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Market Design, Pricing, and the Distribution of Market Power in Finite Markets

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Abstract—This paper analyzes the impact of market design on equilibrium characteristics in finite markets. We study a directed search model in the labor market under two variations: firm-led and worker-led contract terms. The results demonstrate that the preferences of the market sides (employees, firms, and society) regarding market design depend on market size and may either align or diverge. Furthermore, the analysis reveals that for certain market-side ratios, the expected number of concluded contracts increases by more than 10% upon switching to a more efficient design.

Keywords: market design, directed search, finite markets, labour market

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1. INTRODUCTION

The concept of directed search, which underpins a broad class of research, offers a distinct perspective on market structure and interaction, providing an explanation for several key empirical phenomena. One such phenomenon is the coexistence of agents on opposite sides of the market who could potentially trade but remain unmatched due to market frictions. Examples include the simultaneous existence of vacancies and unemployment in the labor market, unmatched individuals in the marriage market, and unserved buyers alongside sellers with unsold inventories in the goods market.

The core idea defining the directed search framework is that the matching process between agents from opposite sides of the market is not random; rather, it emerges as a result of strategic interaction.

(a) In the first stage, agents on one side of the market (*Proposers*) simultaneously and independently publicly announce the terms under which they are willing to contract with any agent from the opposite side of the market (*Respondents*).

(b) In the second stage, agents on the *Respondents* side, *having observed all proposed contracts*, simultaneously and independently decide which agent on the *Proposers* side they wish to approach. It is at this point that "directed search" occurs, in the sense that the *Respondents*' decisions are made strategically, based on the terms offered by the *Proposers* and the anticipated behavior of other *Respondents*.

(c) In the third stage, interaction occurs locally within the resulting subgroups (or submarkets), each consisting of a *Proposer* and all *Respondents* who chose to apply to that specific agent. A matching mechanism determines which pair, if any, forms a contract within each subgroup. While such mechanisms may involve complex formats like various types of auctions, interest often lies in a non-strategic random matching rule. This corresponds, for example, to a scenario where a seller posts a price and subsequently selects a buyer at random from the pool of applicants.

The foundational work most closely related to this research is [1], which examines a product market with homogeneous firms and buyers, where each firm holds a single unit of inventory and matching occurs via a random mechanism between a firm and one of its potential customers. Subsequently, numerous advancements and extensions have been proposed, a comprehensive review of which is provided in [2]. However, generalizations that preserve the possibility of obtaining closed-form solutions are virtually non-existent. One such exception is [3], where the equilibrium is derived analytically for a setting in which firms are permitted to hold an arbitrary, positive volume of inventory.

Similar motifs arise in the stable matching problem. It is well known that the Gale–Shapley algorithm ([4]) identifies two solutions that constitute the upper and lower elements of the lattice of stable matchings. This implies that the version of the algorithm where firms act as the proposing side always yields a stable matching that is firm-optimal and worker-pessimal among all possible stable matchings. The converse holds when workers are the proposing side. In contrast, in the model considered here, depending on the market size, firms — as well as workers — may find it beneficial to forgo the proposer role. Furthermore, in a balanced market with an equal number of agents on both sides, the possibility of either side gaining an advantage by changing the design is absent.

Despite these considerations, existing literature has yet to address the question of how equilibrium characteristics depend on a key element of the basic market structure: namely, the assignment of "buyer" and "seller" roles. However, this question plays a crucial role in policy determination and the consequences of its implementation. For instance, an online platform connecting students and tutors can, by altering its architecture and user interaction design, structure its operations either through tutors posting resumes and students responding to them, or through students creating public requests and interested tutors responding. Depending on the estimated number of users on each side of the market, the platform — which typically earns a commission for creating a successful match — may implement specific functionalities to maximize the expected number of stable pairs formed and, ultimately, the expected commission. A similar question is relevant for service platforms (e.g., *profi.ru*), job search sites (e.g., *hh.ru*), and many others. It is important to note that, in practice, platforms often implement a combination of these options rather than a single pure variant. Nevertheless, understanding how equilibrium characteristics relate to one another in these two limiting cases within a stylized model can shed light on the structure of optimal platform design in more realistic settings.

As part of the analysis in this work, the problem is formulated in terms of a labor market with homogeneous firms, each possessing a single vacancy and willing to pay a maximum wage $wage_{\max}$, and homogeneous employees seeking employment provided a minimum wage $0 \leqslant wage_{\min} < wage_{\max}$ is guaranteed. A comparative analysis of equilibrium characteristics was conducted as a function of market size — i.e., the number of firms and employees — for cases where contract terms, in the form of wages, are proposed by employees through resume publication versus cases where wages are announced by firms via job postings.

The primary contribution of this study consists in characterizing the preference structures of various market participants — employees, firms, and society at large — as a function of market size. It is demonstrated that:

1) From a social welfare perspective, defined as the expected number of market transactions, it is preferable for the side with the smaller population to publish contract terms in the first stage, as this configuration yields a higher objective value than the alternative.

2) From the perspective of individual agents, when the numerical disparity between the two sides is relatively small, each side maximizes its own surplus by delegating the obligation of publishing contract terms to the opposing side, thereby assuming the role of *Respondents* in the second stage. Conversely, in scenarios where the number of firms and employees differs significantly, it becomes more advantageous for each side to act as the *Proposer* and publish the contract terms themselves.

3) Due to the inherent asymmetry and the fact that the threshold for "significant disparity" varies for each side based on their respective sizes, the set of all admissible market sizes is partitioned into distinct regions based on the various combinations of agent preferences regarding the *Proposer* role.

Furthermore, the extent of market inefficiency is quantified, demonstrating that at specific market scales, a mere adjustment of the market design can increase the expected number of executed contracts by more than 10%.

Another finding of the analysis pertains to the heterogeneity of preference motivations: a scenario is identified in which one market side may experience a decrease in its relative share of the aggregate surplus but nonetheless achieve a net gain in monetary terms. This occurs because improved coordination among market agents increases the aggregate surplus to such an extent that both sides realize a monetary benefit.

Consequently, these results provide a framework for determining the optimal design of online platforms or other market environments, contingent upon the objective function of the decision-maker.

2. MODEL

2.1. Market Description

Consider a labor market consisting of $n_f \geqslant 2$ homogeneous firms, each possessing exactly one job vacancy, and $n_w \geqslant 2$ job-seeking employees. Let the set of market sides be denoted by $S = \{\text{Firms, Workers}\}$. We assume that the minimum wage at which an employee is willing to work is $wage_{\min} \geqslant 0$, and the maximum wage a firm is willing to pay is $wage_{\max} > wage_{\min}$. Let the difference $wage_{\max} - wage_{\min}$ be denoted as $\Delta wage$. The market is structured as follows.

In the first stage, agents on one side of the market, the *Proposers* $\in S$, simultaneously and independently set a wage vector. For instance, if firms are the *Proposers*, each firm publishes a job description specifying a fixed wage. If workers are the *Proposers*, each worker publishes a resume specifying their desired wage. In either case, the announced wage cannot be subsequently altered and becomes a binding part of the resulting contract upon successful negotiations.

In the second stage, after observing the contract terms offered by the *Proposers*, the agents on the other side of the market (*Respondents* $\in S$) simultaneously and independently decide which single counterparty from the *Proposers'* side they wish to attempt to form a contract. If firms published job descriptions in the first stage, potential employees respond to them in the second stage, with each employee being able to apply to exactly one vacancy. Conversely, if employees initially published resumes, firms invite them for employment in the second stage, with each firm being able to approach exactly one employee.

In the third stage, the *Proposers* who initially posted the contract terms review the responses received. If an agent on this side receives at least one response, they select a counterparty for the transaction uniformly at random from among the applicants and successfully execute the contract. *Proposers* who receive no responses, as well as *Respondents* whose application was not accepted, remain unmatched and realize zero utility / profit.

All agents act rationally and strategically. Each agent maximizes their payoff, which is defined as $wage - wage_{\min}$ for employees and $wage_{\max} - wage$ for firms, in the event of a successful contract.

We denote the model where firms post vacancies in the first stage as FP (Firms Posting). In this model, firms are the *Proposers* and workers are the *Respondents*. We denote the model where employees post resumes in the first stage as WP (Workers Posting). In this case, workers are the *Proposers* and firms are the *Respondents*. It is worth noting that each version of the model independently represents an adaptation of the canonical directed search model [1] within the labor market context. In the following section, we derive the symmetric subgame perfect Nash equilibrium for each model and compare their equilibrium characteristics.

2.2. Equilibrium

In this section, we move away from the "firm/worker" terminology and reformulate the problem in terms of the *Proposers* and *Respondents* sides introduced in the introduction. Let the number of agents on the *Proposers* side be $n_p \geqslant 2$, and on the *Respondents* side be $n_r \geqslant 2$. Let the utility of each agent from concluding a contract (before accounting for costs or the price) be $val_p \geqslant 0$ for *Proposers* and $val_r \geqslant 0$ for *Respondents*. Depending on the structure of the utility functions, the payoffs for *Proposers* and *Respondents* take one of two forms:

$$\begin{cases} gain_p(val) = val - val_p, \\ gain_r(val) = val_r - val, \end{cases}$$

if, upon contract conclusion, a monetary transfer val flows in the direction of *Respondents* $\Longrightarrow$ *Proposers* (implying $val_r > val_p$), and

$$\begin{cases} gain_p(val) = val_p - val, \\ gain_r(val) = val - val_r \end{cases}$$

in the reverse case.

Theorem 1. *In the model under consideration, there exists a unique uncoordinated subgame perfect symmetric equilibrium in which each Proposer offers a contract price in the first stage equal to*

$$val = \frac{val_p\left(\frac{n_r}{n_p}\right)\left(1-\frac{1}{n_p}\right)^{n_r} + val_r\left(1-\left(1-\frac{1}{n_p}\right)^{n_r-1}\frac{n_r}{n_p}-\left(1-\frac{1}{n_p}\right)^{n_r}\right)}{\frac{n_r}{n_p}\left(1-\frac{1}{n_p}\right)^{n_r}+1-\left(1-\frac{1}{n_p}\right)^{n_r-1}\frac{n_r}{n_p}-\left(1-\frac{1}{n_p}\right)^{n_r}},$$

and each Respondent in the second stage chooses a Proposer to approach uniformly at random with probability $\gamma = \frac{1}{n_p}$.

Proof. The proof follows the logic presented in [2] for Proposition 5, subject to differences in notation. Its key steps are provided below.

The pure strategy of each agent j among the *Proposers* is

$$val_j \in \left[\min(val_r, val_p), \max(val_r, val_p)\right].$$

Let $\mathbf{val} = (val_1, \ldots, val_{n_p})$ denote the strategy profile of the *Proposers*.

The search strategy for agent i among the *Respondents* is the vector

$$\gamma_{i*} = (\gamma_{i1}, \ldots, \gamma_{in_p}),$$

where γ_{ij} represents the probability that *Respondent* i chooses to attempt a contract with *Proposer* j. Let

$$\underset{n_r\times n_p}{\Gamma} = (\gamma_{1*}, \ldots, \gamma_{n_r*})^T = \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1n_p} \\ \vdots & \ddots & \vdots \\ \gamma_{n_r1} & \cdots & \gamma_{n_rn_p} \end{pmatrix}$$

denote the strategy profile of the *Respondents*.

An equilibrium is a pair $(\mathbf{val}, \Gamma)$ such that no agent has an incentive to deviate; specifically, a symmetric equilibrium is one where $val_j = val$ and $\gamma_{ij} = \frac{1}{n_p}$. To verify whether a given pair $(\mathbf{val}, \Gamma)$ constitutes an equilibrium, we must examine the outcome following a unilateral deviation by one of the *Proposers*. We begin with the symmetric vector $\mathbf{val} = (val, \ldots, val)$ and, without loss of generality, assume that the first *Proposer* deviates from strategy val to strategy val^{dev}. Thus, the profile becomes $\mathbf{val} = (val^{dev}, val, \ldots, val)$. We must determine whether this deviation is profitable, assuming a symmetric equilibrium in the subgame following the deviation.

Suppose that for any agent i among the *Respondents*, the probability of attempting a contract with the deviating *Proposer* is $\gamma^{dev} = \gamma_{i1}(val^{dev}, val)$. The symmetric subgame perfect equilibrium is characterized by val and $\gamma^{dev}(val^{dev}, val)$ satisfying the following conditions:

(1) $val^{dev} = val$ maximizes the utility function of the potentially deviating *Proposer*, given no deviations by other agents;

(2) $(\gamma^{dev})^*(val^{dev}, val)$ constitutes a subgame equilibrium for any val^{dev} and val;

(3) on the equilibrium path, we have $\gamma_{ij} = \frac{1}{n_p}$, while after a deviation, *Respondents* visit the deviating *Proposer* with probability $\gamma^{dev} = \gamma^{dev}(val^{dev}, val)$, and all other *Proposers* uniformly at random with probability $\gamma^{nondev} = \frac{1-\gamma^{dev}}{n_p-1}$.

The probability that at least one *Respondent* attempts to contract with the deviating *Proposer* is equal to

$$\alpha_p^{dev} = 1 - \left(1 - \gamma^{dev}\right)^{n_r}.$$

Let the probability that a *Respondent* successfully concludes a contract, given they approach the deviating *Proposer*, be $\alpha_r^{dev} = \alpha_r^{dev}(val^{dev}, val)$. Note that $n_r \gamma^{dev} \alpha_r^{dev} = \alpha_p^{dev}$, as the left-hand side represents the expected number of *Respondents* with whom the deviating *Proposer* concludes a contract, while the right-hand side represents the expected number of successful contracts formed by the deviator. Thus,

$$\alpha_r^{dev} = \frac{1 - \left(1 - \gamma^{dev}\right)^{n_r}}{n_r \gamma^{dev}}.$$

The expected utility of the deviating *Proposer* is equal to

$$V_p^{dev}(val^{dev}, val) = \alpha_p^{dev} gain_p(val^{dev}) = \left(1 - \left(1 - \gamma^{dev}\right)^{n_r}\right) gain_p(val^{dev}).$$

The expected utility of a *Respondent* attempting to contract with the deviating *Proposer* is equal to

$$V_r^{dev}(val^{dev}, val) = \alpha_r^{dev} gain_r(val^{dev}) = \frac{1 - \left(1 - \gamma^{dev}\right)^{n_r}}{n_r \gamma^{dev}} gain_r(val^{dev}).$$

The expected utility of a *Respondent* attempting to contract with any of the non-deviating *Proposers* is equal to

$$V_r^{nondev}(val^{dev}, val) = \frac{1 - \left(1 - \gamma^{nondev}\right)^{n_r}}{n_r \gamma^{nondev}} gain_r(val).$$

Given the above, the first-order condition (FOC) for maximizing the utility $V_p^{dev}(val^{dev}, val)$ of the potentially deviating *Proposer* (assuming no deviations by others) is:

$$0 = \frac{\partial V_p^{dev}}{\partial val^{dev}} = sgn(val_r - val_p)\alpha_p^{dev} + gain_p(val^{dev}) \frac{\partial \alpha_p^{dev}}{\partial val^{dev}}$$
$$= sgn(val_r - val_p)\alpha_p^{dev} + n_r(1 - \gamma^{dev})^{n_r - 1} gain_p(val^{dev}) \frac{\partial \gamma^{dev}}{\partial val^{dev}}.$$

In a symmetric mixed-strategy subgame equilibrium, *Respondents* must be indifferent between attempting a contract with the deviating *Proposer* or any of the non-deviating ones. Thus, γ^{dev} satisfies the indifference condition:

$$\frac{1 - (1 - \gamma^{dev})^{n_r}}{n_r \gamma^{dev}} gain_r(val^{dev}) = \frac{1 - (1 - \gamma^{nondev})^{n_r}}{n_r \gamma^{nondev}} gain_r(val).$$

For any $\gamma^{dev} \in (0, 1)$, one can obtain the implicit derivative $\frac{\partial \gamma^{dev}}{\partial val^{dev}}$ and substitute it into the FOC derived above for the deviating *Proposer*'s expected utility. Simplifying the expression by substituting $\gamma^{dev} = \frac{1}{n_p}$ and $val^{dev} = val$ proves that the deviation is not profitable if and only if val takes the value defined in the statement of this theorem.

The theorem is proven.

We now present the formulas for the key equilibrium characteristics that will be used in the subsequent comparative analysis of market designs.

Corollary 1. *In equilibrium, the unconditional probability of matching for each agent on the Respondents side is given by*

$$\nu(\gamma, n_r) = \sum_{i=1}^{n_p} \gamma\mu(\gamma, n_r) = \mu(\gamma, n_r) = \frac{1 - \left(1 - \frac{1}{n_p}\right)^{n_r}}{\frac{n_r}{n_p}},$$

where $\mu(\gamma, n_r)$ is the probability that a Respondent successfully forms a contract with their chosen Proposer, provided that this Respondent selects the specified Proposer with probability one, while the other $n_r - 1$ Respondents select that same Proposer with the equilibrium probability γ.

The equilibrium expected utility for each agent on the Respondents side is given by

$$\mathbf{E}\left[u_r\right] = gain_r \nu(\gamma, n_r) = \frac{|val_r - val_p| \left(1 - \frac{1}{n_p}\right)^{n_r} \left(1 - \left(1 - \frac{1}{n_p}\right)^{n_r}\right)}{\frac{n_r}{n_p}\left(1 - \frac{1}{n_p}\right)^{n_r} + 1 - \left(1 - \frac{1}{n_p}\right)^{n_r - 1} \frac{n_r}{n_p} - \left(1 - \frac{1}{n_p}\right)^{n_r}}$$

and is bounded as follows:

$$\mathbf{E}\left[u_r\right] \in |val_r - val_p| \left(\left(1 - \frac{1}{n_p}\right)^{n_r}, \left(1 - \frac{1}{n_p}\right)^{n_r - 1}\right).$$

The equilibrium expected Respondents Surplus (RS) is given by

$$\mathbf{E}\left[RS\right] = n_r \mathbf{E}\left[u_r\right].$$

The equilibrium expected utility for each agent on the Proposers side is given by

$$\mathbf{E}\left[u_p\right] = gain_p n_r \gamma\mu(\gamma, n_r)$$

$$= \frac{|val_r - val_p| \left(1 - \left(1 - \frac{1}{n_p}\right)^{n_r}\right)\left(1 - \left(1 - \frac{1}{n_p}\right)^{n_r - 1} \frac{n_r}{n_p} - \left(1 - \frac{1}{n_p}\right)^{n_r}\right)}{\frac{n_r}{n_p}\left(1 - \frac{1}{n_p}\right)^{n_r} + 1 - \left(1 - \frac{1}{n_p}\right)^{n_r - 1} \frac{n_r}{n_p} - \left(1 - \frac{1}{n_p}\right)^{n_r}}$$

and is bounded as follows:

$$\mathbf{E}\left[u_p\right] \in |val_r - val_p| \left(1 - \left(1 - \frac{1}{n_p}\right)^{n_r - 1} \frac{n_r}{n_p} - \left(1 - \frac{1}{n_p}\right)^{n_r}, \right.$$
$$\left. \left(1 - \left(1 - \frac{1}{n_p}\right)^{n_r - 1} \frac{n_r}{n_p} - \left(1 - \frac{1}{n_p}\right)^{n_r}\right)\left(1 + \frac{1}{n_p - 1}\right)\right).$$

The equilibrium expected Proposers Surplus (PS) is given by

$$\mathbf{E}\left[PS\right] = n_p \mathbf{E}\left[u_p\right].$$

The equilibrium expected Total Surplus (TS) is given by

$$\mathbf{E}\left[TS\right] = \mathbf{E}\left[PS\right] + \mathbf{E}\left[RS\right] = n_p \mathbf{E}\left[u_p\right] + n_r \mathbf{E}\left[u_r\right]$$
$$= |val_r - val_p|\, n_r \nu(\gamma, n_r) = |val_r - val_p|\, n_p \left(1 - \left(1 - \frac{1}{n_p}\right)^{n_r}\right).$$

Several points should be noted here.

First, using the aforementioned upper bound of the Respondent's expected utility in conjunction with the lower bound of the Proposer's expected utility yields the exact expression for the expected total surplus.

Second, under the assumption that the arguments n_p and n_r of the expected total surplus function are real-valued, the first partial derivative with respect to each argument equals the lower bound of the expected utility function for Proposers and Respondents, respectively.

The equilibrium expected Number of Deals (ND) is given by

$$\mathbf{E}\left[ND\right] = \frac{1}{|val_r - val_p|}\mathbf{E}\left[TS\right].$$

The expected Share of Deals (SD), representing the proportion of realized contracts relative to the maximum possible number, is given by

$$\mathbf{E}\left[SD\right] = \max\left(\frac{1}{n_p}, \frac{1}{n_r}\right)\mathbf{E}\left[ND\right].$$

The share of the expected Respondents Surplus within the expected Total Surplus is bounded as

$$\frac{\mathbf{E}\left[RS\right]}{\mathbf{E}\left[TS\right]} \in \left(\frac{n_r\left(1-\frac{1}{n_p}\right)^{n_r}}{n_p\left(1-\left(1-\frac{1}{n_p}\right)^{n_r}\right)}, \frac{n_r\left(1-\frac{1}{n_p}\right)^{n_r-1}}{n_p\left(1-\left(1-\frac{1}{n_p}\right)^{n_r}\right)}\right).$$

The equilibrium contract price can be expressed as

$$val = var_r + \left(val_p - val_r\right)\frac{\mathbf{E}\left[RS\right]}{\mathbf{E}\left[TS\right]}.$$

The proof is based on substituting the equilibrium price into the corresponding expressions followed by algebraic manipulations.

3. MARKET DESIGN ANALYSIS

Having described the equilibrium market characteristics in the general case in the previous section, we now return to the firm-employee model to examine how market size influences the preferences of each side regarding its design.

3.1. Market Efficiency and Social Preferences

We begin our analysis with one of the key aspects: the efficiency of the market mechanism. The following metrics can be used interchangeably as parameters characterizing efficiency: the expected number of contracts in equilibrium, the expected share of contracts, or the expected total surplus. These quantities were described individually in Corollary 1. Here, our primary interest lies in the differences in these values as a function of the market design.

Definition 1. The difference in the expected number of contracts between the FP and WP models as a function of market sizes is given by

$$\Delta_{ND}\left(n_f, n_w\right) = \mathbf{E}\left[ND \middle| Proposers = Firms\right] - \mathbf{E}\left[ND \middle| Proposers = Workers\right]$$
$$= n_f\left(1-\left(1-\frac{1}{n_f}\right)^{n_w}\right) - n_w\left(1-\left(1-\frac{1}{n_w}\right)^{n_f}\right).$$

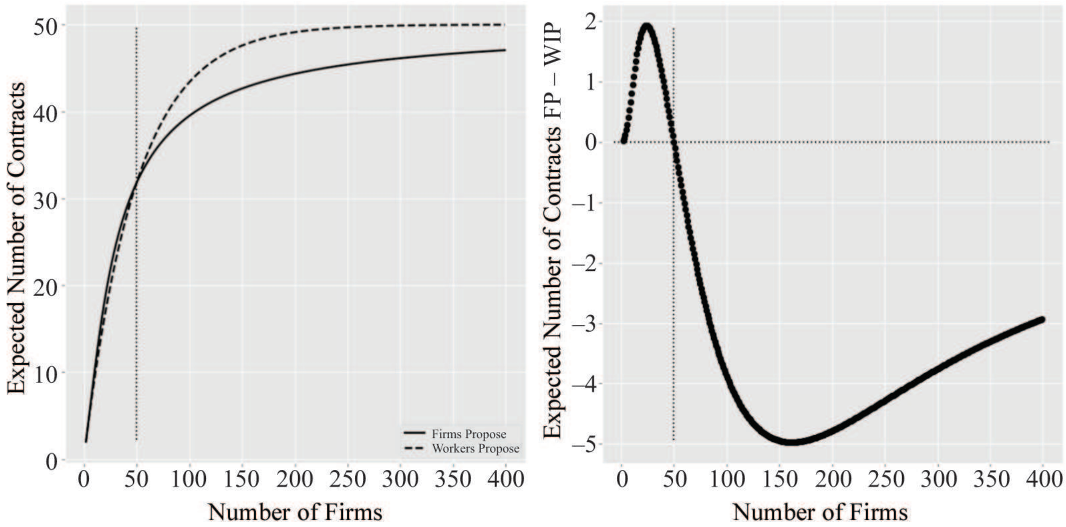

Fig. 1. Expected equilibrium number of contracts in the FP and WP models as a function of the number of firms for $n_w = 50$, $wage_{\min} = 100$, $wage_{\max} = 200$.

Definition 2. The difference in the expected share of concluded contracts relative to the maximum possible number between the FP and WP models, as a function of market sizes, is given by

$$\Delta_{SD}(n_f, n_w) = \mathbf{E}\left[SD \middle| Proposers = Firms\right] - \mathbf{E}\left[SD \middle| Proposers = Workers\right]$$
$$= \max\left(\frac{1}{n_f}, \frac{1}{n_w}\right)\Delta_{ND}(n_f, n_w).$$

Definition 3. The difference in the expected total surplus between the FP and WP models as a function of market sizes is given by

$$\Delta_{TS}(n_f, n_w) = \mathbf{E}\left[TS \middle| Proposers = Firms\right] - \mathbf{E}\left[TS \middle| Proposers = Workers\right]$$
$$= (wage_{\max} - wage_{\min})\,\Delta_{ND}(n_f, n_w).$$

It is worth noting that all three difference functions described above have the same sign for identical values of their arguments. For clarity and ease of interpretation, we refer to the graphical illustration in Fig. 1 and then formulate the analytical properties of these functions.

Theorem 2. *Consider the function $\Delta_{ND}(n_f, n_w)$ of the difference in the expected number of contracts between the FP and WP models. This function has the following properties:*

1) $\Delta_{ND}(n_f, n_w) \geqslant 0 \iff n_f \leqslant n_w$, *with* $\Delta_{ND}(n_f, n_w) = 0 \iff n_f = n_w$,

2) $\Delta_{ND}(n_f; n_w)$ *as a function of* n_f *for a fixed* n_w

(a) *is increasing on the interval* $[2, \theta_1)$, *where* $2 < \theta_1 < n_w$, *for* $n_w \geqslant 5$ (*for* $2 \leqslant n_w \leqslant 4$, *we define* θ_1 *to be* 2),

(b) *is decreasing on the interval* $[\theta_1, \theta_2)$, *where* $n_w < \theta_2 < \infty$,

(c) *is increasing on the interval* $[\theta_2, \infty)$.

The approximate values of θ_1 *and* θ_2 *are given by*

$$\begin{cases} \theta_1 \simeq \left(\dfrac{1}{k_2}\right) n_w \simeq 0.4747 n_w, \\ \theta_2 \simeq \left(\dfrac{1}{k_1}\right) n_w \simeq 3.2565 n_w, \end{cases}$$

where k_1 and k_2 are the smaller and larger positive roots of the equation

$$1-(1+k)\exp(-k)-\exp\left(-\frac{1}{k}\right)=0. \tag{1}$$

3) *In the limit as either argument tends to infinity, the function vanishes for any fixed value of the other argument.*

Proof. First and foremost, observe that

$$\Delta_{ND}(n_f=2;n_w)=2\left(1-\left(1-\frac{1}{2}\right)^{n_w}\right)-n_w\left(1-\left(1-\frac{1}{n_w}\right)^2\right)$$
$$=2\left(1-\frac{1}{2^{n_w}}\right)-n_w\left(1-\left(1-\frac{2}{n_w}+\frac{1}{n_w^2}\right)\right)=2-\frac{1}{2^{n_w-1}}-n_w\left(\frac{2}{n_w}-\frac{1}{n_w^2}\right)$$
$$=2-\frac{1}{2^{n_w-1}}-2+\frac{1}{n_w}=\frac{1}{n_w}-\frac{1}{2^{n_w-1}}>0,\quad \forall n_w\geqslant 3$$

since the exponential function grows faster than the linear one. Also, note that $\Delta_{ND}(n_w;n_w)=0$ due to symmetry.

We extend the definition of $\Delta_{ND}(n_f;n_w)$ to be a function of a real argument n_f for a given parameter n_w.

Calculating the first derivative yields

$$\Delta_{ND}^{'}(n_f;n_w)=1-\left(1-\frac{1}{n_f}\right)^{n_w}-\frac{n_w}{n_f}\left(1-\frac{1}{n_f}\right)^{n_w-1}+n_w\ln\left(1-\frac{1}{n_w}\right)\left(1-\frac{1}{n_w}\right)^{n_f}.$$

Using the following approximations

$$\ln\left(1-\frac{1}{a}\right)\simeq-\frac{1}{a},$$
$$\left(1-\frac{1}{a}\right)^b\simeq\exp\left(-\frac{b}{a}\right),$$

we can simplify the expression for the first derivative as follows:

$$\Delta_{ND}^{'}(n_f;n_w)\simeq1-\left(1-\frac{1}{n_f}\right)^{n_w}-\frac{n_w}{n_f}\left(1-\frac{1}{n_f}\right)^{n_w-1}-\left(1-\frac{1}{n_w}\right)^{n_f},$$
$$\Delta_{ND}^{'}(n_f;n_w)\simeq1-\exp\left(-\frac{n_w-1}{n_f}\right)\left(1+\frac{n_w-1}{n_f}\right)-\exp\left(-\frac{n_f}{n_w}\right).$$

Applying the approximation $\frac{n_w-1}{n_f}\simeq\frac{n_w}{n_f}$, we introduce the substitution $\frac{n_w}{n_f}=k>0$.

$$\Delta_{ND}^{'}(n_f;n_w)\simeq1-(1+k)\exp(-k)-\exp\left(-\frac{1}{k}\right).$$

The first-order conditions defining the extrema of $\Delta_{ND}(n_f;n_w)$ are given by

$$\Delta_{ND}^{'}(n_f;n_w)=0\iff1-(1+k)\exp(-k)-\exp\left(-\frac{1}{k}\right)=0.$$

The specified equation has exactly two solutions satisfying the condition $k>0$ (verified via WolframAlpha):

$$\begin{cases}k_1^*\simeq0.307074,\\ k_2^*\simeq2.106541.\end{cases}$$

Thus, the extrema expressed as functions of the parameter n_w take the form

$$\begin{cases} n_{f_1}^* \simeq 3.25654 n_w, \\ n_{f_2}^* \simeq 0.47471 n_w. \end{cases}$$

It should be noted that numerical analysis reveals that the correction $n_{f_1}^* \simeq 3.25654 n_w - 1$ provides a slightly better approximation of the minimum when Δ_{ND} is treated as a function of an integer argument. However, for the sake of consistency, the uncorrected version will be used throughout the main text.

The second derivative is given by

$$\Delta_{ND}^{''}(n_f; n_w) = -\frac{n_w (n_w - 1)}{n_f^3} \left(1 - \frac{1}{n_f}\right)^{n_w - 2} + n_w \ln^2 \left(1 - \frac{1}{n_w}\right) \left(1 - \frac{1}{n_w}\right)^{n_f}.$$

Simplifying this expression via the logarithm approximation and comparing it with zero yields:

$$\Delta_{ND}^{''}(n_f; n_w) < 0 \iff \frac{1}{n_w} \left(1 - \frac{1}{n_w}\right)^{n_f} < \frac{n_w (n_w - 1)}{n_f^3} \left(1 - \frac{1}{n_f}\right)^{n_w - 2},$$

$$\Delta_{ND}^{''}(n_f; n_w) < 0 \iff \left(1 - \frac{1}{n_f}\right)^{n_w - 2} \frac{1}{n_f^3} > \left(1 - \frac{1}{n_w}\right)^{n_f - 1} \frac{1}{n_w^3}.$$

Taking the natural logarithm of both sides, applying the approximation once more, and performing algebraic simplifications, we obtain:

$$\Delta_{ND}^{''}(n_f; n_w) < 0 \iff 3 \ln(n_f) + \frac{n_w - 2}{n_f} - \frac{n_f - 1}{n_w} - 3 \ln(n_w) < 0.$$

Substituting the parameterized expression for the extrema into the second-derivative inequality, we obtain the following:

$$\Delta_{ND}^{''}\left(n_{f_2}^*(n_w); n_w\right) < 0 \iff 3 \ln(0.475 n_w) + \frac{n_w - 2}{0.475 n_w} - \frac{0.475 n_w - 1}{n_w} - 3 \ln(n_w) < 0,$$

$$\Delta_{ND}^{''}\left(n_{f_2}^*(n_w); n_w\right) < 0 \iff -0.6034 - \frac{-1.5253}{n_w} - \frac{-2.1065}{n_w - 1} < 0 \ \forall n_w \geqslant 2.$$

Thus, the point $n_{f_2}^*$ corresponds to a maximum of the function.

Performing a similar exercise for the point $n_{f_1}^*$, we obtain

$$\Delta_{ND}^{''}\left(n_{f_1}^*(n_w); n_w\right) > 0 \iff 3 \ln(3.2565 n_w) + \frac{n_w - 2}{3.2565 n_w} - \frac{3.2565 n_w - 1}{n_w} - 3 \ln(n_w) < 0,$$

$$\Delta_{ND}^{''}\left(n_{f_1}^*(n_w); n_w\right) > 0 \iff 7.1056 > \frac{7.2565}{n_w} + \frac{0.3071}{n_w - 1} \ \forall n_w \geqslant 2.$$

Thus, the point $n_{f_1}^*$ corresponds to a minimum of the function.

It should be noted that

$$\begin{cases} n_{f_1}^* \simeq 3.25654 n_w > n_w, \\ n_{f_2}^* \simeq 0.47471 n_w < n_w, \end{cases}$$

$\forall n_w \geqslant 2$, i.e., the maximum of $\Delta_{ND}(n_f, n_w)$ is located to the left of n_w, while the minimum is to its right.

Let us now consider the asymptotic behavior.

$$\lim_{n_f\to\infty}\Delta_{ND}\left(n_f,n_w\right)=\lim_{n_f\to\infty}n_f\left(1-\left(1-\frac{1}{n_f}\right)^{n_w}\right)-\lim_{n_f\to\infty}n_w\left(1-\left(1-\frac{1}{n_w}\right)^{n_f}\right)=0,$$
$$\lim_{n_f\to\infty}n_w\left(1-\left(1-\frac{1}{n_w}\right)^{n_f}\right)=n_w,$$
$$\lim_{n_f\to\infty}n_f\left(1-\left(1-\frac{1}{n_f}\right)^{n_w}\right)=n_w.$$

In conclusion, a few points deserve mention.

First, since the original argument n_f of the function $\Delta_{ND}\left(n_w;n_w\right)$ is an integer rather than a real number, the identified minimum and maximum points require an adjustment to the nearest integer that yields the minimum or maximum value of Δ_{ND}.

Second, although approximations were used in the theorem's formulation and its proof, numerical analysis shows that the difference between the true integer extremum and the rounded approximate value does not exceed 1 for any value of n_w.

This completes the proof of the theorem.

Corollary 2. *The function $\Delta_{TS}\left(n_f,n_w\right)$, representing the difference in the expected total surplus between the FP and WP models, shares the same properties as described in Theorem 2 for the expected contract number difference $\Delta_{ND}\left(n_f,n_w\right)$.*

Proof. By definition,

$$\Delta_{TS}\left(n_f,n_w\right)=\left(wage_{\max}-wage_{\min}\right)\Delta_{ND}\left(n_f,n_w\right).$$

Thus, with respect to the properties in Theorem 2, these functions are equivalent as they differ only by a positive constant factor.

Corollary 3. *The function $\Delta_{SD}\left(n_f,n_w\right)$, representing the difference in the expected share of concluded contracts relative to the maximum possible number between the FP and WP models, exhibits the same properties as described in Theorem 2 for the expected number of contracts, subject to an adjustment of the maximum point coordinate λ_1. The approximate value of the point λ_1 is given by*

$$\lambda_1\simeq k_1n_w\simeq 0.3071n_w,$$

where k_1 is the smaller of the two positive roots of equation 1.

Proof. In the case $n_f\geqslant n_w$, by the definition of the function $\Delta_{SD}\left(n_f,n_w\right)$, we have:

$$\Delta_{SD}\left(n_f,n_w\right)=\frac{1}{n_w}\Delta_{ND}\left(n_f,n_w\right),$$

i.e., scaling by $\frac{1}{n_w}$ affects neither the zeros of Δ_{ND} nor its behavior as a function of n_f for a fixed n_w.

In the case $n_f<n_w$, the argument follows the same logic as in the proof of Theorem 2.

The above discussion leads to the following formulation.

Proposition 1. *From the perspective of social welfare and market efficiency, measured by any of the methods given in definitions 1, 2, and 3, it is preferable that the contract terms in the first stage be published by the side of the market with fewer participants.*

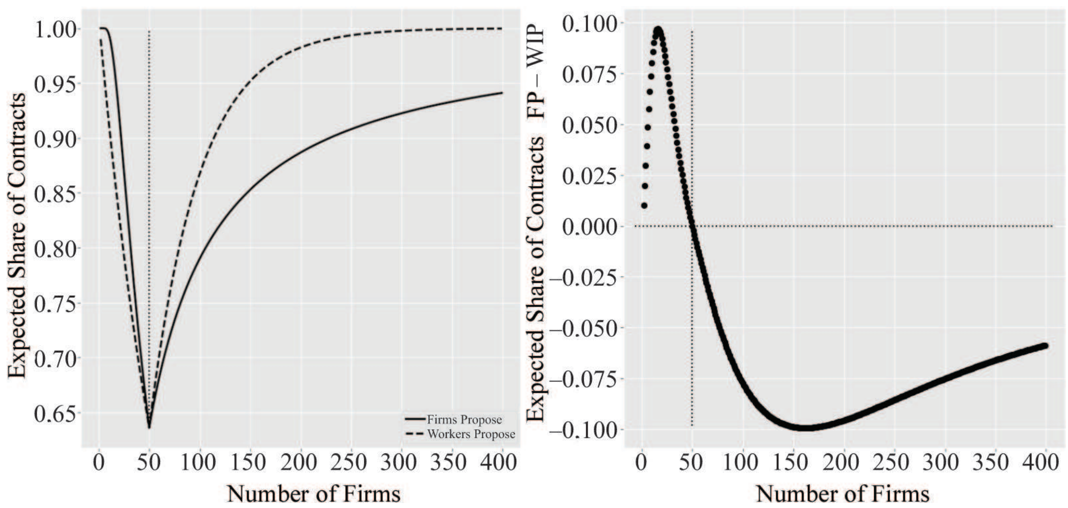

Fig. 2. Expected equilibrium share of concluded contracts in the FP and WP models as a function of the number of firms for $n_w = 50$, $wage_{\min} = 100$, $wage_{\max} = 200$.

At the same time, if we move beyond a binary comparison of relative efficiency between FP and WP for fixed market sizes and instead examine how the superiority of one design over another evolves as market sizes change, it is more appropriate to consider relative metrics, such as the expected share of concluded contracts in equilibrium, a graphical illustration of which is presented in Fig. 2.

It is worth noting that market sizes $n_f = n_w$ are the least efficient in terms of the expected share of concluded contracts in both models (FP and WP); furthermore, due to this extreme inefficiency, society cannot gain any advantage by switching the market design. Conversely, for any other market size, one of the designs is strictly preferable from a social standpoint. It should be noted, however, that in the case of a correlated equilibrium, a market with an equal number of agents on both sides is the most efficient, as it ensures not only a 100% share of concluded contracts but also the absence of unmatched agents on either side.

This situation is analogous to using the ROC-AUC (Area Under the Receiver Operating Characteristic Curve) metric for evaluating the performance of binary classification models in machine learning. A model with an ROC-AUC of 0.5 represents the worst possible case, as it corresponds to random guessing and cannot be improved. Conversely, a model with a very low ROC-AUC, such as 0.01, which implies an inverse ranking of classes, can be significantly improved to an ROC-AUC of $(1 - 0.01)$ by simply flipping the class labels.

Let us now consider the potential gains in market efficiency achievable through a change in its design.

Corollary 4. *The maximum and minimum values of the function $\Delta_{SD}(n_f; n_w)$, representing the difference in the expected share of concluded contracts relative to the maximum possible number between the FP and WP models as a function of n_f for a fixed n_w, are approximately equal in absolute value to*

$$\Delta_{SD}(k_1 n_w; n_w) \simeq -\Delta_{SD}\left(\frac{n_w}{k_1}; n_w\right) \simeq 1 - \frac{1}{k_1} + \frac{1}{k_1}\exp(-k_1) - \exp\left(-\frac{1}{k_1}\right) \simeq 10.04\%$$

and are attained at the points

$$\begin{cases} \lambda_1 \simeq k_1 n_w \simeq 0.3071 n_w, \\ \theta_2 \simeq \left(\dfrac{1}{k_1}\right) n_w \simeq 3.2565 n_w, \end{cases}$$

where k_1 is the smaller of the two positive roots of equation 1.

Thus, for certain market sizes, the gain in efficiency solely due to a change in design can exceed 10% in terms of the expected share of concluded contracts.

Remark 1. It is important to note that, due to the approximations used, the approximate maximum and minimum values of the function $\Delta_{SD}(n_f; n_w)$ are independent of the parameter n_w. The true maximum and minimum values do not possess this property. However, the absolute discrepancy does not exceed 0.4% for $n_w \geqslant 50$ and diminishes as n_w increases further.

3.2. Firms' Preferences

Let us now examine the preferences of firms regarding the market design as a function of its size. The characteristics analyzed include the expected profit of an individual firm and the expected aggregate surplus of all firms, as well as the dependence of these variables on the market mechanism design.

Definition 4. The difference in a firm's expected profit between the FP and WP models, as a function of market sizes, is given in approximate form by

$$\Delta_{u_F}(n_f, n_w) = \mathbf{E}\left[u_p \middle| Proposers = Firms\right] - \mathbf{E}\left[u_r \middle| Proposers = Workers\right]$$
$$\simeq \Delta wage\left(1 - \left(1 - \frac{1}{n_f}\right)^{n_w - 1}\frac{n_w}{n_f} - \left(1 - \frac{1}{n_f}\right)^{n_w} - \left(1 - \frac{1}{n_w}\right)^{n_f}\right).$$

Remark 2. In the FP model, firms act as *Proposers*, whereas in the WP model, they act as *Respondents*; therefore, to obtain the exact difference function, one must use the corresponding values from Corollary 1. However, a full-scale analysis of the exact function is not feasible. Consequently, we employ the aforementioned approximation, which lies between the lower and upper bounds of the exact function derived using the corresponding bounds from Corollary 1.

Definition 5. The difference in the expected total surplus of firms between the FP and WP models, as a function of market sizes, is given in approximate form by

$$\Delta_{FS}(n_f, n_w) = \mathbf{E}\left[PS \middle| Proposers = Firms\right] - \mathbf{E}\left[RS \middle| Proposers = Workers\right] = n_f \Delta_{u_F}(n_f, n_w).$$

A graphical illustration for a particular case is depicted in Fig. 3 for the expected firms' profits and in Fig. 4 for the expected firms' aggregate surplus. Let us now formulate the analytical properties of these functions.

Theorem 3. *Consider the function $\Delta_{u_F}(n_f, n_w)$, representing the difference in a firm's expected profit between the FP and WP models. This function has the following properties.*

1) $\Delta_{u_F}(n_f; n_w)$, *as a function of n_f for a fixed n_w, has exactly two roots, θ_1 and θ_2, the approximate values of which are given by*

$$\begin{cases} \theta_1 \simeq \left(\dfrac{1}{k_2}\right) n_w \simeq 0.4747 n_w, \\ \theta_2 \simeq \left(\dfrac{1}{k_1}\right) n_w \simeq 3.2565 n_w, \end{cases}$$

where k_1 and k_2 are the smaller and larger positive roots of equation 1, respectively. Furthermore,

$$\begin{cases} \Delta_{u_F}(n_f; n_w) > 0 \iff n_f < \theta_1 \vee n_f > \theta_2, \\ \Delta_{u_F}(n_f; n_w) < 0 \iff \theta_1 < n_f < \theta_2. \end{cases}$$

2) $\Delta_{u_F}(n_f; n_w)$, *as a function of n_f for a fixed n_w,*

(a) *is increasing on the interval* $[2, \xi_1)$, *where* $2 < \xi_1 < \theta_1$, *for* $n_w \geqslant 9$ (*for* $2 \leqslant n_w \leqslant 8$ *we define* ξ_1 *to be* 2),

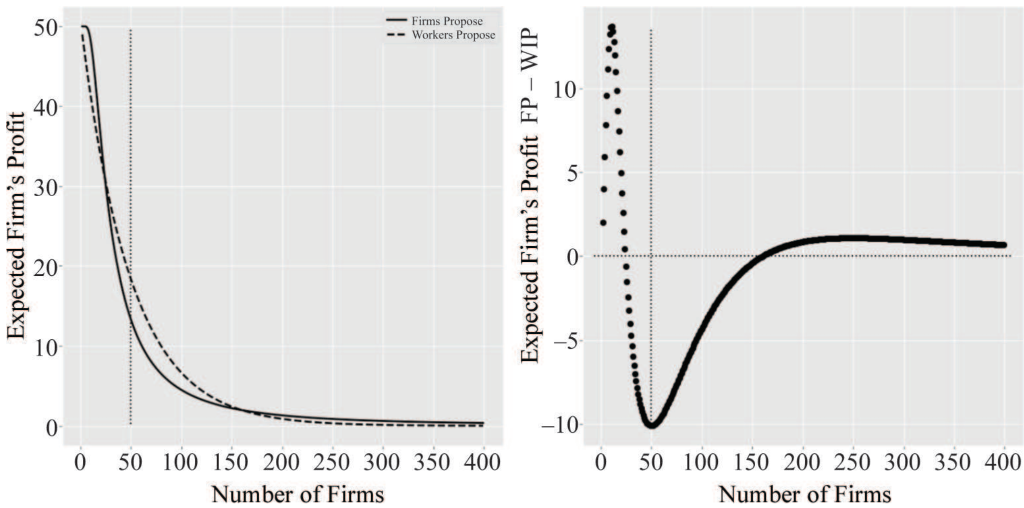

Fig. 3. Expected equilibrium firm's profit in the FP and WP models as a function of the number of firms for $n_w = 50$, $wage_{\min} = 100$, $wage_{\max} = 200$.

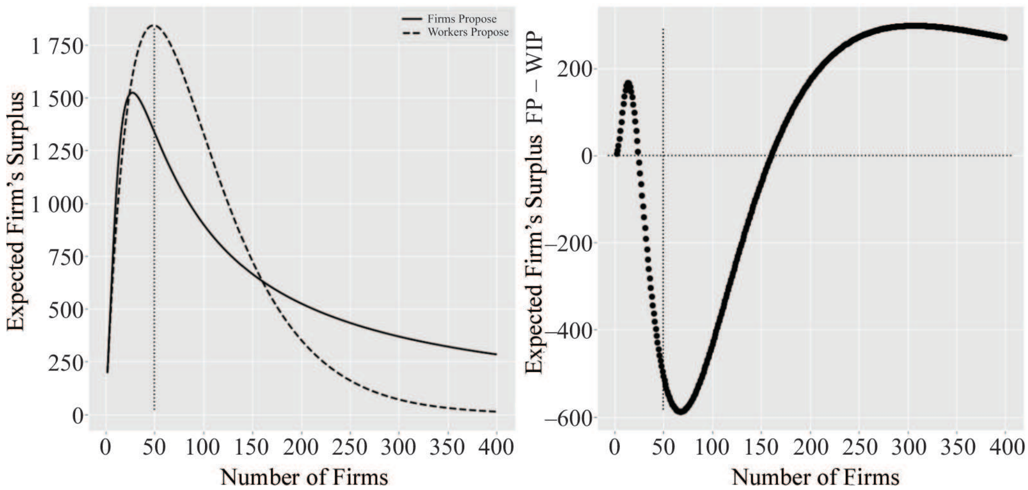

Fig. 4. Expected equilibrium firms' surplus in the FP and WP models as a function of the number of firms for $n_w = 50$, $wage_{\min} = 100$, $wage_{\max} = 200$.

(b) *is decreasing on the interval* $[\xi_1, \xi_2)$, *where* $\xi_2 = n_w$,

(c) *is increasing on the interval* $[\xi_2, \xi_3)$, *where* $\theta_2 < \xi_3 < \infty$,

(d) *is decreasing on the interval* $[\xi_3, \infty)$.

The approximate values of ξ_1, ξ_2, *and* ξ_3 *are given by*

$$\begin{cases} \xi_1 \simeq p_1 n_w \simeq 0.1975 n_w, \\ \xi_2 = p_2 n_w = n_w, \\ \xi_3 \simeq p_3 n_w \simeq 5.0639 n_w, \end{cases}$$

where p_1, p_2, *and* p_3 *are roots of the equation*

$$3\ln(p) + \frac{1}{p} - p = 0 \tag{2}$$

in increasing order. It is worth noting that $p_1 = \frac{1}{p_3}$.

3) *In the limit as either argument tends to infinity, the function vanishes for any fixed value of the other argument.*

Proof. We extend the definition of $\Delta_{u_F}(n_f; n_w)$ to treat n_f as a real-valued argument for a given parameter n_w.

The proof of the first part of this theorem is covered by the proof of Theorem 2. This follows from the fact that the approximation of the function $\Delta_{u_F}(n_f; n_w)$ given in Definition 4 is identically equal to the approximation of the first derivative of $\Delta_{ND}(n_f, n_w)$ used in the proof of Theorem 2. Consequently, the zeros of the first derivative of $\Delta_{ND}(n_f; n_w)$ and the zeros of $\Delta_{u_F}(n_f; n_w)$ coincide.

Subject to the approximations used, the first derivative of the function $\Delta_{u_F}(n_f; n_w)$ is equal to the second derivative of the function $\Delta_{ND}(n_f, n_w)$, the expression for the zeros of which has already been derived in the proof of Theorem 2.

$$\Delta^{'}_{u_F}(n_f; n_w) = 0 \iff 3\ln\left(\frac{n_f}{n_w}\right) + \frac{n_w}{n_f} - \frac{n_f}{n_w} - \frac{2}{n_f} + \frac{1}{n_w} = 0.$$

Within the framework of the approximation, neglecting the term $-\frac{2}{n_f} + \frac{1}{n_w}$ and substituting $\frac{n_f}{n_w} = p$, we obtain the equation

$$\Delta^{'}_{u_F}(n_f; n_w) = 0 \iff 3\ln(p) + \frac{1}{p} - p = 0.$$

Note that $p = 1$ is a root. Furthermore, if some p^* is a root, then $\frac{1}{p^*}$ is also a root. The approximate value of the root other than unity (obtained via WolframAlpha) is 0.1975. Using the interval method, it can be shown that ξ_1 and ξ_3 are local maxima, whereas ξ_2 is a local minimum.

This completes the proof of the theorem.

Corollary 5. *The function $\Delta_{FS}(n_f, n_w)$, representing the difference in the expected aggregate surplus of firms between the FP and WP models as a function of market sizes, exhibits the same properties as described in Theorem 3 for the function $\Delta_{u_F}(n_f, n_w)$, adjusted for the coordinates of the extrema ϕ_1, ϕ_2, and ϕ_3. The approximate values of these extrema are given by*

$$\begin{cases} \phi_1 \simeq \left(\frac{1}{r_3}\right) n_w \simeq \frac{1}{3.7779} n_w \simeq 0.2647 n_w, \\ \phi_2 \simeq \left(\frac{1}{r_2}\right) n_w \simeq \frac{1}{0.7388} n_w \simeq 1.3536 n_w, \\ \phi_3 \simeq \left(\frac{1}{r_1}\right) n_w \simeq \frac{1}{0.1608} n_w \simeq 6.2189 n_w, \end{cases}$$

where r_1, r_2, and r_3 are the roots of the equation

$$1 - \left(1 + r + r^2\right)\exp(-r) - \left(1 - \frac{1}{r}\right)\exp\left(-\frac{1}{r}\right) = 0$$

given in increasing order.

Proof. The proof is similar to that of Theorem 3.

The economic interpretation of the results obtained above can be stated as follows.

Proposition 2. *In a market where the sizes of the two sides do not differ significantly:*

$$\frac{1}{k_2} \simeq 0.4747 \leqslant \frac{n_f}{n_w} \leqslant \frac{1}{k_1} \simeq 3.2565,$$

firms prefer to take the side of Respondents, leaving the responsibility of publishing the contract terms to the workers. Conversely, if the market is significantly unbalanced in either direction, it is more preferable for the firms to publish the terms themselves as Proposers.

3.3. Workers' Preferences

We now turn our attention to the analysis of workers' preferences regarding the market design. In this regard, both the expected utility of an individual worker and the expected aggregate surplus of all workers are of particular interest; however, the primary focus lies in the differences in these values depending on the market design.

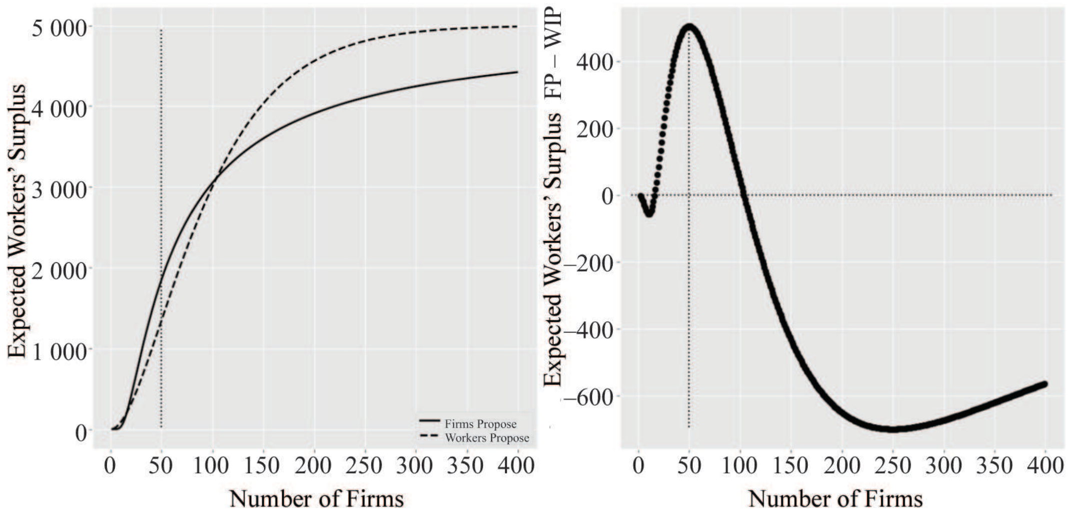

Fig. 5. Expected equilibrium workers' surplus in the FP and WP models as a function of the number of firms for $n_w = 50$, $wage_{\min} = 100$, $wage_{\max} = 200$.

Definition 6. The difference in a worker's expected utility between the FP and WP models, as a function of market sizes, is given in approximate form by

$$\Delta_{u_W}(n_f, n_w) = \mathbf{E}\left[u_r \middle| Proposers = Firms\right] - \mathbf{E}\left[u_p \middle| Proposers = Workers\right]$$
$$\simeq \Delta wage\left(\left(1 - \frac{1}{n_f}\right)^{n_w} + \left(1 - \frac{1}{n_w}\right)^{n_f - 1}\frac{n_f}{n_w} + \left(1 - \frac{1}{n_w}\right)^{n_f} - 1\right).$$

Definition 7. The difference in the expected total surplus of workers between the FP and WP models, as a function of market sizes, is given in approximate form by

$$\Delta_{WS}(n_f, n_w) = \mathbf{E}\left[RS \middle| Proposers = Firms\right] - \mathbf{E}\left[PS \middle| Proposers = Workers\right] \simeq n_w \Delta_{u_W}(n_f, n_w).$$

A graphical illustration for a particular case is presented in Fig. 5. The functions introduced above possess the following analytical properties.

Theorem 4. *Consider the function $\Delta_{u_W}(n_f, n_w)$, representing the difference in a worker's expected utility between the FP and WP models. This function has the following properties.*

1) *$\Delta_{u_W}(n_f; n_w)$, as a function of n_f for a fixed n_w, has exactly two roots, λ_1 and λ_2, the approximate values of which are given by*

$$\begin{cases} \lambda_1 \simeq k_1 n_w \simeq 0.3071 n_w, \\ \lambda_2 \simeq k_2 n_w \simeq 2.1065 n_w, \end{cases}$$

where k_1 and k_2 are the smaller and larger positive roots of equation 1. Furthermore,

$$\begin{cases} \Delta_{u_W}(n_f; n_w) < 0 \iff n_f < \lambda_1 \vee n_f > \lambda_2, \\ \Delta_{u_W}(n_f; n_w) > 0 \iff \lambda_1 < n_f < \lambda_2. \end{cases}$$

2) *$\Delta_{u_W}(n_f; n_w)$, as a function of n_f for a fixed n_w,*

(a) *is decreasing on the interval $[2, \xi_1)$, where $2 < \xi_1 < \lambda_1$, for $n_w \geqslant 9$ (for $2 \leqslant n_w \leqslant 8$ we define ξ_1 to be 2),*

(b) *is increasing on the interval* $[\xi_1, \xi_2)$, *where* $\xi_2 = n_w$,
(c) *is decreasing on the interval* $[\xi_2, \xi_3)$, *where* $\lambda_2 < \xi_3 < \infty$,
(d) *is increasing on the interval* $[\xi_3, \infty)$.
The approximate values of ξ_1, ξ_2, *and* ξ_3 *are given by*

$$\begin{cases} \xi_1 \simeq p_1 n_w \simeq 0.1975 n_w, \\ \xi_2 = p_2 n_w = n_w, \\ \xi_3 \simeq p_3 n_w \simeq 5.0639 n_w, \end{cases}$$

where p_1, p_2, *and* p_3 *are roots of the equation 2 in increasing order.*

3) *In the limit as either argument tends to infinity, the function vanishes for any fixed value of the other argument.*

The proof is similar to that of Theorem 3.

Corollary 6. *The function* $\Delta_{WS}(n_f, n_w)$, *representing the difference in the expected aggregate surplus of workers between the FP and WP models as a function of market sizes, exhibits the same properties as described in Theorem 4 for the function* $\Delta_{u_W}(n_f, n_w)$.

The economic interpretation of the results obtained above can be stated as follows.

Proposition 3. *In a market where the sizes of the two sides do not differ significantly:*

$$k_1 \simeq 0.3071 \leqslant \frac{n_f}{n_w} \leqslant k_2 \simeq 2.1065,$$

workers prefer to take the side of Respondents, leaving the responsibility of publishing the contract terms to the firms. Conversely, if the market is significantly unbalanced in either direction, it is more preferable for the workers to publish the terms themselves as Proposers.

3.4. Market Power and Equilibrium Wage

Another important perspective illustrating the comparative advantage of firms over workers (or vice versa) and reflecting their market power is the expected share of one side's surplus in the expected aggregate surplus. The higher this share, the more favorable the position of one market side relative to the other. It should be noted, however, that this analysis focuses solely on the division of the "pie" rather than its absolute size.

It should be noted that the economic significance of the degree of market power is rooted in the nature of the equilibrium market price. As significance in Corollary 1, the equilibrium wage, being a weighted average of $wage_{\min}$ and $wage_{\max}$, is expressed as a strictly monotonic linear function of the share of one side's expected surplus in the total expected surplus. For convenience, and without loss of generality, the following analysis is conducted using the normalized version.

Definition 8. The difference in the share of the workers' expected surplus in the total expected surplus between the FP and WP models, as a function of market sizes, is approximately given by

$$\Delta_{\frac{WS}{TS}}(n_f, n_w) \simeq \frac{\mathbf{E}\left[RS \middle| Proposers = Firms\right]}{\mathbf{E}\left[TS \middle| Proposers = Firms\right]} - \frac{\mathbf{E}\left[PS \middle| Proposers = Workers\right]}{\mathbf{E}\left[TS \middle| Proposers = Workers\right]}$$

$$\simeq \frac{n_w \left(1 - \frac{1}{n_f}\right)^{n_w}}{n_f \left(1 - \left(1 - \frac{1}{n_f}\right)^{n_w}\right)} + \frac{n_f \left(1 - \frac{1}{n_w}\right)^{n_f}}{n_w \left(1 - \left(1 - \frac{1}{n_w}\right)^{n_f}\right)} - 1.$$

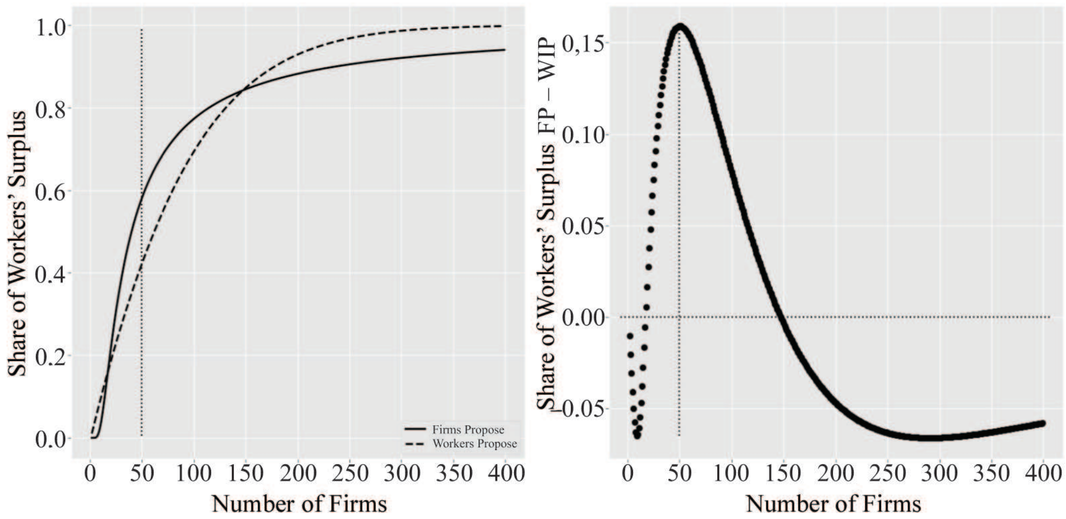

Fig. 6. The share of the workers' expected surplus in the total expected surplus in the FP and WP models as a function of the number of firms for $n_w = 50$, $wage_{\min} = 100$, $wage_{\max} = 200$.

By employing an additional approximation, it may be rewritten as:

$$\Delta_{\frac{WS}{TS}}(x) \simeq \frac{1}{x}\left(\frac{\exp\left(-\frac{1}{x}\right)}{1-\exp\left(-\frac{1}{x}\right)}\right) + x\left(\frac{\exp(-x)}{1-\exp(-x)}\right) - 1,$$

where $x = \frac{n_f}{n_w}$.

A graphical illustration of the function under discussion for a particular case is presented in Fig. 6. The introduced function possesses the following analytical properties.

Theorem 5. *Consider the function $\Delta_{\frac{WS}{TS}}(n_f, n_w)$, representing the difference in the share of the workers' expected surplus in the total expected surplus between the FP and WP model. This function has the following properties.*

1) $\Delta_{\frac{WS}{TS}}(n_f; n_w)$*, as a function of n_f for a fixed n_w, has exactly two roots, α_1 and α_2, the approximate values of which are given by*

$$\begin{cases} \alpha_1 \simeq s_1 n_w \simeq 0.3337 n_w, \\ \alpha_2 \simeq s_2 n_w \simeq 2.9963 n_w, \end{cases}$$

where s_1 and s_2 are the smaller and larger positive roots of the equation

$$\frac{1}{s}\left(\frac{\exp\left(-\frac{1}{s}\right)}{1-\exp\left(-\frac{1}{s}\right)}\right) + s\left(\frac{\exp(-s)}{1-\exp(-s)}\right) - 1 = 0.$$

Furthermore,

$$\begin{cases} \Delta_{\frac{WS}{TS}}(n_f; n_w) < 0 \iff n_f < \alpha_1 \vee n_f > \alpha_2, \\ \Delta_{\frac{WS}{TS}}(n_f; n_w) > 0 \iff \alpha_1 < n_f < \alpha_2. \end{cases}$$

2) $\Delta_{\frac{WS}{TS}}(n_f; n_w)$*, as a function of n_f for a fixed n_w,*

(a) *is decreasing on the interval $[2, \omega_1)$, where $2 < \omega_1 < \alpha_1$, for $n_w \geqslant 11$ (for $2 \leqslant n_w \leqslant 10$ we define ω_1 to be 2),*

(b) *is increasing on the interval $[\omega_1, \omega_2)$, where $\omega_2 = n_w$,*

(c) *is decreasing on the interval $[\omega_2, \omega_3)$, where $\alpha_2 < \omega_3 < \infty$,*

(d) *is increasing on the interval $[\omega_3, \infty)$.*

The approximate values of ω_1, ω_2, *and* ω_3 *are given by*

$$\begin{cases} \omega_1 \simeq t_1 n_w \simeq 0.1697 n_w, \\ \omega_2 = t_2 n_w = n_w, \\ \omega_3 \simeq t_3 n_w \simeq 5.8925 n_w, \end{cases}$$

where t_1, t_2, *and* t_3 *are positive roots of the equation*

$$\frac{t + \exp\left(-\frac{1}{t}\right) - t\exp\left(-\frac{1}{t}\right)}{\left(\exp\left(-\frac{1}{t}\right) - 1\right)^2 t^3} + \frac{1-t}{\exp(t) - 1} - \frac{t}{\left(\exp(t) - 1\right)^2} = 0$$

in increasing order.

3) *In the limit as either argument tends to infinity, the function vanishes for any fixed value of the other argument.*

The proof is similar to that of Theorem 3.

Remark 3. Since the shares of the workers' and firms' expected surpluses in the total expected surplus sum to 1, the difference function for the firms' surplus share is given by $1 - \Delta_{\frac{WS}{TS}}(n_f, n_w)$. This function possesses corresponding properties, and its graph is a reflection of the graph of $\Delta_{\frac{WS}{TS}}(n_f; n_w)$ across the horizontal axis.

The economic interpretation of the results obtained above can be stated as follows.

Proposition 4. *In a market where the sizes of the two sides do not differ significantly:*

$$s_1 \simeq 0.3337 \leqslant \frac{n_f}{n_w} \leqslant s_2 \simeq 2.9963,$$

workers, from the perspective of maximizing their share of the expected total surplus, prefer to take the side of Respondents, leaving the responsibility of publishing the contract terms to the firms. Conversely, if the market sizes differ significantly in either direction, workers find it more preferable to take the side of Proposers to maximize their share.

The preferences of firms regarding the maximization of their share of the expected total surplus are, in this case, exactly the opposite.

Remark 4. It is important to highlight the key distinction between Propositions 3 and 4. In Proposition 3, workers are concerned with maximizing their utility or surplus in monetary terms. In contrast, in Proposition 4, workers choose the market design to maximize their share of the expected total surplus for given market sizes. A consequence of the different thresholds for these two objectives is the emergence of market size intervals where these goals come into conflict with each other.

3.5. Comparative Analysis of the Market Participants' Preferences

Having examined the preferences of each market side individually (workers, firms, and society as a whole), we now consolidate the results of this analysis.

Figure 7 graphically illustrates the combined results of Propositions 1–4 regarding the preferences of the market sides. Based on this, the following conclusions can be drawn.

First, if the parties' preferences are considered strictly in monetary terms, the $\frac{n_f}{n_w}$ axis is divided into six intervals. Within two of these, specifically $\frac{n_f}{n_w} \in (0.3071, 0.4747)$ and $\frac{n_f}{n_w} \in (2.1065, 3.2565)$, the preferences of all parties coincide; in the remaining intervals, they come into conflict.

Second, incorporating the parties' preferences regarding the maximization of their own share of total surplus reveals a significant effect. For $\frac{n_f}{n_w} \in (0.3071, 0.3337)$, from the perspective of maximizing monetary value, it is more beneficial for society, firms, and workers alike if firms take

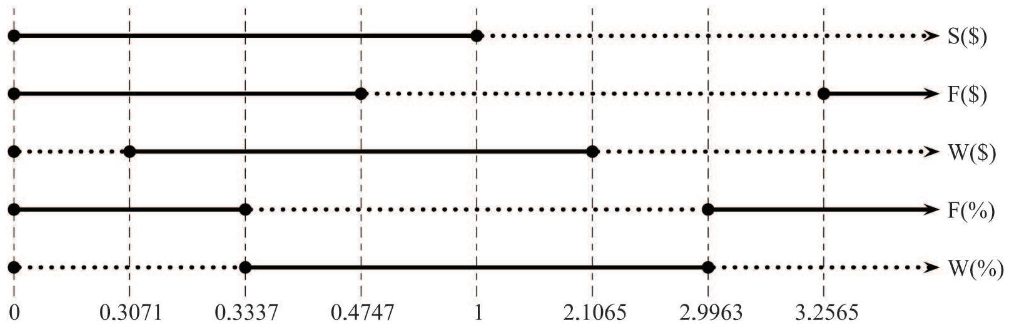

Fig. 7. Market design preferences of firms, workers, and society as a function of the side ratio $\frac{n_f}{n_w}$. (The solid line represents the preference for firms to act as *Proposers*, while the dashed line represents the preference for workers to act as *Proposers.*)

the role of *Proposers*. However, from the standpoint of maximizing their share of the total surplus, workers would prefer to be the proposing side themselves. This implies a situation where one party's pursuit of a larger "slice of the pie" undermines the monetary interests of all market participants, including that very party. A similar effect is observed for firms when $\frac{n_f}{n_w} \in (2.9963, 3.2565)$.

4. CONCLUSION

The conducted analysis describes the structure of preferences of various market participants regarding market design as a function of market size. It was demonstrated that this structure is non-homogeneous: depending on the firm-to-worker ratio, the preferences of the parties may shift several times, occasionally coinciding with one another. The system is most sensitive to changes in design at the ratios $\frac{n_f}{n_w} \simeq 0.3071$ and $\frac{n_f}{n_w} \simeq \frac{1}{0.3071}$, where the gain in the expected share of concluded contracts can exceed 10%. Another key result is the demonstration of the heterogeneity in the choice of objective variables when determining agent preferences. It was shown that a market side may lose a portion of its relative share of the aggregate surplus due to a change in design, yet still gain in monetary terms due to an increase in the total surplus resulting from more efficient agent coordination.

Overall, the findings underscore the importance of market design as a fundamental element of market structure. These results can be applied, for instance, by various online platforms to optimize their objective functions.

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