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TOPICAL ISSUE

Investigation of Feasible and Marginal Operating Regimes of Electric Power Systems

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Abstract—The paper is devoted to the analysis of the feasibility domain of electric power systems. The problems of calculating feasible and marginal regimes of power systems, analyzing the geometry of the feasibility domain, and generating samples in this region are considered. Parallels are drawn with the works of B.T. Polyak on the analysis of the image of a quadratic map, modification of the Newton method and the development of methods for generating asymptotically uniform samples in areas with complex geometry. Particular attention is paid to Newton's method with the transversality condition (TENR), its application for constructing a boundary oracle procedure and utilization for generating samples in the power system feasibility domain.

Keywords: admissible domain of power systems, power flow equations, quadratic mapping image, Newton's method, sampling

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1. INTRODUCTION

The development of theory, models and methods for calculating optimal and marginal operating regimes of the power system remains a challenging and relevant research topic due to the widespread distribution of distributed renewable energy sources, changing patterns of electrical energy consumption and digital transformation in the energy sector. Control of modern power systems requires fast and reliable methods for estimating static stability margins, which are characterized by the distance to the boundary of the feasibility domain. In addition, the growing integration of distributed renewable energy sources is prompting a reassessment of the criteria for optimal grid operating regimes, which shifts frequently operated regimes closer to the boundary. The concept of a feasibility domain for electric power system — a region in the multidimensional space of nodal power injections (the right-hand side of power flow equations) such that this system of equations has at least one real solution — bridges between various problems in energy sector.

Further in the introduction, we discuss the links between the analysis of feasible and marginal operating regimes of a power system with the image of a quadratic mapping, D-partition, Newton's method and its modifications, as well as methods for generating asymptotically uniform samples in areas with complex geometry.

1.1. Image of Quadratic Mapping

The steady-state operation regimes are described by power flow equations, reflecting fundamental Ohm's and Kirchhoff's laws they provide the relation between complex voltages and power at the nodes of power system (1)–(2). These equations are quadratic with respect to variable V. In papers [1, 2] B.T. Polyak proposed sufficient conditions for convexity of the image of quadratic

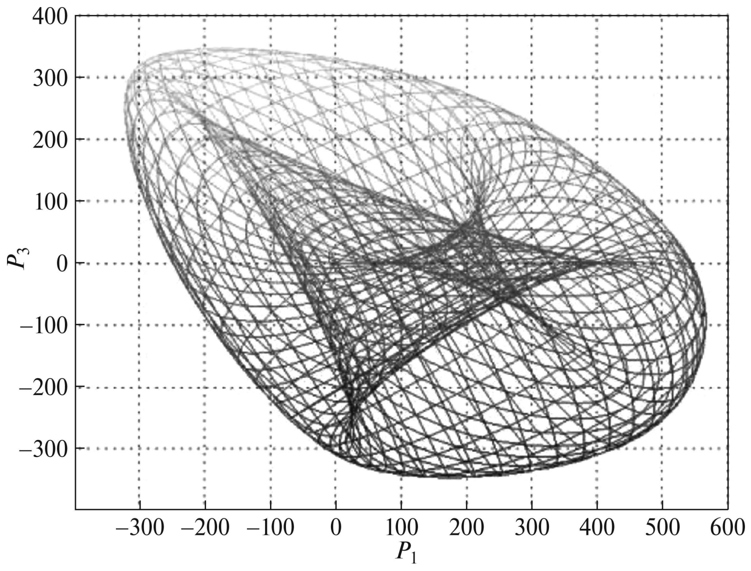

Fig. 1. Convex region of feasible regimes from [4].

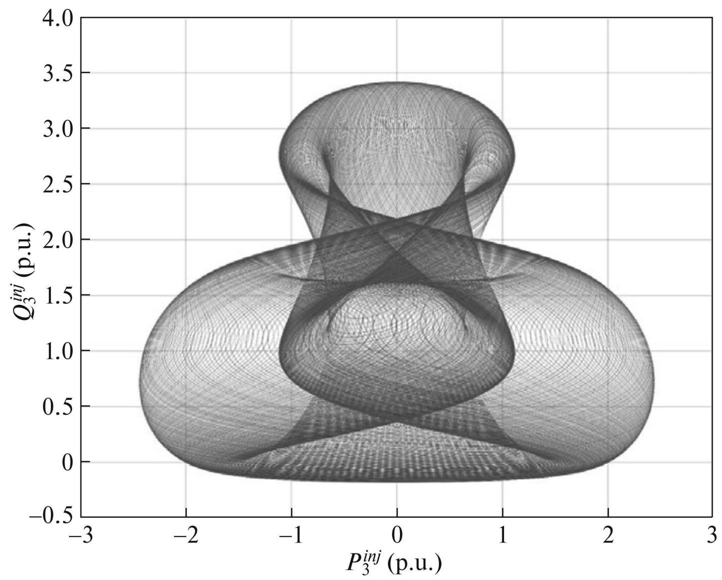

Fig. 2. Non-convex region of feasible regimes from [5].

mapping $f : \mathbb{R}^n \to \mathbb{R}^m$ at $m = 2, 3$, later in [3] a randomized approach for certifying convexity/nonconvexity of the image of quadratic mapping was proposed.

Indeed, feasibility domain of power system has a pretty complicated geometry. Moreover, this complexity holds regardless the number of nodes, one may observe complicated structure for the systems of 3–5 nodes ("nodes" are typical called "buses" in power systems analysis). In 1 presents the cross-section of the feasibility domain for 3 bus system. The area is convex, though it's demonstrated in [4] that it looses convexity for perturbed right-had side of power flow equation.

Examples of more exotic feasibility domains can be found in [5], one of cross-sections of this type for 5-bus system is presented in Fig. 2.

Looking at Figs. 1–2, it is straightforward to recognize a complex internal structure of the feasibility domain. Due to the nonlinearity of the system of power flow equations there are internal bifurcation curves, such that crossing them corresponds to either a change in the number of solutions or the disappearance of solutions, and therefore the admissible state of the system. These equations can have multiple isolated solutions, representing either a stable or unstable equilibrium of a dynamic power system model. The presence of multiple solutions was previously ignored by researchers; their efforts were mainly focused on identifying only one real solution rather than on all isolated solutions. The work [6] is apparently the first to study the phenomenon of multiplicity of solutions to the control system and propose a method for constructing all critical points of the region of admissible modes. Critical points are understood as points of bifurcation curves, the intersection of which changes the number of solutions to the system of power flow equations. Such an analysis of the feasibility domain has much in common with D-decomposition method developed in the works of B.T. Polyak [7, 8]. For linear dynamical system D-decomposition curve splits the parameter space into regions with different numbers of stable roots of characteristic polynomial, while bifurcation curve (or surface in higher dimensional spaces) separates the regions wit different number of solutions to the power flow equations in thee feasibility domain.

1.2. The Role of Newton Method

Traditionally, Newton–Raphson method is applied for calculating steady state regimes. The introduction of additional variables characterizing stability margins makes the system underdetermined. In this case, the problem of calculating regimes appears in the context of B.T. Polyak's papers [9, 10].

One of the most promising methods for fast calculations of marginal regimes is Newton's method with the transversality condition (TENR, Transversality Enforced Newton–Raphson) [11], where, in addition to an additional variables, the condition of degeneracy of the Jacobian matrix is added.

The TENR method is conceptually similar to, but mathematically different from, traditional methods based on the standard Newton method. In TENR, the standard system of power flow equations is complemented by the transversality condition. This constraint regularizes the initially degenerate system at the marginal point and ensures the convergence of Newton's method. In addition, TENR allows the steady state calculation to take into account any technical constraints, which can be represented either as equalities or inequalities. From a computational point of view, a key advantage of TENR is its simple form of writing transversality conditions, which does not require explicit tracking and initialization of zero eigenvectors of the Jacobian. This simplification results in a smaller system of nonlinear equations and also allows for easier initialization of the algorithm.

The TENR method has a number of advantages: the algorithm is numerically stable in the immediate vicinity of the boundary, as well as at the feasibility boundary; it weakly depends on the starting point; decomposition of the Jacobian matrix by singular values has been implemented, which allows us to analyze the sensitivity of the power system and identify the most "effective nodes" for applying control actions. Based on TENR, it is possible to solve the problem of estimating the transfer capability margins [12], as well as online assessment of voltage stability margins [13]. The method has been tested on a number of IEEE benchmark systems as well as on a model of the power system of the Russian Far East [14].

1.3. Sampling in Feasibility Domain

Knowledge about the feasibility domain geometry of power system and its boundaries allows us to make fast estimation of the stability margins and to calculate optimal emergency control actions. The challenge of ensuring reliable and secure real-time operation of power systems is increasing as the current operating regime rapidly changes due to uncertainties associated with increased renewable generation, less predictable demand and various unexpected circumstances. Therefore, to avoid any undesirable system behavior or large-scale power outage, real-time evaluation of voltage stability margins is required. Such an assessment is a challenging task that requires significant computational resources, mainly due to the constantly changing state of operation. Both during the planning and operational stage, safe operation of the network requires voltage stability, which is the ability of the power system to maintain acceptable voltage levels on all buses after exposure to disturbances [15].

Modern power systems are more vulnerable in terms of stability because they operate close to the boundary of the feasibility region. Voltage instability occurs in electrical networks when the operating mode approaches the point of collapse or the point of saddle-node bifurcation, after which the real solution to the steady-state equations vanishes or the number of solutions to the system of steady-state equations changes. You can clarify your description of the feasibility domain via sampling, i.e. generating parameters of feasible modes. Such parameter sets are also useful for tuning machine learning algorithms. One of the directions of B.T. Polyak's research was the development of methods for generating asymptotically uniformly distributed samples in complex domains [16, 17].

This paper provides a detailed description of the TENR method as the most effective tool for calculating the marginal states of a power system, and also shows how to use TENR to build a boundary oracle procedure to generate samples in the feasibility domain.

The paper is organized as follows: Section 2 presents the problem formulation. Section 3 describes the TENR method and discusses the strategy for choosing the optimal step size for its implementation. Section 4 is devoted to the problem of generating samples in the feasibility domain of power system. Section 5 provides numerical examples illustrating the effectiveness of the TENR method both for calculating marginal regimes (boundary points of the feasibility domain) and for generating samples.

2. PROBLEM STATEMENT

Power system marginal states (marginal operating regimes) assessment is closely related to the power flow analysis (so-called regime). A regime is a state of the power system that can be characterized by quantitative indicators: power, voltage, current, phase angles of the EMF vectors, and others. A regime can be categorized as transient or steady-state, depending on the rate of their change. A steady-state is one in which the parameters remain constant over the considered time interval or change relatively slowly [18]. Since a regime has quantitative characteristics, it can be calculated and evaluated. The calculation of the steady-state regime (power flow analysis) involves determining all parameters of the steady-state regime given the known system parameters (circuit diagrams, line impedance, etc.) and some specified regime parameters [19]. The set of equations based on the equivalent circuits of the power system, as well as Ohm's and Kirchhoff's laws, constitutes the mathematical model of the steady-state regimes of power systems.

In the theory of electrical systems, there are numerous available mathematical models, each with its advantages and disadvantages. In this work, the model used is the system of power balance equations presented in a rectangular form. The voltage is represented as a complex: $\hat{V}_i = V_i^r + jV_i^m \in \mathbb{C}$. G_{ij} and B_{ij} are the real and imaginary parts of the complex admittance $\hat{Y}_{ij} = G_{ij} + jB_{ij} \in \mathbb{C}$. The system consists of n buses, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of buses excluding the balancing (slack) bus $\mathcal{S}$; the set of PQ buses (load buses) is denoted as $\mathcal{L}$; and the set of PV buses (generator buses) is denoted as $\mathcal{G}$. For each $i \in \mathcal{N}$, the values of the nodal active power injections can be computed as follows [18, 20, 21]:

$$\sum_{k=1}^{n} \Big\{ V_i^r (G_{ik} V_k^r - B_{ik} V_k^m) + V_i^m (G_{ik} V_k^m + B_{ik} V_k^r) \Big\} = P_i(x) - \lambda (P_{\mathrm{gen},i} - P_{\mathrm{load},i}), \tag{1}$$

where the vector $\mathbf{x} \in \mathbb{R}^n$ is a set of variables (the magnitude of voltages and phase angles at the buses for each bus in the system). Similarly, for each $i \in \mathcal{L}$, one can write the equation for the nodal reactive power injections:

$$\sum_{k=1}^{n} \Big\{ V_i^m (G_{ik} V_k^r - B_{ik} V_k^m) - V_i^r (G_{ik} V_k^m + B_{ik} V_k^r) \Big\} = Q_i(x) - \lambda (Q_{\text{gen},i} - Q_{\text{load},i}). \tag{2}$$

Subscripts "gen" and "load" denote the levels of generation and load at the buses, respectively. The parameter λ is a coefficient used to "stress" the system, meaning that loads are gradually increased. When $\lambda = \lambda_{\max}$, the system reaches its marginal state. Unlike the representation in polar coordinates, the formulation in Cartesian coordinates requires an additional set of equations to account for the voltage limitations at the PV buses. Thus, for each $i \in \mathcal{G}$,

$$(V_i^r)^2 + (V_i^m)^2 - |\hat{V}_i|_{\text{ref}}^2 = 0, \tag{3}$$

where $|\hat{V}_i|_{\text{ref}}$ is the reference voltage magnitude at the specific bus.

The standard system of power flow equations can be generally expressed as follows:

$$\mathcal{F}(x, \lambda) = 0, \tag{4}$$

where $\mathcal{F}$ represents k nonlinear equations, including both power balance equations (such as in (1) and (2)) and various technological constraints presented as equalities. The parameter λ is the loading coefficient that characterizes the system's proximity to the steady-state equation solvability boundary.

The finding of marginal states means to find such $\lambda_{\max}$ that the solution of the system (4) exists for all $0 \leqslant \lambda \leqslant \lambda_{\max}$ but do not exist when $\lambda > \lambda_{\max}$.

From a mathematical perspective, finding of marginal states involves solving the system of equations (4) under the condition that the Jacobian matrix is singular:

$$g(x) = \det \ \nabla_x \mathcal{F}(x, \lambda) = 0. \tag{5}$$

It follows that, to find the marginal state (stability boundary), it is necessary to solve the system of equations (4) together with the additional condition that accounts for the singularity of the Jacobian matrix (5).

3. TRANSVERSALITY ENFORCED NEWTON–RAPHSON METHOD

To solve the system of power flow equations, numerical iterative methods must be employed, which improve the approximation of the initial variables with each iteration. One of the most common and accessible methods is the Newton–Raphson method.

It should be noted that the classical Newton–Raphson method has several drawbacks, including convergence dependence on the chosen initial conditions and poor convergence in close proximity to the stability boundary. The reason for this is the poor conditioning of the Jacobian matrix. Therefore, the standard Newton method provides a consistently underestimated assessment of the power system's stability margin. If the numerical method remains stable at the feasibility boundary, the distance to the marginal state can be determined more accurately.

There is a method that addresses these issues — the TENR method.

During the marginal state analysis, when λ reaches its maximum value $\lambda_{\max}$, the Jacobian matrix of the power flow system becomes singular. Under these circumstances, the Newton method's computational step $J^{-1}\mathcal{F}(x)$ increases, keeping the classical method numerically unstable. As a result, the method may fail to converge or require too many iterations to achieve a result. In the TENR method, an additional condition that accounts for the Jacobian matrix's singularity at the feasibility boundary is added to the base system of equations, with λ also treated as a variable. Thus, the solution domains of the original and the augmented systems of equations coincide.

Within the TENR method, the condition accounting for the Jacobian's singularity at the stability boundary is called the transversality condition $g(\mathbf{x})$. Numerous possible variations of the condition $g(\mathbf{x})$ are available, as presented in [11]. The least computationally expensive approach is based on singular value decomposition.

In general form, the system of equations for finding the limit modes can be written as follows:

$$\begin{aligned} \mathcal{F}(x, \lambda) &= 0, \\ g(x) &= 0 \end{aligned} \tag{6}$$

The system (6) can be numerically solved using the standard Newton method. Undertaking a linearization of equation utilizing the first-order Taylor series within the realms of x and λ results in:

$$\begin{bmatrix} \mathcal{F} \\ g \end{bmatrix} + \begin{pmatrix} \nabla_{\mathbf{x}}\mathcal{F} & \nabla_{\lambda}\mathcal{F} \\ (\nabla_{\mathbf{x}} g)^{\top} & 0 \end{pmatrix} \begin{bmatrix} \Delta x \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{7}$$

In the TENR method, the so-called extended Jacobian matrix $\mathcal{J}(x, \lambda)$ is used in the calculations:

$$\mathcal{J}(x, \lambda) = \begin{pmatrix} \nabla_{x}\mathcal{F} & \nabla_{\lambda}\mathcal{F} \\ (\nabla_{\mathbf{x}} g)^{\top} & 0 \end{pmatrix}. \tag{8}$$

The increments of the unknowns Δx and $\Delta\lambda$ are determined as follows:

$$\begin{bmatrix} \Delta x \\ \Delta\lambda \end{bmatrix} = -\begin{pmatrix} \nabla_{\mathbf{x}}\mathcal{F} & \nabla_{\lambda}\mathcal{F} \\ (\nabla_{\mathbf{x}} g)^{\top} & 0 \end{pmatrix}^{-1} \begin{bmatrix} \mathcal{F} \\ g \end{bmatrix}. \tag{9}$$

Using the calculated increments of the variables, the values of the variables at the next step are determined as follows:

$$\mathcal{N}(x, \lambda) := \begin{bmatrix} x \\ \lambda \end{bmatrix} - \alpha \left[\begin{pmatrix} \nabla_{x}\mathcal{F} & \nabla_{\lambda}\mathcal{F} \\ (\nabla_{x} g)^{\top} & 0 \end{pmatrix}^{-1} \begin{bmatrix} \mathcal{F} \\ g \end{bmatrix} \right]. \tag{10}$$

The parameter α determines the step size in the Newton method, which must be chosen to be sufficiently small. The calculation is performed iteratively until the established convergence criterion is reached:

$$\|\mathcal{N}^{(\kappa)}(x, \lambda) - \mathcal{N}^{(\kappa-1)}(x, \lambda)\| \leqslant \epsilon, \quad \kappa = 1, 2, \ldots, \tag{11}$$

where κ is the iteration counter and ϵ is the desired calculation accuracy.

Newton–Raphson method has quadratic convergence if the initial design point is chosen in close proximity to the actual solution. However, the Newton–Raphson method may diverge. To prevent such situations, it is necessary to optimally select the Newton iteration step size. The methodology for choosing the optimal step size is presented below.

3.1. Optimal Step-Size Strategy

The Newton–Raphson method is highly sensitive to the initial approximation. In some cases, an improper choice of the initial guess can lead to a large number of iterations or the method may fail to converge altogether. To ensure faster convergence and global convergence with any reasonable initial approximation, the system of equations should be supplemented with a damping coefficient α, as introduced in (10). One of the most efficient and computationally simple methods is to adjust α at each iteration.

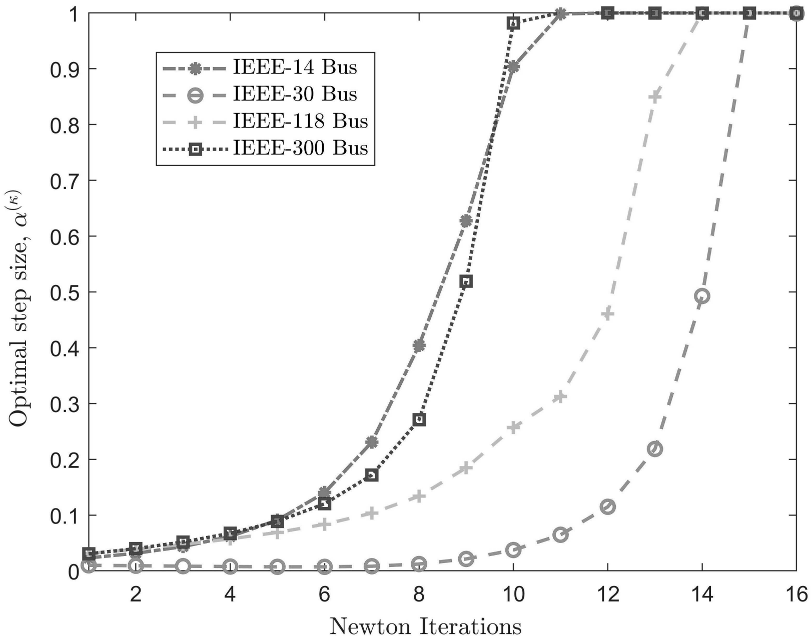

Fig. 3. Optimal step size $\alpha^{(\kappa)}$ selection for each iteration of the TENR method for IEEE test systems.

The original system of equations can be written in a compact form as:

$$\mathcal{H}(z) = \begin{cases} \mathcal{F}(x,\lambda) & = 0 \\ g(x) & = 0. \end{cases} \tag{12}$$

The vector z represents the set of variables $z = [x, \lambda]^\top$. In the Cartesian formulation, the equations considered in (4) are a set of quadratic equations, while the transversality condition $g(x) = 0$ is a linear equation, especially when the transversality condition is expressed through the singular value decomposition of the Jacobian. Therefore, for the point $z^{(\kappa)}$ and the vector $\Delta z^{(\kappa)}$, the system (12) can be approximated by a second-order Taylor expansion as

$$\mathcal{H}(z^{(\kappa)} + \Delta z^{(\kappa)}) \approx \mathcal{H}(z^{(\kappa)}) + [\nabla_z \mathcal{H}(z^{(\kappa)})]\ \Delta z^{(\kappa)} + \frac{1}{2}(\Delta z^{(\kappa)})^\top [\nabla_{zz}\mathcal{H}(z^{(\kappa)})]\ \Delta z^{(\kappa)}. \tag{13}$$

Since all the equations in (4) are quadratic, and $g(x)$ is linear, (13) holds exactly. The optimal step size $\alpha^{(\kappa)}$ in the direction of the vector $\Delta z^{(\kappa)}$ is found by solving the following minimization problem:

$$\mathcal{H}(\alpha) = \left\{ \mathcal{H}(z^{(\kappa)}) + \alpha[\nabla_z \mathcal{H}(z^{\kappa})]\ \Delta z^{(\kappa)} + \frac{\alpha^2}{2}(\Delta z^{(\kappa)})^\top [\nabla_{zz}\mathcal{H}(z^{(\kappa)})]\ \Delta z^{(\kappa)} \right\}, \tag{14}$$

$$\alpha^{(\kappa)} = \underset{\alpha}{\operatorname{argmin}}\ \frac{1}{2}\|\mathcal{H}(\alpha)\|_2^2. \tag{15}$$

The optimization problem (15) can be solved explicitly by applying the first-order optimality condition.

Figure 3 shows the values of $\alpha^{(\kappa)}$ at each iteration of the TENR method for the IEEE test systems consisting of 14, 30, 118, and 300 buses. These test cases include the standard problem of determining the marginal state under initial conditions, where all bus voltage magnitudes are 1, and the corresponding angles are 0. It can be observed that the initially proposed step size strategy leads to small values of α. However, as the algorithm approaches the solution, the step size gradually increases. This behaviour can be explained by the fact that, in the initial iterations, the first-order Taylor approximation of the equations in (12) poorly satisfies the equality. As the algorithm progresses, this approximation becomes more accurate, leading to an increase in the step size α.

4. SAMPLING PARAMETERS OF FEASIBLE REGIMES

For regions with complicated geometry (non-convex, represented by nonlinear equations), which certainly includes the region of feasible power system modes, a working method for obtaining asymptotically uniform samples is based on the use of a version of the Monte Carlo method, namely Markov Chain Monte Carlo (MCMC) [22]. One of the most famous and effective MCMC-type algorithm is called Hit-and-Run (HR), originally proposed in [23], and later rediscovered and analyzed in detail in [24]. Unfortunately, even for simple ill-posed domains (for example, level sets of ill-conditioned functions), the HR method does not work, or at least is computationally inefficient [25].

The variety of applications as well as drawbacks of existing random walk methods open up wide scope for improvement of random walk algorithms. In particular, in the works of B.T. Polyak, it was presented an attempt to use barrier functions (well known in the analysis of interior point methods for convex optimization) and combine them with random walks based on Markov chains. As a result the Barrier Monte Carlo method [26] was proposed, whose mixing properties in some cases turned out to be preferable to the HR method. However, the complexity of each iteration remained quite high (in particular, at each iteration it is required to calculate $\left(\nabla^2 F(x)\right)^{-1/2}$, where $F(x)$ is the barrier function for the region Q). Moreover, this approach cannot accelerate the convergence of the distribution of the resulting points to a uniform one for areas similar to simplexes. Finally, in [27] the idea of Billiard Walk was presented, and theorems on the asymptotic uniformity of generated samples have been proved for the convex and non-convex cases. In contrast to the Ball Walk method, where each subsequent point is selected uniformly random at the intersection of the ball centered at the current point and the region under consideration, and the Hit-and-Run method, where the next point is randomly selected uniformly on a random chord drawn through the current point, the Billiard Walk method is based on a billiard trajectory of random length, released from the current point in a random direction.

The Hit-and-Run method and its improved modification, the Billiard Walk method, provide a useful tool for generating samples in the feasibility domain. The only requirement for a domain is that it must have a boundary oracle procedure and, in the case of a Billiard Walk, a way to recover the normal to the boundary.

Let us describe the application of the Hit-and-Run method and the boundary oracle procedure necessary for its implementation for the power system feasibility domain. The generated samples are located in the multidimensional space of nodal power injections $S_i = P_i + jQ_i$, $i = 1, \ldots, n$, which includes active power P_i and reactive power Q_i.

1. Choose initial regime S^0, $k = 0$. It can be arbitrary feasible point or so-called flat start: $V_i = 1$, $P_i = 0$, $Q_i = 0$; $i = 1, \ldots, n$.
2. Generate random direction d^k, which is uniform random on the unit sphere in R^{2n}. Components of d correspond to increments of active and reactive powers in the righ-had side of the equations (1)–(2).
3. Calculate marginal states in the directions d^k and $-d^k$ as well as corresponding $\overline{\lambda}$, $\underline{\lambda}$ via TENR method.
4. Update $k = k + 1$ and specify the next sample as $S^k = S^{k-1} + td$, where scalar t is uniform random in $[-\underline{\lambda}, \overline{\lambda}]$.
5. Save S^k and correspondng regime parameters. Go to Step 2.

For implementation of Billiard Walk algorithm eigenvector corresponding to zero eigenvalue of the Jacobian should be used as a normal to the boundary.

5. NUMERICAL EXPERIMENTS

Let's illustrate the effectiveness of the TENR method and its modifications for stability margin assessment and generating samples in the power system feasibility domain. Several power systems models from the IEEE collection [28, 29], widely used in academic research, were chosen as examples. The TENR method is integrated into the open-source software package PESOL [30].

5.1. Determination of Marginal States

The accuracy comparison of stability margin assessment was conducted between TENR and three of the most common limit state estimation methods integrated into various software packages: Continuation Power Flow (CPF), Power System Analysis Toolbox (PSAT), and MATPOWER. The comparison results of the λ values, characterizing the stability margins, are presented in Table.

Comparison of the stability margin obtained using the TENR method with analogues (without taking into account voltage constraints)

IEEE-scheme	λ_{TENR}	λ_{CPF}	λ_{PSAT}	λ_{MAT}
9 buses	1.486	1.486	1.481	1.483
14 buses	3.061	3.061	3.059	3.056
30 buses	1.958	1.957	1.959	1.838
57 buses	0.893	0.892	0.891	0.890
118 buses	2.188	2.187	2.187	2.184
300 buses	0.430	0.430	0.429	0.425

The comparison results between the TENR method and its direct competitors show that the stability margin calculated using TENR is not lower than the values obtained using other methods in all considered cases. For some cases, TENR indicated a slightly higher actual stability margin than other methods. The main advantage of TENR is the calculation speed and scalability (calculation of power systems with thousands of buses). A detailed comparison of the calculation speed of TENR with its direct competitors is presented in [14].

5.2. Generation of Parameters for Marginal States

In this work, a five-bus scheme, shown in Fig. 4, is considered as an example. This system is a modified example first presented in [31]: Bus 1 is the slack bus with voltage $\hat{V}_1 = 1.0$. The adopted description in power engineering of complex voltage in polar form $|V|e^{j\delta}$ is used here, where the

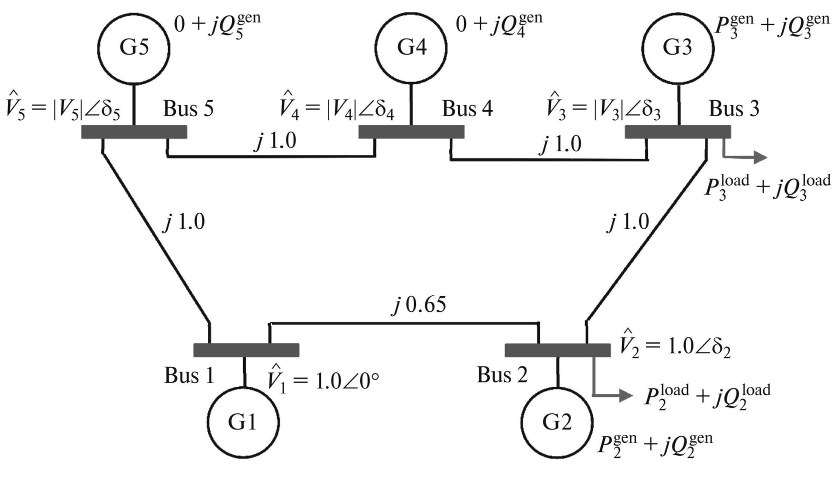

Fig. 4. 5-bus power system test case.

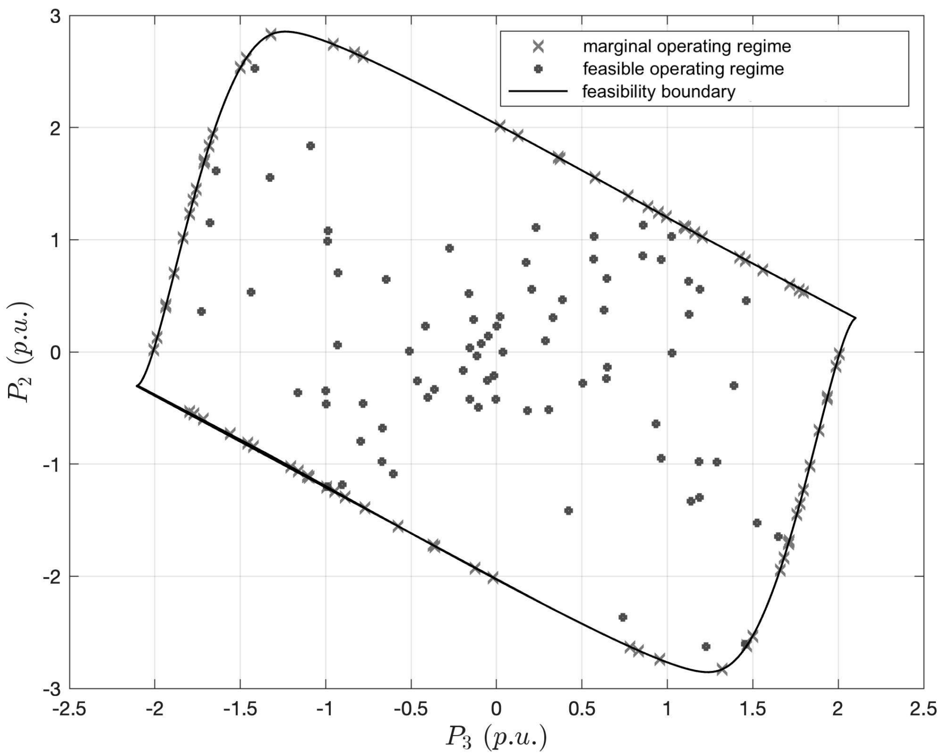

Fig. 5. Cross-section of the feasible region by P_2–P_3 parameter plane with a fixed value $Q_3 = 2$ *p.u.* (80 feasible points).

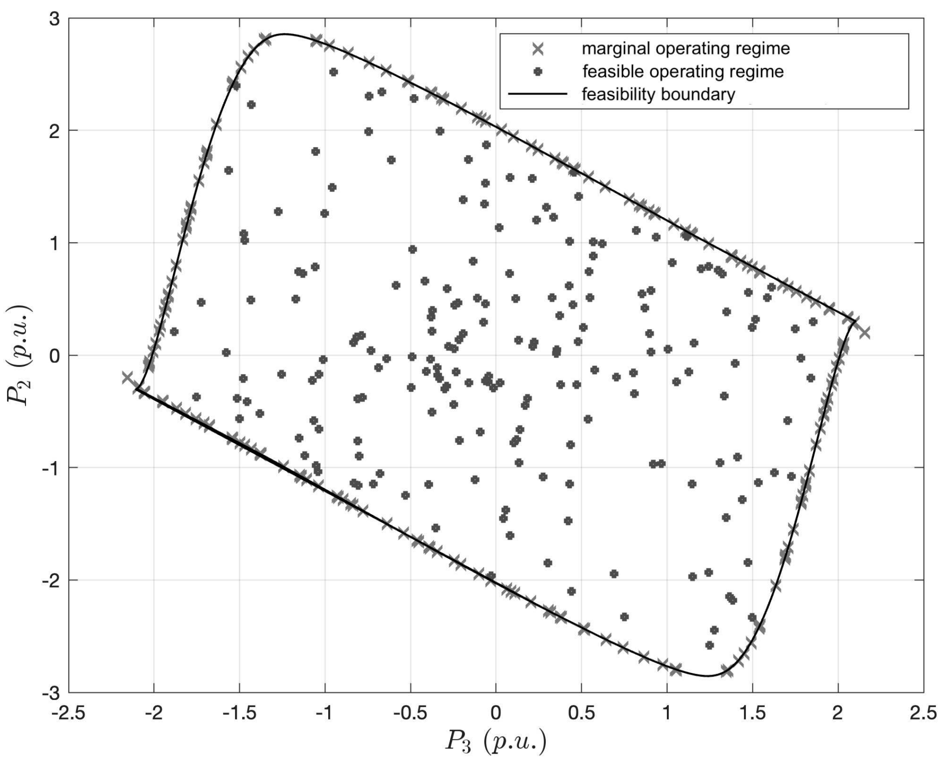

Fig. 6. Cross-section of the feasible region by P_2–P_3 parameter plane with a fixed value $Q_3 = 2$ *p.u.* (200 feasible points).

phase angle is presented following the magnitude in the form $V\angle\delta$. The other buses in the system are PV buses with a fixed voltage value of 1.0 (p.u.), except for Bus 3, which is a PQ bus with a complex voltage value of $\hat{V}_3 = |V_3|e^{j\delta_3}$. It is also assumed that synchronous compensators with zero active power are installed at Buses 4 and 5. Consequently, the solution space is limited to the parameters P_2, P_3, Q_3.

Consider a cross-section of the feasibility region by the P_2–P_3 parameter plane, with the remaining parameters of the right-hand side of the power flow equations fixed as indicated above, $Q_3 = 2$. The results of generating 80 and 200 feasible regimes are shown in Figs. 5 and 6. The dots correspond to the internal points of the feasible operating region, while the crosses represent the limit operating regimes. The figures show that 200 generated feasible operating modes are sufficient for solving practical optimization problems, and the limit modes fairly densely cover the boundary of the permissible region.

6. CONSLUSION

This paper describes the main difficulties encountered in calculating the parameters of critical regimes in power systems and presents the TENR method, which currently appears to be the most effective method for marginal state assessment. Moreover, for the first time, the use of the TENR method for constructing a boundary oracle procedure and generating points within the feasible operating region has been presented and tested.

Surprisingly, the tasks of analyzing feasible and marginal states in power systems draw their solutions from the works of B.T. Polyak. His results on the convexity of the image of quadratic mappings, modifications of the Newton method, and detailed descriptions of random walk schemes for generating points in regions with complex geometry have proven extremely useful for power engineering. The authors do not doubt that researchers will discover many more such connections and bridges in the future.

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