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Modification of the Electrodynamic Three-Axis Attitude Stabilization Method for an Artificial Earth Satellite in the Orbital Frame

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Abstract—This paper considers the three-axis attitude stabilization problem of an artificial Earth satellite in an arbitrary (indirect) position in the orbital frame. The problem is solved using an electrodynamic control system that generates the Lorentz torque and the magnetic interaction torque. The control torques contain restoring, dissipative, and compensating components. A new method for forming dissipative torques is proposed, which can be implemented using the available control actuators. The Lyapunov function method is applied to obtain sufficient conditions for the asymptotic stability of the satellite's program motion without imposing constraints on the geomagnetic field model or the satellite's orbit inclination. A combined electrodynamic control is presented to reduce the stabilization time and decrease the amplitude of transient oscillations. The results of numerical experiments are provided to demonstrate the effectiveness and performance of the modified satellite stabilization method.

Keywords: satellite, electrodynamic control, three-axis stabilization, modified control, Lyapunov function, asymptotic stability

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1. INTRODUCTION

The interaction of an artificial Earth satellite with the geomagnetic field is a source of torques, which are successfully used to solve a wide range of problems of controlled satellite motion dynamics about its center of mass [1, 2]. This class of problems includes three-axis satellite attitude stabilization in the orbital frame, considered below. The interaction of the geomagnetic field with magnetic coils mounted on the satellite is regarded as a simple, reliable, and efficient way of solving this problem [2–7] and more complex ones requiring other program modes of the satellite's angular motion [8]. Despite the global controllability and observability of magnetic control systems [9], it is necessary to consider their drawback, i.e., the impossibility of generating a control torque in the direction of the geomagnetic field induction vector [4]. The same drawback appears in another well-known control system based on the interaction of an electric charge with the geomagnetic field [10–14]. However, combining the capabilities of these two principles into a unified electrodynamic control system (EDCS) allows achieving better performance compared to each of the two systems used separately [15].

EDCSs were applied to solve several satellite dynamics problems: one-axis and three-axis satellite attitude stabilization in the orbital and Koenig frames, dual-spin satellite stabilization, and others [16]. However, many of the above problems were solved using particular assumptions, e.g., concerning the choice of the geomagnetic field model, the shape and inclination of the satellite's orbit.

In three-axis attitude stabilization problems, the program mode is usually the direct equilibrium position of the satellite in the orbital frame, i.e., the position where its principal central axes of inertia coincide with those of the orbital frame. At the same time, a topical problem is to stabilize a satellite in indirect equilibrium positions, which is fundamentally more complex since in such positions, the significant gravitational torque becomes a disturbance. The problem of stabilizing an Earth-surveying satellite in an oblique position in the orbital frame in a contingency emergency mode was solved in [17]; in particular, the issues of digital control of the magnetic actuator at all transition stages of the satellite attitude control system to the emergency mode and subsequent long-term maintenance of this mode were studied in detail.

Also, there are various known methods for implementing passive or combined satellite attitude stabilization systems without propellant or energy consumption (or with small consumption), as they involve gravitational and aerodynamic torques simultaneously [18–20].

Such a satellite attitude stabilization approach naturally leads to the implementation of indirect equilibrium positions of the satellite in the orbital frame as operation modes. In the case, the problems solved include selecting the most suitable nominal attitude modes, avoiding undesirable resonance effects [21], as well as stabilizing the chosen nominal modes using additional means such as flywheels and active magnetic systems [22, 23]. Consequently, other methods involving the Earth's magnetic field may also be useful. The problem of stabilizing a satellite in indirect equilibrium positions using an EDCS was considered in the paper [24]. The control laws were constructed in accordance with the classical approach, i.e., creating restoring, dissipative, and compensating torques in the control system [25]. Within this approach, the dissipative torque is generated as a linear function of the satellite's relative angular velocity.

In this paper, we propose two modifications of the dissipative torque and their adaptive combination. The high performance of the combined method for forming the dissipative torque is demonstrated. No constraints are imposed on the choice of a particular geomagnetic field model or the satellite's orbit inclination. The study is based on a nonlinear mathematical model, employing the Lyapunov function method to prove the asymptotic stability of the program motion.

2. ELECTRODYNAMIC CONTROL OF SATELLITE'S ANGULAR MOTION

2.1. Satellite's Program Motion

Consider the motion of a satellite as a rigid body about its center of mass. The satellite moves in a Keplerian circular orbit of arbitrary inclination i and radius R in the Earth's magnetic field (EMF). By assumption, the satellite is equipped with a controlled electrostatic charge Q, which is distributed over a certain volume V with a density σ, and a controlled intrinsic magnetic moment $\vec{I}$.

To describe the satellite attitude, we use the following right orthogonal Cartesian frames (Fig. 1).

1) $O_E X^* Y^* Z^*$, the inertial frame where the $O_E X^*$ axis is directed to the ascending node of the orbit and the $O_E Z^*$ axis is along the Earth's diurnal rotation axis.

2) $C\xi\eta\zeta$, the orbital frame with origin at the satellite's center of mass where the $C\xi$ axis (the unit vector $\vec{\xi}_0$) is along the tangent to the orbit in the direction of motion, the $C\eta$ axis (the unit vector $\vec{\eta}_0$) is along the orbit normal, and the $C\zeta$ axis (the unit vector $\vec{\zeta}_0$) is along the local vertical.

3) $Cxyz$, the body-fixed frame whose axes are along the principal central axes of inertia of the satellite.

The relative position of the frames $Cxyz$ and $C\xi\eta\zeta$ is determined by the direction cosine matrix $\mathbf{A} = \{a_{ij}\}_{i,j=1}^{3}$; its elements can be expressed through the "aircraft" angles ψ, θ, and φ according to the following representations of the unit vectors $\vec{\xi}_0$, $\vec{\eta}_0$, and $\vec{\zeta}_0$ of the orbital frame in the

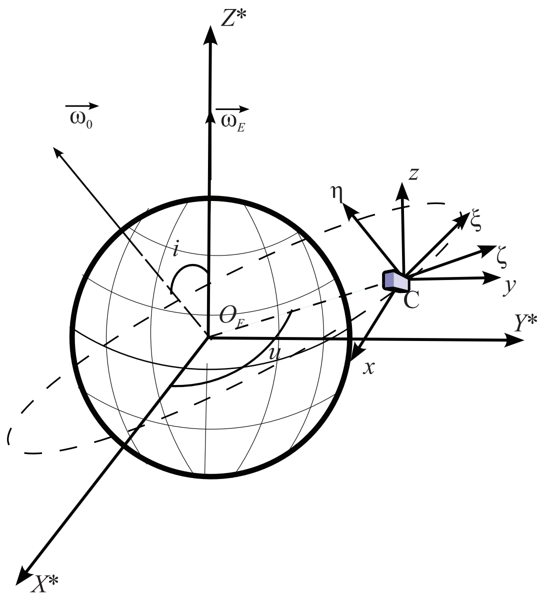

Fig. 1. Frames.

frame $Cxyz$:

$$\vec{\xi}_0 = (\cos\psi\cos\theta,\ \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi,\ \sin\psi\sin\varphi + \cos\psi\sin\theta\cos\varphi)^\top,$$
$$\vec{\eta}_0 = (\sin\psi\cos\theta,\ \cos\psi\cos\varphi + \sin\psi\sin\theta\sin\varphi,\ \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi)^\top,$$
$$\vec{\zeta}_0 = (-\sin\theta,\ \sin\varphi\cos\theta,\ \cos\varphi\cos\theta)^\top.$$

This study takes into account the rotation of the orbital frame relative to the inertial one with the angular velocity $\vec{\omega}_0 = \omega_0\vec{\eta}_0$ and the Earth's rotation with the angular velocity $\vec{\omega}_{\mathrm{E}}$ (see Fig. 1). Therefore, the absolute angular velocity $\vec{\omega}$ of the satellite can be written as

$$\vec{\omega} = \vec{\omega}' + \vec{\omega}_0, \tag{1}$$

where $\vec{\omega}'$ is its angular velocity relative to the orbital frame. The velocity vector $\vec{v}_C$ of the satellite's center of mass relative to the rotating Earth is expanded in terms of the unit vectors $\vec{\xi}_0$, $\vec{\eta}_0$, and $\vec{\zeta}_0$ as follows:

$$\vec{v}_C = \left(R\left(\omega_0 - \omega_{\mathrm{E}}\cos i\right),\ R\omega_{\mathrm{E}}\sin i\,\cos u,\ 0\right)^\top,$$

where $u = \omega_0 t$ is the latitude argument, which will be used below as a dimensionless independent variable.

The mass distribution of the satellite is characterized by the inertia tensor $\mathbf{J} = \mathrm{diag}(A, B, C)$ in the principal central axes of inertia $Cxyz$.

The program attitude of the satellite in the orbital frame is defined by the matrix $\mathbf{A}_0 = \{a_{ij}(\varphi_0, \theta_0, \psi_0)\}_{i,j=1}^{3}$. Accordingly, it is required to design control torques ensuring the existence of an asymptotically stable program motion of the satellite about its center of mass such that

$$\varphi = \varphi_0, \quad \theta = \theta_0, \quad \psi = \psi_0, \quad \vec{\omega}' = \vec{0}. \tag{2}$$

2.2. Control Torques and Controlled Vectors

When the satellite moves relative to the EMF with magnetic induction $\vec{B}$, the Lorentz torque $\vec{M}_L$ [26] and the magnetic interaction torque $\vec{M}_M$ are excited. They have the form

$$\vec{M}_L = \vec{P} \times \vec{T}, \qquad \vec{M}_M = \vec{I} \times \mathbf{A}^\top \vec{B}, \tag{3}$$

where $\vec{P} = Q\vec{\rho}_0$, $\vec{\rho}_0$ is the radius vector of the satellite's charge center relative to its center of mass, and $\vec{T} = \mathbf{A}^\top(\vec{v}_C \times \vec{B})$. From this point onwards, the controlled vectors $\vec{P}$ and $\vec{I}$ are supposed to be given in the frame $Cxyz$. The torques (3) are the control ones in the electrodynamic control system for the satellite's angular motion.

Note that for a given satellite orbit, the magnetic induction $\vec{B}$ in (3) is a known function of the coordinates of a near-Earth space point. Assume that the EMF is uniform in the part of space occupied by the satellite, and the vector $\vec{B} = B_\xi\vec{\xi}_0 + B_\eta\vec{\eta}_0 + B_\zeta\vec{\zeta}_0$ takes the same value at points of this space as at the satellite's center of mass (point C).

In the program mode (2) of the satellite's rotational motion, the vectors $\vec{T}$ and $\vec{B}$ take the form

$$\vec{T}_0 = \mathbf{A}_0^\top(\vec{v}_C \times \vec{B}), \qquad \vec{B}_0 = \mathbf{A}_0^\top\vec{B}.$$

In the electrodynamic control system, the variation laws of the controlled vectors $\vec{P}$ and $\vec{I}$ must be chosen to implement the program mode of the satellite's rotational motion. Let us design each of them as a sum of the restoring, dissipative, and compensating components:

$$\vec{P} = \vec{P}_{rest} + \vec{P}_{diss} + \vec{P}_{comp}\,, \qquad \vec{I} = \vec{I}_{rest} + \vec{I}_{diss} + \vec{I}_{comp}. \tag{4}$$

In view of (4), the control torques (3) become

$$\begin{aligned} \vec{M}_L &= (\vec{P}_{rest} + \vec{P}_{diss} + \vec{P}_{comp}) \times \vec{T}, \\ \vec{M}_M &= (\vec{I}_{rest} + \vec{I}_{diss} + \vec{I}_{comp}) \times \mathbf{A}^\top\vec{B}. \end{aligned} \tag{5}$$

3. MATHEMATICAL MODEL

3.1. The System of Equations Describing Satellite's Motion

Under the gravitational (disturbing) torque and the control torques (3), the motion of the satellite (a rigid body) about its center of mass obeys the dynamic Euler equations

$$\frac{d}{dt}(\mathbf{J}\vec{\omega}) + \vec{\omega} \times (\mathbf{J}\vec{\omega}) = \vec{M}_{GR} + \vec{M}_L + \vec{M}_M, \tag{6}$$

where $\vec{M}_{GR} = 3\omega_0^2(\mathbf{A}^\top\vec{\zeta}_0) \times (\mathbf{J}\mathbf{A}^\top\vec{\zeta}_0)$ is the gravitational torque [1].

Substituting the absolute angular velocity (1), the gravitational torque, and the control torques into (6) yields

$$\begin{aligned} \frac{d}{dt}\left[\mathbf{J}\left(\vec{\omega}_0\mathbf{A}^\top\vec{\eta}_0 + \vec{\omega}'\right)\right] &+ \left[\vec{\omega}_0\mathbf{A}^\top\vec{\eta}_0 + \vec{\omega}'\right] \times \left[\mathbf{J}\left(\vec{\omega}_0\mathbf{A}^\top\vec{\eta}_0 + \vec{\omega}'\right)\right] \\ &= 3\omega_0^2(\mathbf{A}^\top\vec{\zeta}_0) \times (\mathbf{J}\mathbf{A}^\top\vec{\zeta}_0) + (\vec{P}_{rest} + \vec{P}_{diss} + \vec{P}_{comp}) \times \vec{T} \\ &\quad + (\vec{I}_{rest} + \vec{I}_{diss} + \vec{I}_{comp}) \times \mathbf{A}^\top\vec{B}. \end{aligned} \tag{7}$$

To close the system of differential equations (7), we consider them together with the kinematic Poisson equations

$$\frac{d\vec{\xi}_0}{dt} = \vec{\xi}_0 \times \vec{\omega} - \omega_0\vec{\zeta}_0, \quad \frac{d\vec{\eta}_0}{dt} = \vec{\eta}_0 \times \vec{\omega}, \quad \frac{d\vec{\zeta}_0}{dt} = \vec{\zeta}_0 \times \vec{\omega} + \omega_0\vec{\xi}_0. \tag{8}$$

3.2. Disturbing Torque Compensation

The program mode (2) of the satellite's angular motion is generally not a direct equilibrium position of the satellite in the orbital frame. Therefore, the gravitational torque and the expression $\omega_0^2(\mathbf{A}^\top\vec{\eta}_0)\times(\mathbf{J}\mathbf{A}^\top\vec{\eta}_0)$ from equations (7) do not vanish in the satellite's program motion. To compensate for the total disturbing torque

$$\vec{M}_d = \omega_0^2\left(3(\mathbf{A}^\top\vec{\zeta}_0)\times(\mathbf{J}\mathbf{A}^\top\vec{\zeta}_0) - (\mathbf{A}^\top\vec{\eta}_0)\times(\mathbf{J}\mathbf{A}^\top\vec{\eta}_0)\right),$$

we will utilize an approach based on creating the corresponding compensating components of the electrodynamic parameters [27] to ensure the equality $\vec{M}_d + \vec{P}_{comp}\times\vec{T} + \vec{I}_{comp}\times(\mathbf{A}^\top\vec{B}) = \vec{0}$. The restoring and compensating components of the controlled vectors $\vec{P}$ and $\vec{I}$ for implementing electrodynamic control to stabilize the program motion of the satellite are given by

$$\vec{P}_{rest} = Qk_L\vec{T}_0, \quad \vec{P}_{comp} = \frac{\vec{M}_d\cdot(\mathbf{A}^\top\vec{B})}{|\vec{B}|\,|\vec{T}|}\left(\mathbf{V}^\top(0,0,1)^\top\right),$$

$$\vec{I}_{rest} = k_M\vec{B}_0, \quad \vec{I}_{comp} = \mathbf{V}^\top\left(0, \frac{\vec{M}_d\cdot\left(\left(\mathbf{A}^\top\vec{B}\right)\times\vec{T}\right)}{|\vec{B}|^2\,|\vec{T}|}, -\frac{\vec{M}_d\cdot\vec{T}}{|\vec{B}|\,|\vec{T}|}\right)^\top.$$

Here, k_L and k_M are tunable control parameters (constants or functions of time), and $\mathbf{V} = \{v_{ij}\}_{i,j=1}^3$ is a known matrix with the elements

$$v_{11} = B_x/|\vec{B}|, \quad v_{12} = B_y/|\vec{B}|, \quad v_{13} = B_z/|\vec{B}|,$$

$$v_{21} = T_x/|\vec{T}|, \quad v_{22} = T_y/|\vec{T}|, \quad v_{23} = T_z/|\vec{T}|,$$

$$v_{31} = \frac{B_yT_z - B_zT_y}{|\vec{B}||\vec{T}|}, \quad v_{32} = \frac{B_zT_x - B_xT_z}{|\vec{B}||\vec{T}|}, \quad v_{33} = \frac{B_xT_y - B_yT_x}{|\vec{B}||\vec{T}|}.$$

The choice of the dissipative components $\vec{P}_{diss}$ and $\vec{I}_{diss}$ of the controlled vectors will be justified below.

3.3. Decreasing the Satellite's Relative Angular Velocity

Satellite attitude stabilization implies not only achieving the desired orientation but also ensuring its asymptotic stability. For this purpose, dissipative components are provided in the controlled vectors (4); they can be chosen in various ways. For example, numerous passive devices, including well-established composite schemes in satellites like Vertistat, spherical magnetic dampers (both fluid and eddy-current), and hysteresis rods, generate dissipative torques proportional to the satellite's angular velocity (see [28] and the references therein).

In electrodynamic control systems, dissipative torques are typically created using semi-passive methods. In addition to linear control solutions [15, 29], nonlinear controls with respect to angular velocity are effective in many cases [30].

In control of mechanical systems, as well as in satellite angular motion control, angular momentum-based control laws are widely used [31–34]. In particular, a well-known satellite stabilization solution involves damping proportional to the satellite's angular momentum [6].

In previous works, we applied dissipative components with the control parameters h_L and h_M of the form

$$\vec{P}_{diss} = Qh_L\vec{\omega}'\times\vec{T}, \quad \vec{I}_{diss} = h_M\vec{\omega}'\times(\mathbf{A}^\top\vec{B}). \tag{9}$$

In this study, along with formulas (9), we propose three new variants of forming dissipative components. In the first variant, the relative angular velocity is replaced by $\vec{K} = \mathbf{J}\vec{\omega}'$, the angular momentum of the satellite in relative motion:

$$\vec{P}_{diss} = Qh_L \left(A^{-1}\mathbf{J}\vec{\omega}'\right) \times \vec{T}; \quad \vec{I}_{diss} = h_M \left(A^{-1}\mathbf{J}\vec{\omega}'\right) \times (\mathbf{A}^\top \vec{B}). \tag{10}$$

In the second variant of forming the dissipative component of the controlled vectors, the unit vector of $\vec{K}$ appears instead of $\vec{K}$:

$$\vec{P}_{diss} = \frac{Qh_L}{|\vec{K}|} \left(A^{-1}\mathbf{J}\vec{\omega}'\right) \times \vec{T}; \quad \vec{I}_{diss} = \frac{h_M}{|\vec{K}|} \left(A^{-1}\mathbf{J}\vec{\omega}'\right) \times (\mathbf{A}^\top \vec{B}). \tag{11}$$

Here, stabilization will be achieved formally over an infinite time. Therefore, to implement the corresponding control algorithm, the norm of the vector $\vec{K}$ near zero is set equal to a given small constant Δ, as soon as $|\vec{K}|$ decreases to this constant. In other words, we introduce the modified norm

$$|\vec{K}|_{mod} = \frac{|\Delta - |\vec{K}|| + \Delta + |\vec{K}|}{2}. \tag{12}$$

The third variant is a combination of (10) and (11); it will be justified in Section 5.

Note that compared to (9), the new variants (10) and (11) of forming the dissipative components and their combination do not require additional measurement devices in the control system.

By substituting (5) into (7), we obtain the following system of differential equations, to be considered together with (8):

$$\begin{aligned} &\frac{d}{dt}\left[\mathbf{J}\left(\vec{\omega}' + \omega_0\vec{\eta}_0\right)\right] + (\vec{\omega}' + \omega_0\vec{\eta}_0) \times (\mathbf{J}\vec{\omega}') + \omega_0\vec{\omega}' \times (\mathbf{J}\vec{\eta}_0) \\ &= Qk_L\vec{T}_0 \times \vec{T} + \vec{P}_{diss} \times \vec{T} + k_M\vec{B}_0 \times \left(\mathbf{A}^\top\vec{B}\right) + \vec{I}_{diss} \times \left(\mathbf{A}^\top\vec{B}\right). \end{aligned} \tag{13}$$

Thus, the problem is to determine conditions on the parameters k_L, k_M, h_L, and h_M ensuring the asymptotic stability of the program motion (2).

4. MOTION STABILITY ANALYSIS

Let us derive asymptotic stability conditions for the program motion (2) using the variant (10) as more general compared to (9). Assuming the smallness of the angles $\varphi_1 = \varphi - \varphi_0$, $\theta_1 = \theta - \theta_0$, and $\psi_1 = \psi - \psi_0$ and their time derivatives, we expand the control torques on the right-hand side of equation (13) into power series in these small variables, discarding all terms of degree above 1, similar to the approach used in [24, 35]:

$$\begin{aligned} M^{(1)}_{L\nu_m} &= l_{m1}(t)(k_L\varphi_1 + h_L\dot{\varphi}_1) + l_{m2}(t)(k_L\theta_1 + \delta h_L\dot{\theta}_1) + l_{m3}(t)(k_L\psi_1 + \varepsilon h_L\dot{\psi}_1), \\ M^{(1)}_{M\nu_m} &= b_{m1}(t)(k_M\varphi_1 + h_M\dot{\varphi}_1) + b_{m2}(t)(k_M\theta_1 + \delta h_M\dot{\theta}_1) + b_{m3}(t)(k_M\psi_1 + \varepsilon h_M\dot{\psi}_1). \end{aligned}$$

Here $m = 1, 2, 3$, $\nu_1 = x$, $\nu_2 = y$, $\nu_3 = z$, $\delta = B/A$, and $\varepsilon = C/A$. We compile the symmetric matrices $\mathbf{S}_L(t) = \{l_{ij}\}_{i,j=1}^3$ and $\mathbf{S}_M(t) = \{b_{ij}\}_{i,j=1}^3$ with the elements l_{ij} and b_{ij}, respectively (see the Appendix). Then equations (13) turn into the differential equations of the perturbed motion in the three-axis satellite attitude stabilization problem:

$$\mathbf{J}\ddot{\vec{y}} + \mathbf{D}(t)\dot{\vec{y}} + \mathbf{G}\dot{\vec{y}} + \mathbf{H}(t)\vec{y} + \vec{F}(t, \vec{y}, \dot{\vec{y}}) = \vec{0}, \tag{14}$$

where $\vec{y}=\begin{pmatrix}\varphi_1\\ \theta_1\\ \psi_1\end{pmatrix}$, $\mathbf{G}=\omega_0\left(A-B+C\right)\begin{pmatrix}0&0&1\\0&0&0\\-1&0&0\end{pmatrix}$ is a constant matrix, $\mathbf{D}(t)=-A^{-1}\mathbf{J}\left(h_L\mathbf{S}_L(t)+h_M\mathbf{S}_M(t)\right)$ and $\mathbf{H}(t)=-\left(k_L\mathbf{S}_L(t)+k_M\mathbf{S}_M(t)\right)$ are matrices with time-varying coefficients, and the vector $\vec{F}$ nonlinearly depends on $\vec{y}$ and $\dot{\vec{y}}$.

Let us introduce the control parameters k_{L0}, k_{M0}, h_{L0}, and h_{M0} (positive constants) as follows:

$$k_{L0}=k_LQ|\vec{T}(t)|^2,\quad k_{M0}=k_M|\vec{B}(t)|^2,\quad h_{L0}=h_LQ|\vec{T}(t)|^2,\quad h_{M0}=h_M|\vec{B}(t)|^2.$$

Denoting their ratios by $\varepsilon_1=k_{L0}/k_{M0}$, $\varepsilon_2=h_{L0}/h_{M0}$, and $\mu=h_{M0}/k_{M0}$, we obtain

$$\mathbf{D}(t)=-\mu k_{M0}\left(A^{-1}\mathbf{J}\mathbf{U}_M(t)+\varepsilon_2A^{-1}\mathbf{J}\mathbf{U}_L(t)\right),$$
$$\mathbf{H}(t)=-k_{M0}\left(\mathbf{U}_M(t)+\varepsilon_1\mathbf{U}_L(t)\right),$$

where

$$\mathbf{U}_M(t)=\frac{\mathbf{S}_M(t)}{|\vec{B}(t)|^2},\quad \mathbf{U}_L(t)=\frac{\mathbf{S}_L(t)}{Q|\vec{T}(t)|^2}.$$

The Lyapunov function can be chosen as a quadratic form with a constant $\chi>0$:

$$V(t,\vec{y},\dot{\vec{y}})=\frac{1}{2}\vec{y}^\top\mathbf{D}(t)\vec{y}+\frac{\chi}{2}\dot{\vec{y}}^\top\mathbf{J}\dot{\vec{y}}+\vec{y}^\top\mathbf{J}\dot{\vec{y}}+\frac{\chi}{2}\vec{y}^\top\mathbf{H}(t)\vec{y}. \tag{15}$$

The time derivative of the function (15) along the trajectories of system (14) is given by

$$\begin{aligned}\dot{V}(t,\vec{y},\dot{\vec{y}})|_{(14)}&=-\chi\dot{\vec{y}}^\top\mathbf{D}(t)\dot{\vec{y}}-\vec{y}^\top\mathbf{G}\dot{\vec{y}}-\vec{y}^\top\mathbf{H}(t)\vec{y}-\chi\dot{\vec{y}}^\top\mathbf{G}\dot{\vec{y}}\\&+\frac{1}{2}\vec{y}^\top\dot{\mathbf{D}}(t)\vec{y}+\dot{\vec{y}}^\top\mathbf{J}\dot{\vec{y}}+\frac{\chi}{2}\vec{y}^\top\dot{\mathbf{H}}(t)\vec{y}-\left(\chi\dot{\vec{y}}+\vec{y}\right)^\top\vec{F}(t,\vec{y},\dot{\vec{y}}).\end{aligned} \tag{16}$$

For any vectors $\vec{x},\vec{z}\in\mathbb{R}^3$, the quadratic forms in the expressions (15) and (16) can be estimated as [36, 37]

$$\begin{aligned}&\vec{x}^\top\mathbf{D}(t)\vec{x}\geqslant\mu k_{M0}\mathrm{inf}\lambda_{\min}\left(-A^{-1}\mathbf{J}\mathbf{U}_M(t)-\varepsilon_2A^{-1}\mathbf{J}\mathbf{U}_L(t)\right)\|\vec{x}\|^2,\\&\vec{x}^\top\mathbf{H}(t)\vec{x}\geqslant k_{M0}\mathrm{inf}\lambda_{\min}\left(-\mathbf{U}_M(t)-\varepsilon_1\mathbf{U}_L(t)\right)\|\vec{x}\|^2,\\&\min\{A,B,C\}\|\vec{x}\|^2\leqslant\vec{x}^\top\mathbf{J}\vec{x}\leqslant\max\{A,B,C\}\|\vec{x}\|^2,\\&|\vec{x}^\top\mathbf{J}\vec{z}|\leqslant\max\{A,B,C\}\|\vec{x}\|\|\vec{z}\|,\\&\vec{x}^\top\dot{\mathbf{D}}(t)\vec{x}\leqslant\mu k_{M0}\mathrm{sup}\lambda_{\max}\left(-A^{-1}\mathbf{J}\dot{\mathbf{U}}_M(t)-\varepsilon_2A^{-1}\mathbf{J}\dot{\mathbf{U}}_L(t)\right)\|\vec{x}\|^2,\\&\vec{x}^\top\dot{\mathbf{H}}(t)\vec{x}\leqslant k_{M0}\mathrm{sup}\lambda_{\max}\left(-\dot{\mathbf{U}}_M(t)-\varepsilon_1\dot{\mathbf{U}}_L(t)\right)\|\vec{x}\|^2,\\&|\vec{x}^\top\mathbf{G}\vec{z}|\leqslant\omega_0\left(A-B+C\right)\|\vec{x}\|\|\vec{z}\|,\quad\vec{x}^\top\mathbf{G}\vec{x}\leqslant\omega_0\left(A-B+C\right)\|\vec{x}\|^2,\end{aligned} \tag{17}$$

where $\lambda_{\min}(\mathbf{M})$ and $\lambda_{\max}(\mathbf{M})$ indicate the largest and smallest eigenvalues, respectively, of a matrix $\mathbf{M}$, and the operations sup and inf are performed for all $t\in\mathbb{R}$. In addition, we denote

$$\begin{gathered}d_1=\min\{A,B,C\},\quad d_2=\max\{A,B,C\},\quad d_3=A-B+C,\\d_{41}=\mathrm{inf}\lambda_{\min}\left[-A^{-1}\mathbf{J}\left(\mathbf{U}_M(t)+\varepsilon_2\mathbf{U}_L(t)\right)\right],\ d_{42}=\mathrm{inf}\lambda_{\min}\left(-\mathbf{U}_M(t)-\varepsilon_1\mathbf{U}_L(t)\right),\\d_{51}=\mathrm{sup}\lambda_{\max}\left[-A^{-1}\mathbf{J}\left(\dot{\mathbf{U}}_M(t)+\varepsilon_2\dot{\mathbf{U}}_L(t)\right)\right],\ d_{52}=\mathrm{sup}\lambda_{\max}\left(-\dot{\mathbf{U}}_M(t)-\varepsilon_1\dot{\mathbf{U}}_L(t)\right).\end{gathered} \tag{18}$$

According to (17) and (18), the Lyapunov function satisfies the lower bound

$$V(t,\vec{y},\dot{\vec{y}}) \geqslant \frac{1}{2}\|\vec{y}\|^2\left(\mu k_{M0}d_{41}+\chi k_{M0}d_{42}-d_2a_1\omega_0^2\right)+\frac{1}{2}\|\dot{\vec{y}}\|^2\left(d_1\chi-\frac{d_2}{a_1\omega_0^2}\right),$$

where $a_1 > 0$ is some constant.

Hence, if

$$\chi > \frac{d_2}{d_1a_1\omega_0^2}, \quad \chi > \frac{d_2a_1\omega_0^2-\mu k_{M0}d_{41}}{k_{M0}d_{42}}, \tag{19}$$

the Lyapunov function (15) will be positive definite.

Furthermore, the numbers $c > 0$ and $\delta_0 > 0$ can be assigned so that

$$\dot{V}(t,\vec{y},\dot{\vec{y}})|_{(14)} \leqslant \frac{1}{2}\|\vec{y}\|^2\left(a_2\omega_0^2d_3+\mu k_{M0}d_{51}+\chi k_{M0}d_{52}-2k_{M0}d_{42}\right)$$
$$+\|\dot{\vec{y}}\|^2\left(d_2+\frac{d_3}{2a_2}-\chi\mu k_{M0}d_{41}\right)+c\left(\chi\|\dot{\vec{y}}\|+\|\vec{y}\|\right)\left(\|\dot{\vec{y}}\|^2+\|\vec{y}\|^2\right)$$

for $t \geqslant 0$, $\|\vec{y}\| < \delta_0$, and $\|\dot{\vec{y}}\| < \delta_0$, where $a_2 > 0$ is some constant.

Thus, under the conditions

$$k_{M0} > \frac{a_2\omega_0^2d_3}{2d_{42}-\mu d_{51}-\chi d_{52}}, \quad \chi > \frac{d_2+d_3/(2a_2)}{\mu k_{M0}d_{41}}, \quad \chi < \frac{2d_{42}-\mu d_{51}}{d_{52}}, \tag{20}$$

the derivative of the function (16) along the trajectories of system (14) is negative definite.

Theorem 1. *Assume that for given satellite parameters A, B, C, and Q, orbit parameters R and i, and control parameters k_{L0}, k_{M0}, h_{L0}, and h_{M0}, there exist positive numbers a_1, a_2, and χ satisfying inequalities* (19) *and* (20). *Then the program motion* (2) *of system* (7), (8) *is asymptotically stable.*

The proof of this result is provided in the Appendix.

5. NUMERICAL EXPERIMENTS

To assess the effect of the above modifications of the dissipative torque on the stabilization process of the satellite's angular motion, we conducted a series of numerical experiments for the following parameters: $A = 1000\,\text{kg}\times\text{m}^2$, $B = 1300\,\text{kg}\times\text{m}^2$, $C = 700\,\text{kg}\times\text{m}^2$, and $Q = 15\times10^{-3}\,\text{C}$ (the satellite); $R = 7\times10^6\,\text{m}$ and $i = 30^\circ$ (the orbit); $k_{L0} = 2.5\times10^{-3}\,\text{N}\times\text{m}$, $k_{M0} = 2\times10^{-3}\,\text{N}\times\text{m}$, $h_{L0} = 0.2\,\text{N}\times\text{m}\times\text{s}$, and $h_{M0} = 1.0\,\text{N}\times\text{m}\times\text{s}$ (the control parameters).

The desired satellite attitude was given by the angles $\varphi_0 = 0.2\,\text{rad}$, $\psi_0 = 0.4\,\text{rad}$, and $\theta_0 = 0.6\,\text{rad}$.

In computer simulations, the geomagnetic field was modeled using an octupole approximation, and the Gaussian coefficients were determined according to the IGRF model of the 13th generation [38]. Note that for the electrodynamic control method proposed, the model's order of approximation is not limited: this method can be used with a multipole IGRF model of any order.

For the selected satellite, orbit, and control parameters satisfying the sufficient asymptotic stability conditions, the nonlinear differential equations (7) and (8) were integrated numerically.

Figures 2–5 show the satellite stabilization process for the following initial conditions: $\varphi(0) = -0.2\,\text{rad}$, $\psi(0) = 0.0\,\text{rad}$, $\theta(0) = 0.1\,\text{rad}$, $\Omega_x(0) = \omega_x(0)/\omega_0 = 0.3$, $\Omega_y(0) = \omega_y(0)/\omega_0 = 1.1$, and $\Omega_z(0) = \omega_z(0)/\omega_0 = 0.5$. In all figures, the horizontal axes represent the dimensionless variable u, for which $u = 5$ corresponds to $t = 4638.22\,\text{s}$ or approximately 1 h 17 min. Next, Fig. 2 shows the stabilization process under the modified (10) and original (9) dissipative components of the controlled vectors.

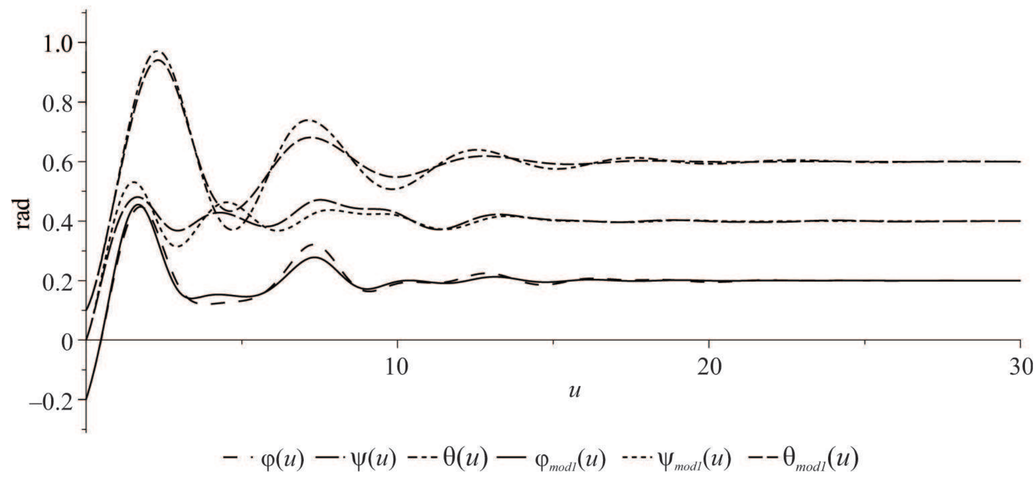

Fig. 2. The dynamics of the satellite attitude angles $\varphi(u)$, $\psi(u)$, and $\theta(u)$ under the variant (9) and $\varphi_{\text{mod1}}(u)$, $\psi_{\text{mod1}}(u)$, and $\theta_{\text{mod1}}(u)$ under the variant (10).

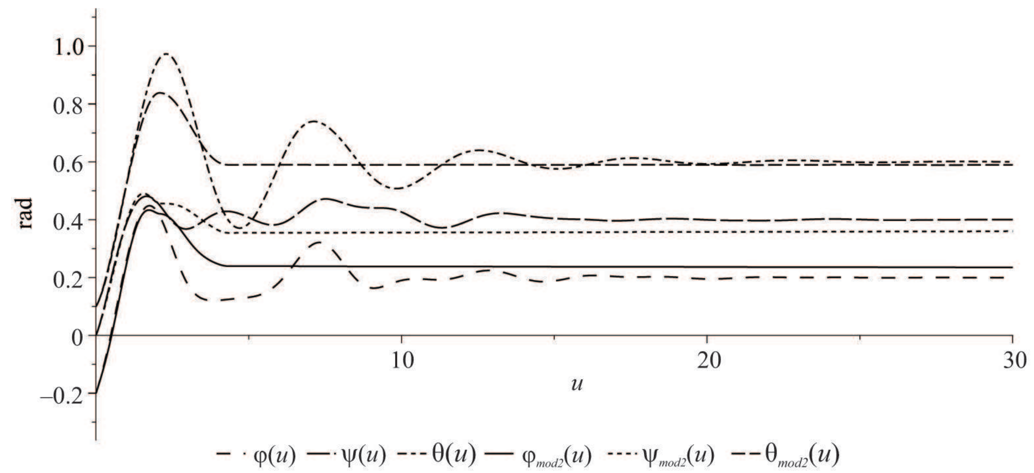

Fig. 3. The dynamics of the satellite attitude angles $\varphi(u)$, $\psi(u)$, and $\theta(u)$ under the variant (9) and $\varphi_{\text{mod2}}(u)$, $\psi_{\text{mod2}}(u)$, and $\theta_{\text{mod2}}(u)$ under the variant (11).

By analogy, Fig. 3 presents the stabilization process under the modified (11) and original (9) dissipative components of the controlled vectors. The value $\Delta = 0.001$ was adopted for the numerical integration of the differential equations in (12).

Note the smoother satellite stabilization transients here. However, the desired attitude was achieved in a rather long time: starting from a certain time instant (Fig. 4), $|\vec{K}|$ became small, leading to a small denominator in formula (11) and a subsequent reduction in control performance.

To overcome this drawback, the approaches (10) and (11) were combined. Figure 5 shows the satellite stabilization process under the combination of the modified dissipative components (10) and (11) of the controlled vectors.

Thus, with combining the variants of forming the dissipative components of the controlled vectors (4), it is possible to stabilize the satellite in the orbital frame in the indirect equilibrium position (2) (in the general case) and, moreover, to obtain relatively smoother and faster-converging transients of the satellite stabilization process using the available control actuators.

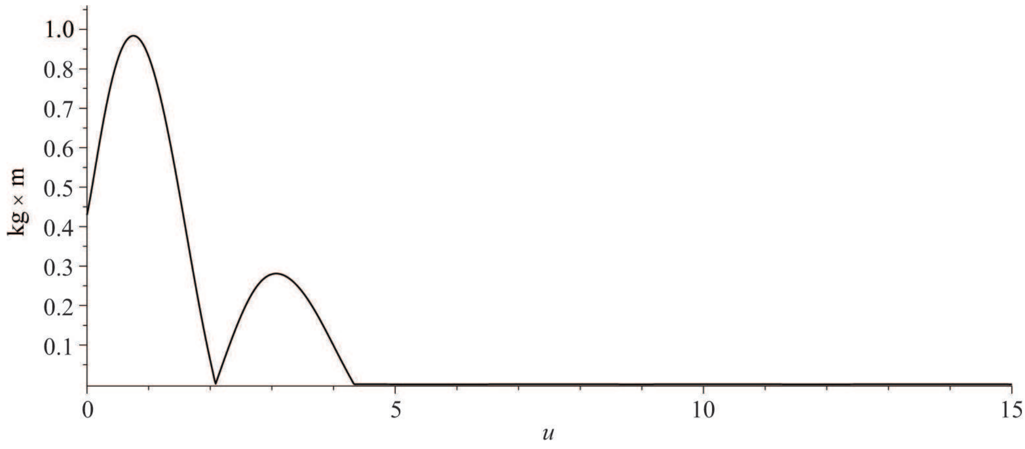

Fig. 4. The magnitude of the vector $\vec{K}$ under the variant (11).

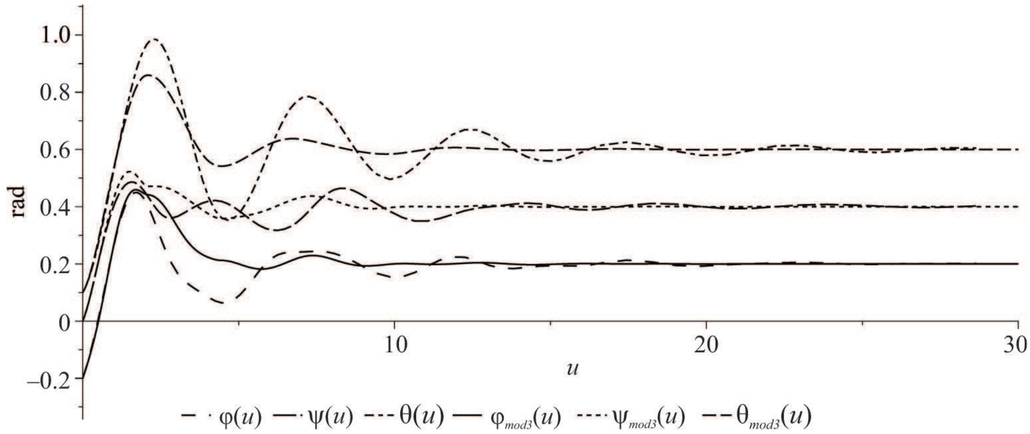

Fig. 5. The dynamics of the satellite attitude angles $\varphi(u)$, $\psi(u)$, and $\theta(u)$ under the variant (9) and $\varphi_{\text{mod3}}(u)$, $\psi_{\text{mod3}}(u)$, and $\theta_{\text{mod3}}(u)$ under the combination of the variants (10) and (11).

During the numerical experiments, we also analyzed the realizability of the control law in terms of the bounded magnitudes of the controlled vectors (4). As established, with this choice of control parameters, the control torques (3) do not exceed the gravitational torque by the order of magnitude.

6. CONCLUSIONS

This paper has addressed the electrodynamic control problem for the angular motion of a charged satellite with an intrinsic magnetic moment, as well as its stabilization problem in the orbital frame in an arbitrary position. Three new variants of forming the dissipative torque have been proposed, which can be implemented using the available control actuators. The first two variants, considered separately, suffer from some drawbacks, which can be eliminated by combining them into a uniform system for forming the dissipative torque (the third variant). Such combined electrodynamic control possesses the advantages of the first and second variants, i.e., reduces the stabilization time and decreases the amplitude of transient oscillations. These performance characteristics are of paramount importance when the satellite carries equipment or devices sensitive to oscillations. The results of numerical experiments have demonstrated the effectiveness and performance of the modified satellite attitude control method.

Using the Lyapunov function method, sufficient stability conditions for the program motion of the satellite have been obtained and expressed explicitly in terms of the satellite, orbit, and control parameters.

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APPENDIX

The symmetric matrices $\mathbf{S}_L(t) = \{l_{ij}\}_{i,j=1}^3$ and $\mathbf{S}_M(t) = \{b_{ij}\}_{i,j=1}^3$ with the elements l_{ij} and b_{ij} (see Section 4) have the following form:

$$
\begin{aligned}
l_{11}(t) &= Q(-B_\zeta(\sin\psi_0 v_{C\xi} - \cos\psi_0 v_{C\eta})(-B_\eta v_{C\xi} + B_\xi v_{C\eta})\sin\theta_0 - B_\zeta^2 v_{C\xi}^2\cos\psi_0 - B_\zeta^2 v_{C\xi}v_{C\eta}\sin\psi_0 \\
&\quad - \cos\theta_0(-B_\eta v_{C\xi} + B_\xi v_{C\eta})^2)\cos\varphi_0 \\
&\quad - B_\zeta(B_\zeta v_{C\xi}(\sin\psi_0 v_{C\xi} - \cos\psi_0 v_{C\eta})\sin\theta_0 \\
&\quad + (B_\xi v_{C\eta} - B_\eta v_{C\xi})(\cos\theta_0 v_{C\xi} - \cos\psi_0 v_{C\xi} - \sin\psi_0 v_{C\eta}))\sin\varphi_0, \\
l_{12}(t) &= l_{21}(t) = Qv_{C\eta}(((B_\eta v_{C\xi} - B_\xi v_{C\eta})\cos\theta_0 + B_\zeta\sin\theta_0(\cos\psi_0 v_{C\eta} \\
&\quad - \sin\psi_0 v_{C\xi}))\sin\varphi_0 - B_\zeta\cos\varphi_0(\cos\psi_0 v_{C\xi} + \sin\psi_0 v_{C\eta}))B_\zeta, \\
l_{13}(t) &= l_{31}(t) = Qv_{C\eta}(((B_\eta v_{C\xi} - B_\xi v_{C\eta})\cos\theta_0 + B_\zeta\sin\theta_0(\cos\psi_0 v_{C\eta} \\
&\quad - \sin\psi_0 v_{C\xi}))\cos\varphi_0 + B_\zeta\sin\varphi_0(\cos\psi_0 v_{C\xi} + \sin\psi_0 v_{C\eta}))B_\zeta; \\
l_{22}(t) &= -Q(-(B_\xi v_{C\eta} - B_\eta v_{C\xi})(B_\zeta\sin\theta_0(-\sin\psi_0 v_{C\xi} + \cos\psi_0 v_{C\eta}) - \cos\theta_0(B_\xi v_{C\eta} - B_\eta v_{C\xi}))\cos\varphi_0 \\
&\quad + ((B_\xi v_{C\eta}^2 - B_\eta v_{C\eta}v_{C\xi})\sin\theta_0 + B_\zeta v_{C\eta}(\cos\psi_0 v_{C\eta} - \sin\psi_0 v_{C\xi})\cos\theta_0 \\
&\quad - \sin\varphi_0(-B_\eta v_{C\xi} + B_\xi v_{C\eta})(\cos\psi_0 v_{C\xi} + \sin\psi_0 v_{C\eta}))B_\zeta), \\
l_{23}(t) &= l_{32}(t) = -Qv_{C\xi}(((B_\eta v_{C\xi} - B_\xi v_{C\eta})\cos\theta_0 \\
&\quad + B_\zeta\sin\theta_0(\cos\psi_0 v_{C\eta} - \sin\psi_0 v_{C\xi}))\cos\varphi_0 \\
&\quad + B_\zeta\sin\varphi_0(\cos\psi_0 v_{C\xi} + \sin\psi_0 v_{C\eta}))B_\zeta, \\
l_{33}(t) &= -Q(-v_{C\xi}((B_\eta v_{C\xi} - B_\xi v_{C\eta})\cos\theta_0 \\
&\quad + B_\zeta\sin\theta_0(-\sin\psi_0 v_{C\xi} + \cos\psi_0 v_{C\eta}))\sin\varphi_0 \\
&\quad + B_\zeta v_{C\eta}(-\sin\psi_0 v_{C\xi} + \cos\psi_0 v_{C\eta})\cos\theta_0 + (-B_\eta v_{C\eta}v_{C\xi} + B_\xi v_{C\eta}^2)\sin\theta_0 \\
&\quad + B_\zeta\cos\varphi_0 v_{C\xi}(\cos\psi_0 v_{C\xi} + \sin\psi_0 v_{C\eta}))B_\zeta; \\
b_{11}(t) &= (-B_\zeta(B_\xi\cos\psi_0 + B_\eta\sin\psi_0)\sin\theta_0 + B_\xi B_\eta\sin\psi_0 \\
&\quad - B_\eta^2\cos\psi_0 - B_\zeta^2\cos\theta_0)\cos\varphi_0 - ((B_\xi B_\eta\cos\psi_0 + B_\eta^2\sin\psi_0)\sin\theta_0 \\
&\quad + B_\zeta(B_\xi\sin\psi_0 - \cos\psi_0 B_\eta + B_\eta\cos\theta_0))\sin\varphi_0, \\
b_{12}(t) &= b_{21}(t) = ((B_\zeta\cos\theta_0 + \sin\theta_0(B_\xi\cos\psi_0 + B_\eta\sin\psi_0))\sin\varphi_0 \\
&\quad - \cos\varphi_0(B_\xi\sin\psi_0 - \cos\psi_0 B_\eta))B_\xi; \\
b_{13}(t) &= b_{31}(t) = B_\xi((B_\zeta\cos\theta_0 + \sin\theta_0(B_\xi\cos\psi_0 + B_\eta\sin\psi_0))\cos\varphi_0 \\
&\quad + \sin\varphi_0(B_\xi\sin\psi_0 - \cos\psi_0 B_\eta)), \\
b_{22}(t) &= -B_\zeta(B_\zeta\cos\theta_0 + \sin\theta_0(B_\xi\cos\psi_0 + B_\eta\sin\psi_0))\cos\varphi_0 - (B_\xi^2\cos\psi_0 \\
&\quad + B_\xi B_\eta\sin\psi_0)\cos\theta_0 + B_\zeta(\sin\theta_0 B_\xi - \sin\varphi_0(B_\xi\sin\psi_0 - \cos\psi_0 B_\eta)), \\
b_{23}(t) &= b_{32}(t) = B_\eta((B_\zeta\cos\theta_0 + \sin\theta_0(B_\xi\cos\psi_0 + B_\eta\sin\psi_0))\cos\varphi_0 \\
&\quad + \sin\varphi_0(B_\xi\sin\psi_0 - \cos\psi_0 B_\eta)), \\
b_{33}(t) &= -B_\eta(B_\zeta\cos\theta_0 + \sin\theta_0(B_\xi\cos\psi_0 + B_\eta\sin\psi_0))\sin\varphi_0 - (B_\xi^2\cos\psi_0 \\
&\quad + B_\xi B_\eta\sin\psi_0)\cos\theta_0 + \sin\theta_0 B_\xi B_\zeta + \cos\varphi_0 B_\eta(B_\xi\sin\psi_0 - \cos\psi_0 B_\eta).
\end{aligned}
$$

Proof of Theorem 1. Resolving inequalities (19) and (20) with respect to χ, we write them as

$$\begin{aligned}
&1)\quad \chi > \frac{d_2}{d_1 a_1 \omega_0^2},\\
&2)\quad \chi > \frac{d_2 a_1 \omega_0^2 - \mu k_{M0} d_{41}}{k_{M0} d_{42}},\\
&3)\quad \chi < \frac{2k_{M0} d_{42} - \mu k_{M0} d_{51} - a_2 \omega_0^2 d_3}{k_{M0} d_{52}} = \frac{2d_{42} - \mu d_{51}}{d_{52}} - \frac{a_2 \omega_0^2 d_3}{k_{M0} d_{52}},\\
&4)\quad \chi > \frac{d_2 + d_3/(2a_2)}{\mu k_{M0} d_{41}},\\
&5)\quad \chi < \frac{2d_{42} - \mu d_{51}}{d_{52}}.
\end{aligned} \tag{A.1}$$

Inequalities 1), 2), and 4) of system (A.1) define a lower bound for the number χ.

Let us denote $\chi_1 = \frac{d_2}{d_1 a_1 \omega_0^2}$, $\chi_2 = \frac{d_2 a_1 \omega_0^2 - \mu k_{M0} d_{41}}{k_{M0} d_{42}}$, and $\chi_4 = \frac{d_2 + d_3/(2a_2)}{\mu k_{M0} d_{41}}$. We emphasize that χ_1 and χ_2 depend on the parameter a_1, and as a_1 increases, $\chi_1(a_1)$ decreases hyperbolically whereas $\chi_2(a_1)$ increases linearly. Thus, to minimize $\max\{\chi_1, \chi_2\}$, the parameter a_1^* is chosen so that $\chi_1(a_1^*) = \chi_2(a_1^*)$.

Inequalities 3) and 5) of system (A.1) define an upper bound for the admissible values of the parameter χ. If $\frac{a_2 \omega_0^2 d_3}{k_{M0} d_{52}} > 0$, then inequality 5) follows from inequality 3). Let the desired bound be denoted by $\chi_3 = \frac{2k_{M0} d_{42} - \mu k_{M0} d_{51} - a_2 \omega_0^2 d_3}{k_{M0} d_{52}}$. Consequently, the system of inequalities (A.1) gets simplified to

$$\max\{\chi_1(a_1^*), \chi_4\} < \chi_3.$$

Similarly, we emphasize that χ_3 and χ_4 depend on the parameter a_2. Here, χ_3 decreases linearly whereas χ_4 hyperbolically. To determine χ, it is necessary to find a nonempty interval $\left[a_2^{(1)}, a_2^{(2)}\right]$ where the inequality $\chi_4 < \chi_3$ holds for every point inside it. From the equation $\chi_3(a_2) = \chi_4(a_2)$ we find the roots $a_2^{(1)}$ and $a_2^{(2)}$.

If the length of the desired interval $\left[a_2^{(1)}, a_2^{(2)}\right]$ tends to zero, it shrinks to a point a_2^*, and the value of a_2^* can be determined from the equality of the slopes of the line $\chi_3(a_2)$ and the tangent to the curve $\chi_4(a_2)$. Consequently, $a_2^* = \sqrt{d_{52}/(2\mu \omega_0^2 d_{41})}$. Such a choice of the constant a_2 can be recommended for numerical analysis of the sufficient asymptotic stability conditions. Finally, the system of inequalities (A.1) reduces to

$$\max\{\chi_1(a_1^*), \chi_4(a_2^*)\} < \chi_3(a_2^*). \tag{A.2}$$

If inequality (A.2) fails because $\chi_1(a_1^*) > \chi_3(a_2^*)$, then one can choose an appropriate value of a_2 by reducing it to $a_2^{(1)}$.

The proof of Theorem 1 is complete.

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