*ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2026, Vol. 87, No. 3, pp. 218–226.*

**ROBUST, ADAPTIVE, AND NETWORK CONTROL**

# Optimal Spanning Trees in Fault Diagnosis and Design of Fuzzy Devices

**D. V. Speranskii**

*Russian University of Transport, Moscow, Russia*
*e-mail: Speranskiy.dv@gmail.com*

Received September 16, 2023
Revised November 28, 2025
Accepted December 30, 2025

**Abstract**—The realizability of a fuzzy graph is conceptualized, and two optimization problems are formulated to construct optimal spanning trees for fuzzy graphs. The optimality criteria are graph realizability and tree minimality (by length), with realizability taken into account. Being multicriteria, these problems fundamentally differ from the analogous problem for crisp graphs. An exact method for solving the above problems is described; it has a rather high computational complexity. A simple heuristic algorithm for constructing a minimal spanning tree of a fuzzy graph is proposed; being computationally less intensive, this method, however, does not ensure absolute minimality of the spanning tree. The method is illustrated with an example.

*Keywords*: fuzzy graph, spanning trees, realizability and tree minimality, exact construction methods, heuristic method

**DOI:** 10.7868/S1608303226030033

## 1. INTRODUCTION

When designing electronic devices, one often needs to connect several contacts electrically. It is desirable to obtain a circuit with a minimum number of conductors. This problem can be solved using the model of a connected undirected graph $G(V, E)$, where $V$ and $E$ denote the sets of vertices and possible connections between vertex pairs, respectively. For each edge $(u, v) \in E$, a weight $w(u, v)$ can be assigned, determining the cost of connecting $u$ and $v$. To minimize the cost of all vertex connections, it is necessary to find an acyclic subset $T \subseteq E$ connecting all vertices with the minimal total weight $w(T) = \sum_{(u,v)\in T} w(u, v)$. Since the set $T$ is acyclic and connects all vertices, it must form a tree, called a minimal spanning tree.

Consider another case: the initial device represents a set of subcircuits, and there is a given set of pairs of subcircuits that can mutually diagnose each other. Assume that these pairs have multichannel links for mutual fault diagnosis. Each multichannel link of a subcircuit pair has a weight equal to its cost. By analogy with the previous problem, it is required to construct an acyclic subset of subcircuits connecting all mutually diagnosable pairs and having minimal weight. Obviously, the solution of this problem also reduces to constructing a minimal spanning tree.

The above problems are of graph theory, and graphs have many optimization-related practical applications in various fields. The extensive literature devoted to their solution largely pertains to classical graph theory, where a graph is deterministic and its description contains no uncertainties. Such graphs are usually called crisp. The numerous results established for crisp graphs, as well as the basic concepts and constructs of graph theory, were presented in the monographs [1, 2]. Therefore, the corresponding references are generally omitted below, assuming the reader's background on the subject.

The problem of constructing spanning trees for classical graphs has been actively studied since the 1920s. For instance, a solution algorithm proposed by O. Borůvka was published in 1926; it was later rediscovered multiple times (K. Florek, S. Perkal, and W. Sołtysiak, occasionally miscalled W. Sollin in the literature). Various approaches were used to solve the problem. For example, based on Borůvka's algorithm, a randomized algorithm with linear (on average) execution time was implemented. E. Minieka, the author of the monograph [3], proposed an original algorithm; this monograph, with Chapter 2 entirely focused on the problem under consideration, also presented the (now widely used) algorithms of R. Prim [4] and J. Kruskal [5]. The algorithms mentioned involve the idea of ordering the graph edges by weight and their incidence.

Following the current proliferation of computer networks and the development of a new mathematical apparatus (e.g., genetic algorithms), the recent publications (for instance, see [6, 7]) have provided new approaches for constructing spanning trees using evolutionary computations.

Unfortunately, the crisp graph model turns out to be inapplicable in many real situations where the operation of an object of study is not precisely described, and such a description cannot be obtained in principle. In this case, new tools are needed to reflect the emerging uncertainties. Such tools were proposed in L. Zadeh's paper [8]: he introduced the concept of a fuzzy set, the foundation of the theory of fuzzy sets, now widely used in applications.

Today, the theory of fuzzy graphs is an actively developing area. The interest of researchers in this theory over the last decade is due to important practical applications. Supplementing the above examples, we also mention the problems of system survivability, assessing the information reliability of complex systems, and modeling heterogeneous systems. The phenomenon of fuzziness arises in many optimization problems of an economic nature as well. For instance, consider the well-known problem of designing a road network for a certain region with minimal construction costs. If the list of regional roads (between cities) and their parameters is precisely known, then the problem has an exact solution using the crisp graph model. Several efficient algorithms have been developed to obtain it [3].

If the reasonability of building roads between particular city pairs is ambiguous, then the resulting uncertainty requires a different model to solve the problem.

To illustrate the method for constructing a spanning tree for a fuzzy graph, we will use the regional road network design problem, owing to its popularity and intuitively clear formulation and interpretation.

In crisp graph theory, the road network design problem reduces to constructing a spanning tree for an original graph. Under uncertainty, this problem can be solved using the fuzzy graph model, and it also reduces to constructing a spanning tree. This paper is devoted to constructing spanning trees for fuzzy graphs that are optimal by criteria different from those adopted for crisp graphs.

## 2. SOME CONCEPTS AND DEFINITIONS

First, we recall the concepts necessary for further considerations. They are all interpreted as presented in A. Kaufmann's monograph [9]. We begin with the concept of a fuzzy set. Let a set $A$ be a subset of a set $E$ $(A \subset E)$. The membership of an element $x \in A$ is written using a function $\mu_A(x)$ that takes any nonnegative value in an ordered set $M$, particularly in the interval [0, 1]. A mathematical object of the form

$$A = \{(x_1|\mu_A(x_1)), (x_2|\mu_A(x_2)), \ldots, (x_n|\mu_A(x_n))\},$$

where $\mu_A(x_i)$ defines the grade of membership of an element $x_i$ in the set $A$, is called a fuzzy subset of the set $E$. Here is the definition of a fuzzy graph [9]. Let $E_1, E_2$ be two sets, and let $x \in E_1$ and $y \in E_2$. The set of all pairs $(x, y)$ defines the Cartesian product $E_1 \times E_2$. A fuzzy subset $G$ such that

$$\forall (x, y) \in E_1 \times E_2 \, \mu_G(x, y) \in M,$$

where $M$ is the set of membership grades of $E_1 \times E_2$, is called a fuzzy graph. If the pairs $(x, y)$ are considered ordered, then the corresponding graph is usually called a directed fuzzy graph. Fuzzy graphs, like crisp ones, are depicted as vertices and edges (arcs) connecting them.

The terms encountered in this paper generally have the same meaning as in crisp graph theory. If some concepts for fuzzy graphs do not coincide with the analogs for crisp graphs, this will be specified explicitly. Let $a(x, y)$ denote the length of an edge $(x, y)$ in an undirected fuzzy graph. Next, we recall the concept of a spanning tree of a graph: it is an acyclic connected subgraph of a given connected graph that includes all its vertices.

## 3. PROBLEM FORMULATION IN SUBSTANTIVE TERMS

First, we comment on the problem of constructing a spanning tree for a fuzzy graph. In a region containing several cities, a road construction project is considered to connect each city pair (not necessarily directly; transit connections are possible). There is a given set of city pairs for which the construction of a directly connecting road is possible (allowed). The lengths of roads between city pairs from this set are known. Also, there is an assessment of the reasonability of including each road in the project (e.g., based on the resulting economic benefit and the construction costs). This assessment is defined as a number from some ordered set $M$ (in particular, from the interval [0, 1]), understood as the grade of membership of the road in the network project under design.

As a mathematical model of the road network, we will use a fuzzy graph where the set of regional cities is the set of vertices, and the set of project roads is the set of its edges. In contrast to a crisp graph where the weight of each edge $(x, y)$ is typically a single number interpreted as its length, the weight for a fuzzy graph is an ordered pair of numbers $(a(x, y), \mu_G(x, y))$: the first corresponds to the edge length, and the second to its grade of membership. Obviously, now the optimality of the project can be assessed by two criteria associated with the components of the pair. Therefore, two optimization problems are considered in the paper; see their substantive formulations below. It is required to design a road network project (construct a spanning tree) connecting all cities of the region so that:

1) The maximum probability of tree construction (the maximum sum of the membership grades of its edges) is reached under the minimum possible sum of road lengths included in the project, using the parameters of the fuzzy graph model of the network project.

2) The minimum sum of all road lengths in the network is reached under the maximum possible sum of their membership grades, using the parameters of the fuzzy graph model of the network project.

Hereinafter, the road network project and its mathematical model (the spanning tree of the fuzzy graph) are used as synonyms.

As is known [3], when solving the analog of this problem for a crisp graph, the optimality criterion is the minimum total road length of the network project. In this case, the problem reduces to constructing a minimal spanning tree for the graph where the cities are the vertices, and the lengths of the edges (roads) connecting city pairs are known. The solution of the two problems posed for a fuzzy graph also obviously reduces to constructing some optimal spanning trees, but in a different interpretation (understanding) of optimality. To the best of our knowledge, the problem in the above or similar statements for a general fuzzy graph has not been studied in the literature.

## 4. REALIZABILITY AND MINIMALITY OF A SPANNING TREE

To clarify the concepts introduced below, we will present the considerations and motivation behind them for fuzzy graphs. If a fuzzy graph $G(X, E)$ is a mathematical model of the network for solving the problem, then the set $E$ consists only of “allowed” edges for which $\mu_G(x, y) > 0$. As mentioned above, the weight of each edge $(x, y)$ is an ordered pair of numbers $a(x, y), \mu_G(x, y)$, where the second number is the grade of membership of $(x, y)$ in the set $E$. This value can always be transformed to the probability $P(x, y)$ of the belonging of the edge $(x, y)$ to the set $E$. Clearly, the membership function of a fuzzy set resembles a probability density. The difference is only that the sum of probabilities over all possible values of a random variable always equals 1, while the sum $S$ of the membership function values of all arcs in a fuzzy graph can be any nonnegative number. In this regard, it becomes possible to transform the membership function of a graph edge to a probability distribution function. This is done simply by normalizing the membership function, i.e., dividing its value for each edge of the fuzzy graph by the value $S \neq 0$. Such a method was used in several publications as well.

In fuzzy set theory, an edge's nonzero grade of membership is interpreted only as a potential possibility of including this edge in the graph. Naturally, the greater the grade is, the higher the possibility will be. However, the factual inclusion of an edge (with a nonzero probability estimate) in the graph is not guaranteed. An analogous situation arises in probability theory: an event may have a nonzero probability of occurrence, but this does not mean it will necessarily happen.

By assumption, a given fuzzy graph is connected: otherwise, a spanning tree would not exist for it. However, in reality, some edges with nonzero membership values may be absent, violating the connectivity property of the graph. To avoid this as much as possible, it is necessary to maximize the membership value for each edge (road) included in the spanning tree (i.e., maximize the factual existence probability of this tree).

For the further presentation, we introduce the following notation: $E = \{(a_1, b_1), \ldots, (a_z, b_z)\}$ is the set of edges of a fuzzy graph $G(X, E)$; $Rect\ (G(X, E))$ is the realizability of the graph $G(X, E)$; $Lect\ (G(X, E))$ is the total length of all edges of the graph $G(X, E)$; $Pbe\ (x, y)$ is the probability of the belonging of an edge $(x, y)$ to the set $E = \{(a_1, b_1), \ldots, (a_z, b_z)\}$; $Mct\ (G(X, E))$ is the minimal spanning tree of the graph $G(X, E)$; and $Lct\ (G(X, E))$ is the length of the spanning tree of the graph $G(X, E)$. Using these designations, we give some definitions.

The realizability of a spanning tree of a fuzzy graph $G(X, E)$ is the sum of the probabilities of the belonging of all edges included in this tree, i.e., $Rect\ (G(X, E)) = \sum_{i=1}^{z} Pbe_G(a_i, b_i)$. Thus, the realizability of a road network project is a probabilistic estimate for the factual existence possibility of a spanning tree for a given fuzzy graph with specified probabilities of the belonging of its edges. Note that the concept of realizability agrees well with common sense: the greater the realizability value is, the higher the chances of constructing a spanning tree will be.

The *length of a spanning tree* of a fuzzy graph is the sum of the lengths of all its edges: $Lect\ (G(X, E)) = \sum_{i=1}^{z} a(a_i, b_i)$. The minimum-length (shortest) spanning tree of a fuzzy graph will be called the *minimal spanning tree* and denoted by $Mct\ (G(X, E))$.

A *spanning tree* that has maximum realizability and, at the same time, the minimum possible length (*optimality by tree length*) under the specified estimates of the probabilities of the belonging of its edges will be called *optimal by realizability.*

Now we provide a rigorous statement of the first problem posed in the previous section: for a given undirected fuzzy graph, it is required to construct a spanning tree that is *optimal by realizability.* The second problem is now stated as follows: for a given undirected fuzzy graph, it is required to construct a *minimal spanning tree.*

## 5. A METHOD FOR FINDING OPTIMAL SPANNING TREES

The specification of a fuzzy graph involves a membership function, and the problem of obtaining an appropriate function arises accordingly. Researchers have been studying this problem since the very introduction of the concept of a fuzzy graph. There are many publications on this topic, e.g., [10–12]. Note that calculating the probability of a certain event with a given accuracy requires conducting a significant number of experiments (in principle, tending to infinity). This is not always reasonable due to high complexity, and one often uses some approximate value. We emphasize that methods for determining the values of necessary parameters to obtain results with a given accuracy are now well-developed in sampling theory. For instance, see the monograph [13] for a detailed coverage of these issues. The problem of constructing a membership function goes beyond the scope of this paper; therefore, by assumption, such a function is available together with a fuzzy graph specification.

To construct a spanning tree optimal by realizability for a fuzzy graph, we propose a method based on the results of solving a similar problem (finding a minimal spanning tree) in classical graph theory. For instance, a procedure for constructing all spanning trees of a crisp graph, as well as a proof of its correctness, was presented in [1, Ch. 7, Sec. 2]. Removing the second component (the membership value) from the weight of each edge in a given fuzzy graph, we obtain the corresponding crisp graph $\hat{G}(X, E)$. This graph is a special case of the original fuzzy graph, where all edges are present regardless of their membership values. Using the above procedure, we construct for $\hat{G}(X, E)$ the set of all its spanning trees, denoted by $Ct(\hat{G}(X, E)) = \{G_1, \ldots, G_k\}$. Next, for each edge of the spanning trees in this set, we restore the second component (membership value) to its weight, thereby making these trees fuzzy. Clearly, after this transformation, the crisp spanning trees of the set $Ct(\hat{G}(X, E))$ remain spanning trees and form the set $Ct(G(X, E)) = \{\bar{G}_1, \ldots, \bar{G}_k\}$ of all spanning trees of the given fuzzy graph $G(X, E)$.

For each fuzzy spanning tree from the set $Ct(G(X, E))$, we calculate its realizability $Rect\ (\bar{G}_i)$, $i = 1, \ldots, k$ and find among them the tree (or several trees) with maximum realizability. Next, for each fuzzy spanning tree from the set $Ct(G(X, E))$, we calculate $Lect\ (\bar{G}_i)$, $i = 1, \ldots, k$, its length. For the given graph, the spanning tree optimal by realizability can be found using the data obtained.

Let all spanning trees of the set $Ct(G(X, E))$ be numbered. We form the set $MaxR$ of the numbers of all trees with maximum realizability and the set $ML$ of the numbers of all trees from the set $Ct(G(X, E))$ with lengths calculated.

Letting $MaxR = \{v_1, \ldots, v_z\}$, we form the set of all pairs ($v_i$, *the length of tree* $v_i$), where $i = 1, \ldots, z$. Among all these pairs, we find the one with the minimum tree length; suppose that the corresponding tree has number $i_0$. Obviously, the spanning tree with number $i_0$ is optimal by realizability. If the set $MaxR$ is a singleton, then the optimal tree has that number. If there are several such trees, we select the tree $j_0$ with the minimum length among the elements ($v_i$, *the length of tree* $v_i$) of the above set. Clearly, the tree with number $j_0$ will be optimal by realizability.

The second optimization problem (minimizing the spanning tree) is solved by analogy. First, we select the minimal spanning tree in the set of pairs ($v_i$, *the length of tree* $v_i$). If this is only one tree with number $i_0$, then this tree with the realizability value $Rect(G_{i_0})$ is optimal (minimal by length). If there are several such trees (with numbers $i_0^1, i_0^2, \ldots, i_0^r$), then we find among them the tree $j_0$ with the maximum realizability value. Obviously, this tree will be a minimal spanning tree.

The method proposed rests on the possibility of constructing the set of all spanning trees of a graph. This is needed sometimes when selecting the "best" spanning tree, but the selection criterion itself is very computationally intensive. In such a situation, solving the optimization problem may become practically infeasible. As noted in [1], the number of all spanning trees of a complete connected undirected graph with $n$ vertices is $n^{n-2}$, the formula derived back in the 19th century. According to the above formula, the number of spanning trees grows very rapidly when increasing $n$. This growth rate is valid not only for complete but also for all other types of graphs. Therefore, an efficient method is needed to generate an exhaustive set of all trees mentioned, without repetitions. Such methods have been proposed, but unfortunately, they do not always yield the solution in practice due to their rather high complexity.

## 6. A HEURISTIC ALGORITHM FOR CONSTRUCTING THE MINIMAL SPANNING TREE

Due to the discussion above, for the problem under consideration, it is desirable to have approximate solution methods more acceptable for practical implementation instead of exact ones.

Note that the problems addressed here for a fuzzy graph fundamentally differ from the analogous problem for a crisp graph in the sense of multicriteria optimization [14]. In most real applications, optimization has to be performed by many criteria; in the general case, they can be formulated as follows:

$$\max\{z_1 = f_1(x), \ldots, z_q = f_q(x)\}, \quad g_i(x) \geqslant 0, \quad i = 1, \ldots, m, \quad x \geqslant 0.$$

Here, the search space of solutions is given by

$$S = \{x \in R^n | g_i(x) \geqslant 0,\ i = 1, \ldots, m,\ x \geqslant 0\}.$$

When solving multicriteria problems, one seeks a solution that is better than the others. However, such a best solution by all criteria does not necessarily exist. This occurs due to various conflicts; obviously, a solution may be best by one criterion and, at the same time, worst by others. Therefore, in multicriteria optimization, Pareto optimal solutions are often constructed [15, 16].

In the problems under consideration, we have two criteria, namely, the realizability value of the spanning tree (to be maximized) and the spanning tree length (to be minimized). Multicriteria optimization is a natural extension of ordinary numerical or combinatorial optimization. Therefore, many well-developed methods have been generalized to this class of problems. We emphasize that the central issue of multicriteria optimization is the construction of an objective function. Over the past decades, several approaches to this problem have been developed, including vector evaluation, Pareto ranking, and the use of weighted sums.

Note that the weighted sum approach has been popular in recent years: the new (overall) objective function is constructed from partial objective functions as a weighted sum of the form

$$F(x) = \sum_{i=1}^{k} w_i f_i(x), \quad w_i \in [0, 1], \quad \sum_{i=1}^{k} w_i = 1.$$

Here, each objective function $f_i(x)$ is assigned a particular weight, and the original problem reduces to the scalar case. Different weights $w_i$ yield different Pareto-optimal solutions. Clearly, in view of the diverse nature of partial objective functions, the choice of appropriate weights is a quite complex task.

The approach proposed below will also reduce the original problem to the scalar case, but with a different method.

According to [2, Ch. 23], several algorithms have been developed for finding the minimal spanning tree for a crisp graph. Prim's and Kruskal's algorithms are among the most popular ones. Both are greedy algorithms: their strategy is to choose the best alternative at the current iteration. Generally speaking, this strategy does not ensure the absolute optimum. However, the algorithms have been proven to construct minimal spanning trees. What is also important, for a crisp graph $G(X, E)$, both algorithms run in an estimated time of $O(X \lg E)$ [2], which is a good performance. Based on the above considerations, an acceptable solution would be reducing the original problem for a fuzzy graph to the same problem for a crisp one and applying Prim's or Kruskal's algorithm.

This idea can be implemented by proposing a suitable method for transforming a given fuzzy graph to a crisp one. It seems that such a transformation should preserve the structure of the original graph (the sets of vertices and edges, as well as the connections between them) but suitably replace the weight of each edge (an ordered pair of values) with a single scalar value. This essentially

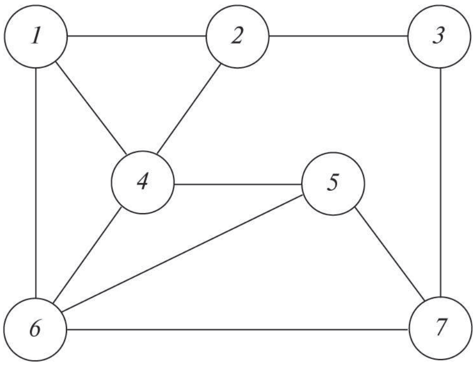

**Fig. 1.** An example of a fuzzy graph.

means that the scalar value should somehow reflect the information about the edge's probability of belonging and its length.

Clearly, such a transformation should not be computationally-intensive, i.e., should be limited to fairly simple operations over weights. From the standpoint of simplicity, arithmetic operations are suitable as one example. Since these components have different and sometimes incompatible natures, addition, subtraction, and division are often of little sense here; and therefore, multiplication is the most suitable operation.

Below, we propose a heuristic algorithm for constructing the minimal spanning tree of a fuzzy graph based on the idea expressed. In a fuzzy graph $G(X, E)$, each edge $(x, y)$ belongs to it with a given probability $P(x, y)$. This fact can be interpreted as follows: under a sufficiently large number $N$ of trials (simulations of transport flows in the network), the entire road $(x, y)$ will be used not $N$ but only $P(x, y)N$ times. Similarly, in each trial (a simulation of a particular transport route), one can conditionally assume that the road $(x, y)$ of length $a(x, y)$ will be used not entirely but partially, to the length $P(x, y)\ a(x, y)$. When conducting $N$ trials, the total path length on road $(x, y)$ will equal $N(P(x, y)\ a(x, y))$. Then, on average per trial, the path traveled will be $P(x, y)\ a(x, y)$. In this case, the above part can be treated as the "length of edge" $(x, y)$ "weighted" considering its absolute length and probability of belonging. Thus, the weight of an edge in the original fuzzy graph $G(X, E)$, representing an ordered pair, will be replaced by a single number, and the graph becomes crisp. Note that the replacement of the length $a(x, y)$ of an edge with $P(x, y)\ a(x, y)$ cannot, of course, claim to be rigorously justified. It is only an attempt to explain the motivation behind the heuristic change of graph parameters.

After obtaining a crisp graph by the heuristic algorithm, we can apply Prim's and Kruskal's algorithms to this graph and construct a minimal spanning tree. After construction, for each edge in the resulting spanning tree, we restore as its weight the ordered pair corresponding to that edge in the original fuzzy graph. The minimal spanning tree of the fuzzy graph obtained in this way will serve as an approximation to the minimal one.

Obviously, the heuristic algorithm does not surely yield an optimal spanning tree by the adopted minimality criterion. Generally speaking, this algorithm constructs a spanning tree that is minimal (by length), but its realizability value may be non-maximum. As advantages, we note the simplicity and low computational complexity of the algorithm.

To assess the effectiveness of this algorithm, we need sufficient statistical data on the results of its application, which are currently unavailable. According to its limited testing, the heuristic algorithm surely constructs the minimal spanning tree, and the deviations of its realizability values from the maximum are relatively insignificant. Let us illustrate the operation of the algorithm with a small example. Consider a fuzzy graph in Fig. 1; its parameters are combined in the table below.

The parameters of a fuzzy graph

| No. | Edge $(x, y)$ | The probability of the belonging of edge, $Pbe(x,y)$ | Edge length $a(x,y)$ | Replacement length $Pbe(x,y) \times a(x,y)$ |
|---|---|---|---|---|
| 1 | (1,2) | 0.099 | 70 | 6.93 |
| 2 | (1,4) | 0.099 | 60 | 5.94 |
| 3 | (1,6) | 0.099 | 50 | 4.95 |
| 4 | (2,3) | 0.096 | 30 | 2.88 |
| 5 | (2,4) | 0.089 | 30 | 2.67 |
| 6 | (3,7) | 0.092 | 50 | 4.60 |
| 7 | (4,5) | 0.089 | 20 | 1.78 |
| 8 | (4,6) | 0.080 | 40 | 3.20 |
| 9 | (5,6) | 0.094 | 40 | 3.76 |
| 10 | (5,7) | 0.091 | 60 | 5.46 |
| 11 | (6,7) | 0.072 | 90 | 6.48 |

A separate column of the table presents the values of $P(x,y)\ a(x,y)$, which replace the edge weights (ordered pairs) of the fuzzy graph with a single number. Using these data, two different spanning trees were constructed by Prim's algorithm. Figure 2 shows the tree constructed starting from vertex 1; Fig. 3, the tree constructed starting from vertex 7. As is easily verified, both trees have the same length, equal to 220, and are minimal for the corresponding crisp graph. A fuzzy spanning tree corresponds to each of these trees. According to the table, we assign the probabilities of belonging to all edges of these graphs; then the corresponding realizability values can be calculated. For the graph in Fig. 2, the realizability value is 0.545; for the graph in Fig. 3, 0.559.

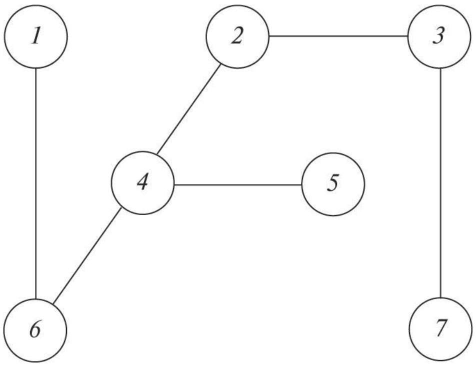

**Fig. 2.** The first variant of the tree.

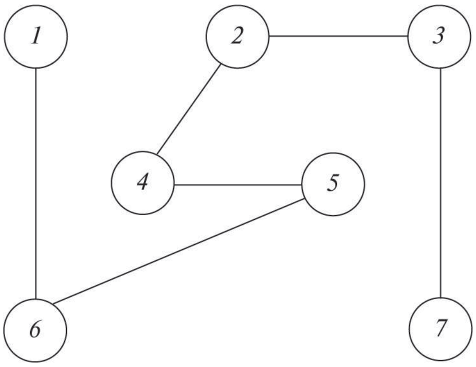

**Fig. 3.** The second variant of the tree.

Therefore, the heuristic algorithm may yield a spanning tree of non-maximal realizability. Note that the minimal spanning tree of the crisp graph obtained by Prim's algorithm based on the table data has the same structure as the one in Fig. 2 and the same length of 220.

## 7. CONCLUSIONS

In this paper, we have conceptualized the realizability of a spanning tree of a fuzzy graph and, using this concept, formulated two optimization problems for constructing optimal spanning trees. Previously, analogous problems in such a statement have not been considered in the literature. The optimality criteria used are the maximum value of tree realizability and the minimality of the spanning tree (by its length). An exact solution method for the two problems has been proposed, which suffers from high complexity. In this regard, a heuristic algorithm has been described to construct a minimal spanning tree considering its realizability. The algorithm is fairly simple and has low computational complexity, but does not ensure maximum realizability. This algorithm is based on reducing the original multicriteria problem to a single-criterion one (spanning tree minimality), which is efficiently solved, e.g., by Prim's and Kruskal's algorithms.

## REFERENCES

1. Christofides, N., *Graph Theory: An Algorithmic Approach*, New York: Academic Press, 1975.
2. Cormen, T.H., Leiserson, C.E., Rivest, R.L., and Stein, C., *Introduction to Algorithms*, 3rd ed., Cambridge, MA: MIT Press, 2009.
3. Minieka, E., *Optimization Algorithms for Networks and Graphs*, New York: Marcel Dekker, 1978.
4. Prim, R.C., Shortest Connection Network and Some General Realization, *Bell System Technical Journal*, 1957, vol. 36, pp. 1389–1401.
5. Kruskal, J.B., On the Shortest Spanning Subtree of a Graph and Traveling Salesman Problem, *Proc. AMS*, 1956, vol. 7, pp. 48–50.
6. Kuz'min, R.A. and Molchanov, V.A., An Algorithm for Constructing Minimal Spanning Trees in Computer Networks, in *Matematika. Mekhanika* (Mathematics. Mechanics), 2023, no. 25, pp. 46–50.
7. Meshkov, A.N., Construction of a Spanning Tree Using Steiner's Genetic Algorithm, *Trudy V Mezhdunarodnoi nauchno-prakticheskoi konferentsii "Aktual'nye voprosy sovremennykh issledovanii"* (Proceedings of the 5th International Scientific and Practical Conference "Topical Issues in Contemporary Research"), July 5, 2023, Penza: Nauka i Prosveshchenie, 2023, pp. 74–76.
8. Zadeh, L.A., Fuzzy Sets, *Information and Control*, 1965, vol. 8, no. 3, pp. 338–353.
9. Kaufmann, A., *Introduction to the Theory of Fuzzy Subsets. Vol. 1*, New York: Academic Press, 1975.
10. Norwich, A.M. and Turksen, I.B., The Construction of Membership Functions, in *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, New York: Academic Press, 1975.
11. Qaid, W.A.A., Methods for Constructing Membership Functions of Fuzzy Sets, *Izvestiya SFedU. Engineering Sciences*, 2013, no. 2(139), pp. 144–153.
12. Fedorova, E.V. and Gryaznov, S.M., Features of Forming Membership Functions in the Construction of Fuzzy Control Systems, *Trudy III Vserossiiskoi nauchno-tekhnicheskoi konferentsii "Intellektual'nye sistemy upravleniya i mekhatronika. 2017"* (Proceedings of the 3rd All-Russian Scientific and Technical Conference "Intelligent Control Systems and Mechatronics. 2017"), 2017, pp. 170–174.
13. Cochran, W.G., *Sampling Techniques*, 3rd ed., New York: John Wiley & Sons, 1977.
14. Keeney, R.L. and Raiffa, H., *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, New York: John Wiley & Sons, 1976.
15. Pareto, V., *Manual di Economica Politica*, Societa Editrice Libraia, Milan, Italia, 1906; translated into English by A.S. Schwier under the title *Manual of Political Economy*, New York: Macmillan, 1971.
16. Gen, M., Cheng, R., and Lin, L., *Network Models and Optimization*, London: Springer-Verlag, 2008.

*This paper was recommended for publication by O.N. Granichin, a member of the Editorial Board*