(t) - l_1\overline{\varepsilon}_1(t)\\
&\Rightarrow |\overline{\varepsilon}_1(t)| \le \frac{E_2}{l_1},\, |\,\overline{\varepsilon}_2(t) - l_1\overline{\varepsilon}_1(t)| \le \frac{\overline{E}_2}{l_1} \Rightarrow l_1 \to \infty :\\
&\overline{\varepsilon}_i(t) \approx 0,\ \overline{\varepsilon}_{i+1}(t) \approx l_i\overline{\varepsilon}_i(t).
\end{aligned}$$

2. In the following stages $i = \overline{2, r-1}$, corrective actions are synthesized according to the following logical scheme:

$$\begin{aligned}
&\dot{\overline{\varepsilon}}_i(t) = \overline{\varepsilon}_{i+1}(t) - \mathrm{sat}_{L_i}(l_i\overline{\varepsilon}_i(t)),\ L_\mathrm{i} > |\overline{\varepsilon}_{i+1}(t)| \Rightarrow \dot{\overline{\varepsilon}}_\mathrm{i}(\mathrm{t}) = \overline{\varepsilon}_{i+1}(t) - l_i\overline{\varepsilon}_i(t) \Rightarrow\\
&\Rightarrow |\overline{\varepsilon}_i(t)| \le \frac{E_{i+1}}{l_i},\, |\,\overline{\varepsilon}_{i+1}(t) - l_i\overline{\varepsilon}_i(t)| \le \frac{\overline{E}_{i+1}}{l_i} \Rightarrow l_i \to \infty :\\
&\overline{\varepsilon}_i(t) \approx 0,\ \overline{\varepsilon}_{i+1}(t) \approx l_i\overline{\varepsilon}_i(t).
\end{aligned}$$

3. After consideration the last subsystem (34)

$$\dot{\overline{\varepsilon}}_r(t) = c^{\mathrm{T}}\overline{\varepsilon}(t) + \overline{\xi}_{r+1}(t) - \mathrm{sat}_{L_r}(l_r\overline{\varepsilon}_r(t)),$$

and taking into account $\overline{\varepsilon}_i(t) \approx 0$ by choosing $L_r > \left|\overline{\xi}_{r+1}(t)\right|$ the relation is obtained

$$\overline{\xi}_{r+1}(t) \approx l_r\overline{\varepsilon}_r(t).$$

The variables $\overline{x}_i,\ i = \overline{1, r}$, corresponding to the derivatives of the signal $\overline{x}_1$, can be calculated using the estimates $\overline{\xi}(t)$ and the signal $\overline{\xi}_{r+1}(t)$ by sequentially solving the equations (32) from top to bottom.

5. EXAMPLE OF THE USE OF THE DEVELOPED ALGORITHMS FOR UAV CONTROL

This section examines the applied problem of controlling the horizontal movement of an unmanned aerial vehicle (UAV) transporting a load suspended on a weightless cable (see Fig. 1). The

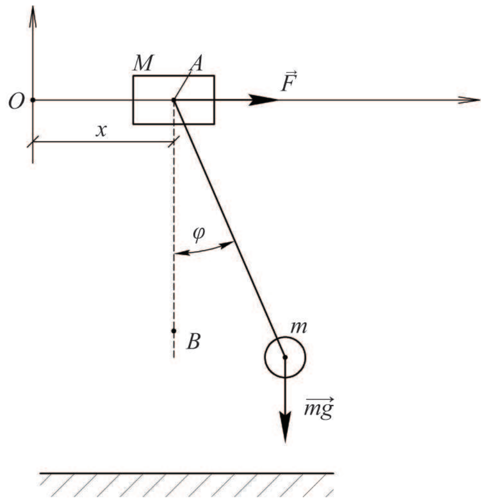

Fig. 1. The qualitative scheme of UAV with the transported cargo.

dynamic equations describing the behavior of the specified object can be represented as

$$\begin{aligned} &\dot{x} = v, \\ &\dot{v} = -\frac{J_{\text{eq}}\alpha_v}{\Delta}v - \frac{m^2l^2g}{\Delta}\varphi - \frac{ml\alpha_\omega}{\Delta}\omega + \frac{J_{\text{eq}}}{\Delta}F, \\ &\dot{\varphi} = \omega, \\ &\dot{\omega} = -\frac{mglM_{\text{eq}}}{\Delta}\varphi - \frac{M_{\text{eq}}\alpha_\omega}{\Delta}\omega - \frac{ml\alpha_v}{\Delta}v + \frac{ml}{\Delta}F, \end{aligned} \tag{35}$$

where x is the horizontal coordinate of the center of mass, v is the horizontal velocity of the UAV center of mass, φ is the angular deviation of the cable with the load, ω is the angular velocity of the cable with the load, $J_{\text{eq}} = J + ml^2$, J is the inertia of the suspended load during its rotation about its center of gravity, $M_{\text{eq}} = m + M$, m, M are the masses of the suspended body and the UAV, respectively, l is the distance from the point of suspension of the load to its center of gravity, $\Delta = M_{\text{eq}}J_{\text{eq}} - m^2l^2$, AB is the line of zero deflection of the cable with the load, OA is the line along which the UAV moves with the load (horizontal line), α_M is a coefficient of viscous friction of the medium for the aircraft, α_m is a coefficient of viscous friction of the medium for the suspended load, α_r is a coefficient taking into account the effect of losses on friction at the point of suspension of the cable, $\alpha_v = \alpha_m + \alpha_M$, $\alpha_\omega = \alpha_m l + \alpha_r$, F is the traction force in in the horizontal direction, the cable is assumed to be weightless compared to the load.

The problem of the horizontal position stabilization of UAV at a given point is stated in this section:

$$\lim_{t\to\infty}|x(t) - x_{\text{d}}| = \lim_{t\to\infty}|\overline{x}(t)| = 0,\ \ \overline{x}(t) = x(t) - x_{\text{d}},\ \ x_{\text{d}} = \text{const} > 0. \tag{36}$$

It is assumed, that only UAV position and velocity can be measured.

Using a change of variables

$$\begin{pmatrix} y \\ v_y \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{J_{\text{eq}}} & 0 & \dfrac{1}{ml} & 0 \\ 0 & -\dfrac{1}{J_{\text{eq}}} & 0 & \dfrac{1}{ml} \end{pmatrix} \begin{pmatrix} x \\ v \\ \varphi \\ \omega \end{pmatrix},$$

one can rewrite the equations (35) in the form

$$\begin{aligned}
&\dot{x} = v,\\
&\dot{v} = -\frac{gl^3m^3}{J_{\text{eq}}\Delta}x - \frac{\alpha_v J_{\text{eq}}^2 + \alpha_\omega l^2m^2}{J_{\text{eq}}\Delta}v - \frac{gl^3m^3}{\Delta}y - \frac{\alpha_\omega l^2m^2}{\Delta}v_y + \frac{J_{\text{eq}}}{\Delta}F,\\
&\dot{y} = v_y,\\
&\dot{v}_y = -\frac{mgl}{J_{\text{eq}}}y - \frac{\alpha_\omega}{J_{\text{eq}}}v_y - \frac{mgl}{J_{\text{eq}}^2}x - \frac{\alpha_\omega}{J_{\text{eq}}^2}v.
\end{aligned}$$

Taking into account (36), the last system is transformed to the representation:

$$\begin{aligned}
&\dot{\overline{x}} = v,\\
&\dot{v} = -\frac{gl^3m^3}{J_{\text{eq}}\Delta}\overline{x} - \frac{\alpha_v J_{\text{eq}}^2 + \alpha_\omega l^2m^2}{J_{\text{eq}}\Delta}v - \frac{gl^3m^3}{\Delta}y - \frac{\alpha_\omega l^2m^2}{\Delta}v_y + \frac{J_{\text{eq}}}{\Delta}F-\\
&\quad - \frac{gl^3m^3}{J_{\text{eq}}\Delta}x_{\text{d}},\\
&\dot{y} = v_y,\\
&\dot{v}_y = -\frac{mgl}{J_{\text{eq}}}y - \frac{\alpha_\omega}{J_{\text{eq}}}v_y - \frac{mgl}{J_{\text{eq}}^2}\overline{x} - \frac{\alpha_\omega}{J_{\text{eq}}^2}v - \frac{mgl}{J_{\text{eq}}^2}x_{\text{d}}.
\end{aligned} \tag{37}$$

According to the proposed approach, unmeasured state variables y, v_y may be treated as disturbances. Considering, that the second and fourth equations of the system contain constant terms $-\frac{gl^3m^3}{J_{\text{eq}}\Delta}x_{\text{d}}$ and $-\frac{mgl}{J_{\text{eq}}^2}x_{\text{d}}$, the complete disturbances model according to (37) can be represented as

$$\dot{\omega} = W\omega + B_e e, \tag{38}$$

where $\omega = \begin{pmatrix} \omega_1 & \omega_2 & \omega_3 \end{pmatrix}^{\text{T}} = \begin{pmatrix} x_{\text{d}} & y & v_y \end{pmatrix}^{\text{T}}$, $W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{mgl}{J_{\text{eq}}^2} & -\frac{mgl}{J_{\text{eq}}} & -\frac{\alpha_\omega}{J_{\text{eq}}} \end{pmatrix}$,

$e = \begin{pmatrix} \overline{x} & v \end{pmatrix}^{\text{T}}$, $B_e = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{mgl}{J_{\text{eq}}^2} & -\frac{\alpha_\omega}{J_{\text{eq}}^2} \end{pmatrix}$.

Using (37)–(38), the transformed system (37) is

$$\begin{aligned}
&\dot{\overline{x}} = v,\\
&\dot{v} = -\frac{gl^3m^3}{J_{\text{eq}}\Delta}\overline{x} - \frac{\alpha_v J_{\text{eq}}^2 + \alpha_\omega l^2m^2}{J_{\text{eq}}\Delta}v + Q\omega + \frac{J_{\text{eq}}}{\Delta}F,\\
&\dot{\omega} = W\omega + B_e e,
\end{aligned} \tag{39}$$

where $Q = \begin{pmatrix} -\frac{gl^3m^3}{J_{\text{eq}}\Delta} & -\frac{gl^3m^3}{\Delta} & -\frac{\alpha_\omega l^2m^2}{\Delta} \end{pmatrix}$.

The difference between the systems (39) and (5) representations is that the disturbances subsystem is non-autonomous (the $B_\omega e$ term is present). However, as will be shown below, this does not change the proposed methodology to feedback synthesis.

We will demonstrate by direct calculations, that the (W, Q) pair is unobservable. To do this, the rank of the observability matrix is calculated:

$$\operatorname{rank}\begin{pmatrix} Q \\ QW \\ QW^2 \end{pmatrix} = \operatorname{rank}\begin{pmatrix} -\dfrac{gm^3l^3}{J_{\mathrm{eq}}\Delta} & -\dfrac{gm^3l^3}{\Delta} & -\dfrac{\alpha_\omega m^2l^2}{\Delta} \\ \dfrac{\alpha_\omega gl^3m^3}{\Delta J_{\mathrm{eq}}^2} & \dfrac{\alpha_\omega gl^3m^3}{\Delta J_{\mathrm{eq}}} & \dfrac{m^2l^2(\alpha_\omega^2 - gmlJ_{\mathrm{eq}})}{J_{\mathrm{eq}}\Delta} \\ \dfrac{gm^3l^3(-\alpha_\omega^2 + gmlJ_{\mathrm{eq}})}{J_{\mathrm{eq}}^3\Delta} & \dfrac{gm^3l^3(-\alpha_\omega^2 + gmlJ_{\mathrm{eq}})}{J_{\mathrm{eq}}^2\Delta} & \dfrac{\alpha_\omega m^2l^2(-\alpha_\omega^2 + 2gmlJ_{\mathrm{eq}})}{J_{\mathrm{eq}}^2\Delta} \end{pmatrix}$$

$$= \operatorname{rank}\begin{pmatrix} -\dfrac{gm^3l^3}{J_{\mathrm{eq}}\Delta} & -\dfrac{gm^3l^3}{\Delta} & -\dfrac{\alpha_\omega m^2l^2}{\Delta} \\ 0 & 0 & -\dfrac{gm^3l^3}{\Delta} \\ 0 & 0 & \dfrac{gm^3l^3}{J_{\mathrm{eq}}\Delta} \end{pmatrix} = \begin{pmatrix} -\dfrac{gm^3l^3}{J_{\mathrm{eq}}\Delta} & -\dfrac{gm^3l^3}{\Delta} & -\dfrac{\alpha_\omega m^2l^2}{\Delta} \\ 0 & 0 & -\dfrac{gm^3l^3}{\Delta} \\ 0 & 0 & 0 \end{pmatrix} = 2.$$

The system (39), using the substitution of functions,

$$\xi = H\omega,\ H = \begin{pmatrix} Q \\ QW \\ H_3 \end{pmatrix},\ H_3 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix},\ \xi = \begin{pmatrix} \xi_1 & \xi_2 & \xi_3 \end{pmatrix}^{\mathrm{T}}$$

is transformed into the following

$$\begin{aligned} \dot{\overline{x}} &= v, \\ \dot{v} &= -\frac{gl^3m^3}{J_{\mathrm{eq}}\Delta}\overline{x} - \frac{\alpha_v J_{\mathrm{eq}}^2 + \alpha_\omega l^2m^2}{J_{\mathrm{eq}}\Delta}v + \xi_1 + \frac{J_{\mathrm{eq}}}{\Delta}F, \\ \dot{\xi} &= \widetilde{W}\xi + \widetilde{B}_e e, \end{aligned} \tag{40}$$

where $\widetilde{W} = HWH^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -\dfrac{gml}{J_{\mathrm{eq}}} & -\dfrac{\alpha_\omega}{J_{\mathrm{eq}}} & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$$\widetilde{B}_e = HB_e = \begin{pmatrix} \dfrac{\alpha_\omega gm^3l^3}{\Delta J_{\mathrm{eq}}^2} & \dfrac{\alpha_\omega^2 m^2l^2}{\Delta J_{\mathrm{eq}}^2} \\ \dfrac{gm^3l^3(-\alpha_\omega^2 + gmlJ_{\mathrm{eq}})}{\Delta J_{\mathrm{eq}}^3} & \dfrac{\alpha_\omega m^2l^2(-\alpha_\omega^2 + gmlJ_{\mathrm{eq}})}{\Delta J_{\mathrm{eq}}^3} \\ 0 & 0 \end{pmatrix}.$$

In the structure (40) is highlighted unobservable subspace with respect to the variable ξ_4. When synthesizing feedback based on a dynamic compensator, the active variable suddenly does not contribute to the output error of the variable, so the active compensator constructs the observed components $\xi^* = (\ \xi_1 \ \ \xi_2\ )^{\mathrm{T}}$ of the the disturbances vector ξ.

The observability of the pair $(W^*,\ Q^*)$ is now revealed by the direct substitution

$$\operatorname{Rank}\begin{pmatrix} Q^* \\ Q^*W^* \end{pmatrix} = \operatorname{Rank}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 2,$$

where

$$W^* = \begin{pmatrix} 0 & 1 \\ -\dfrac{gml}{J_{\rm eq}} & -\dfrac{\alpha_\omega}{J_{\rm eq}} \end{pmatrix}, \ Q^* = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

In the introduced notations, the disturbances model is

$$\dot{\xi}^* = W^*\xi^* + B_e^* e, \ B_e^* = \begin{pmatrix} \dfrac{\alpha_\omega g m^3 l^3}{\Delta J_{\rm eq}^2} & \dfrac{\alpha_\omega^2 m^2 l^2}{\Delta J_{\rm eq}^2} \\ \dfrac{g m^3 l^3(-\alpha_\omega^2 + gmlJ_{\rm eq})}{\Delta J_{\rm eq}^3} & \dfrac{\alpha_\omega m^2 l^2(-\alpha_\omega^2 + gmlJ_{\rm eq})}{\Delta J_{\rm eq}^3} \end{pmatrix}. \tag{41}$$

The horizontal control force is chosen like the following

$$F = -\frac{\Delta}{J_{\rm eq}} Q^* s + \frac{\Delta}{J_{\rm eq}} \overline{u} \tag{42}$$

with dynamic compensator according to (6), (41)

$$\dot{s} = W^* s + B_e^* e - \overline{v}. \tag{43}$$

The equations of the closed-loop system according to (40), (42)–(43):

$$\begin{aligned} &\dot{\overline{x}} = v, \\ &\dot{v}_= -\frac{gl^3m^3}{J_{\rm eq}\Delta}\overline{x} - \frac{\alpha_v J_{\rm eq}^2 + \alpha_\omega l^2 m^2}{J_{\rm eq}\Delta} v + Q^*\varepsilon + \overline{u}, \\ &\dot{\varepsilon} = W\varepsilon + \overline{v}, \end{aligned} \tag{44}$$

where $\varepsilon = \xi^* - s$.

Selecting the control inputs $\overline{u}$, $\overline{v}$ like in the expressions (9)

$$\begin{aligned} &\overline{v} = -LKe + W^* L v, \\ &\overline{u} = \frac{gl^3m^3}{J_{\rm eq}\Delta}\overline{x} + \left(\frac{\alpha_v J_{\rm eq}^2 + \alpha_\omega l^2 m^2}{J_{\rm eq}\Delta} + Q^* L\right) v + Ke, \end{aligned} \tag{45}$$

the equations of the closed-loop system according to (44)–(45) are derived

$$\begin{aligned} &\dot{\overline{x}} = v, \\ &\dot{v} = Ke + Q^*(\varepsilon + Lv), \\ &\dot{\varepsilon} = W^*\varepsilon - LKe + W^* L v, \end{aligned}$$

where $L = \begin{pmatrix} L_1 & L_2 \end{pmatrix}^{\rm T} \in R^2$, $K = \begin{pmatrix} K_1 & K_2 \end{pmatrix}^{\rm T} \in R^2$ are the matrices with constant coefficients.

By introducing the new variable $\overline{\varepsilon} = \varepsilon + Lv$, the last system is rewritten as

$$\begin{aligned} &\dot{\overline{x}} = v, \\ &\dot{v} = Ke + Q^*\overline{\varepsilon}, \\ &\dot{\overline{\varepsilon}} = (W^* + LQ^*)\overline{\varepsilon}. \end{aligned}$$

Since the pair (W^*, Q^*) is observable, the convergence of the variable $\overline{\varepsilon}$ can be assigned at a desired rate.

The numerical values of the plant parameters adopted in the simulation

Parameter	Value
l, [m]	5
J, [kg$\cdot$ m^2]	1.2
α_v, [kg/s]	10
α_ω, [m$^2 \cdot$ kg/s]	20
m, [kg]	3.5
M, [kg]	2
a_1	7.5
a_0	14
$x(t_0)$, [m]	10
$v(t_0)$, [m/s]	5
$\varphi(t_0)$, [rad]	–0.5
$\omega(t_0)$, [rad/s]	10
x_d, [m]	25

For the numerical experiment, the selected parameters are presented in the table.

The characteristic polynomial of the matrix $W^* + LQ^*$:

$$p^2 + \frac{-L_1 J_\mathrm{eq} + \alpha_\omega}{J_\mathrm{eq}} p + \frac{-L_2 J_\mathrm{eq} + gml - L_1\alpha_\omega}{J_\mathrm{eq}} = 0. \tag{46}$$

To assign the coefficients of the last polynomial equal to the desired ones, the elements of the matrix L are chosen. The desired polynomial is

$$p^2 + a_1 p + a_0 = 0, \tag{47}$$

with the parameters from the table and the roots of the equation

$$p_1 = -3.5,\ p_2 = -4.$$

According to the reasoning, from (46)–(47), one can obtain

$$\frac{-L_1 J_\mathrm{eq} + \alpha_\omega}{J_\mathrm{eq}} = a_1 = 7.5, \quad \frac{-L_2 J_\mathrm{eq} + gml - L_1\alpha_\omega}{J_\mathrm{eq}} = a_0 = 14.$$

Substituting the parameters from the table, the feedback coefficients of the dynamic compensator are calculated

$$L_1 = \frac{\alpha_\omega - a_1 J_\mathrm{eq}}{J_\mathrm{eq}} = -7.2745;$$

$$L_2 = \frac{-a_0 J_\mathrm{eq}^2 - (\alpha_\omega - a_1 J_\mathrm{eq})\alpha_\omega + J_\mathrm{eq} gml}{J_\mathrm{eq}^2} = -10.4243.$$

The following feedback matrix K is used in the control inputs (45)

$$K = \begin{pmatrix} -0.12 & -0.7 \end{pmatrix}.$$

The feedback with such parameters ensures stability and assigns time constants

$$\tau_1 = 0.4\ \mathrm{s}^{-1}, \quad \tau_2 = 0.3\ \mathrm{s}^{-1}$$

in the subsystem corresponding to the vector e.

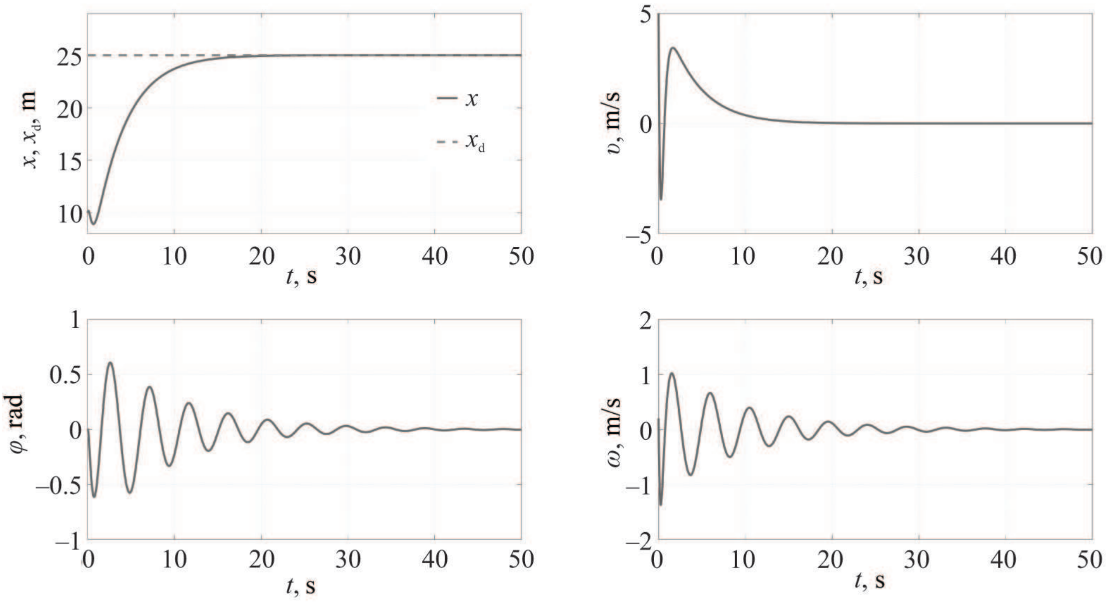

Fig. 2. Transient process for the state space of the plant (31).

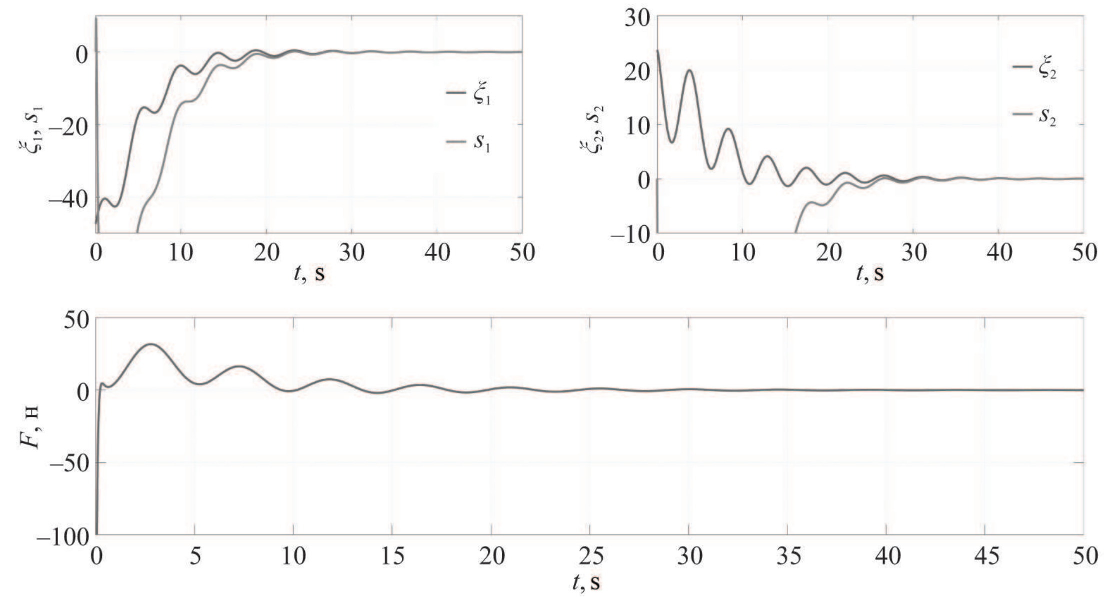

Fig. 3. Transient process for the components of the disturbances vector (36) and dynamic compensator (39).

The simulation results of the system (31) with the dynamic feedback (42)–(43), (45) and the initial conditions from the table are shown on Figs. 2 and 3.

As can be seen from Fig. 3, the current UAV altitude converges to the desired value, and the components of the dynamic compensator vector s converge to the corresponding components of the disturbances vector ξ^*. Note that, despite of the convergence of the output variable to zero, the control force F contains the damped oscillatory component (see Fig. 3). This is due to the fact, that the control input compensates the oscillations of the variables φ, ω, the damping rate of which is determined by the coefficient α_ω.

The control law (42)–(43), (45) was synthesized without correction of this damping, and for this reason, a long tail exists in the transient process (see Figs. 2 and 3). However, due to the controllability of the original system, (37) by correcting with the using the components $\overline{x}, v$ (see equations (37)), it would be possible to correct the damping of the components φ, ω, and, consequently, the length of the specified tail.

6. CONCLUSION

The decomposition methods are proposed for the problem of dynamic compensation of the external disturbances of a given class. The use of methods for synthesizing systems with separable motions within the framework of sliding mode theory and deep feedback loops has allowed us to extend existing dynamic compensation methods to a class of nonlinear systems and ensure robust properties under uncertainty in both, the control plant model and the disturbances generator. It should be noted, that the considered example of UAV control with an oscillating load, which can be considered both as part of the control plant model or as an external disturbance, demonstrates the potential for extending the problems of dynamic compensation of the external disturbances to the tasks, where the disturbances generator is considered as a part of the control plant. Together with the decomposition methods proposed in this paper, this approach also allows designers to decompose the feedback synthesis procedure into smaller subproblems.

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