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TOPICAL ISSUE

Interestingness Indices for Building Neural Networks Based on Concept Lattices

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Abstract—The difficulty of interpreting performance of neural networks is a well-known problem, which is attracting a lot of attention. In particular, neural networks based on concept lattices present a promising direction in this area. Selection of formal concepts for building a neural network has a key effect on the quality of its performance. Criteria for selecting formal concepts can be based on interestingness indices, when concepts with the highest values of a certain index are used to build a neural network. This article studies the influence of the choice of an interestingness index on the neural network performance.

Keywords: neural network architecture, Formal Concept Analysis, interestingness indices, neural networks based on formal concept lattices

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1. INTRODUCTION

The difficulty of interpreting the results when working with neural networks is an important problem, which has recently been the subject of many scientific research. One of the proposed solutions is to build a neural network using a concept lattice. In [1], a neural network with an architecture built in accordance with the concept lattice of the original dataset was presented to increase the stability of the classification. In [2] it was proposed to build a neural network based on concept lattice, where formal concepts based on monotonic and antimonotonic Galois connections were used.

However, since the number of formal concepts for a given dataset can grow exponentially with the size of the input data, the important task is to be able to reduce the number of formal concepts in order to build a neural network without losing the quality of its performance. This can be done in two ways — by selecting the most significant attributes (preprocessing), and by selecting the most important formal concepts (postprocessing). In [3] various methods of selecting the most interesting formal concepts based on their interestingness indices were considered. In [4], measures of the interest of concepts were compared in such aspects as the efficiency of finding and the possibility to apply them to noisy data.

In this paper, a study of four interestingness indices was conducted: *basic level*, *target entropy*, Δ-*stability* and *lift* as a criteria for selecting interesting formal concepts. The article is organized as follows:

— Section 2 provides the basic definitions of the theory of formal concept analysis (FCA);

— Section 3 is devoted to theoretical information about the studied interestingness indices;

— Section 4 provides the formulation of the problem and the formal description of the experiment;

— Section 5 explains the neural network architecture;

— Section 6 shows the results of the experiments and the discussion;

— Section 7 provides the conclusions obtained from the results of the work.

2. FORMAL CONCEPT ANALYSIS

Let us turn to the main definitions from the formal concept analysis (FCA) [5]. Let us consider a set of G objects, a set of M attributes, and a binary relation $I \subseteq G \times M$ such that $(g, m) \in I$, if and only if the object g has the attribute m. Such a triple $K = (G, M, I)$ is called *formal context*. Using *derivation operators*, defined for $A \subseteq G$, $B \subseteq M$ as

$$A' = \{m \in M \mid gIm \text{ for all } g \in A\},$$
$$B' = \{g \in G \mid gIm \text{ for all } m \in B\},$$

one defines *formal concept of context* K as a pair of (A, B) such that $A \in G,\ B \in M,\ A' = B$, $B' = A$. A is called *extent*, B is called *intent* of the concept (A, B). Formal concepts are ordered by relation $\geqslant$

$$(A_1, B_1) \geqslant (A_2, B_2) \iff A_1 \supseteq A_2$$

form a complete lattice called *concept lattice* $L = (G, M, I)$.

Covering relation $\prec$ which corresponds to partial order $\leqslant$ (if it exists) is defined as follows:

$$(A_1, B_1) \prec (A_2, B_2) \iff (A_1, B_1) \leqslant (A_2, B_2)$$

and there is no concept (A_3, B_3) such that $(A_1, B_1) < (A_3, B_3) < (A_2, B_2)$.

3. INTERESTINGNESS INDICES

The following is a formal description of the interestingness indices under study:

3.1. Basic Level

For the first time, a general definition of the basic level of the concept was presented in [6].

Informally, cohesion of a formal concept is a measure of similarity between all pairs of objects from the intent of the concept.

According to the idea of E. Rosh, formalized in [6], the concept (A, B) belongs to the basic level if it satisfies the following conditions:

— (BL_1) (A, B) has high cohesion;

— (BL_2) (A, B) has higher cohesion than its upper neighbors (that is the concepts covering the concept (A, B) in the sense of covering relation $\prec$);

— (BL_3) (A, B) has slightly less cohesion than its lower neighbors (i.e., the concepts covered by the concept (A, B) in the sense of covering relation $\prec$).

In a different form:

$$BL(A, B) = \mathcal{C}(\alpha_1(A, B), \alpha_2(A, B), \alpha_3(A, B)), \tag{1}$$

where: $\mathcal{C}(\alpha_1, \alpha_2, \alpha_3) = \alpha_1 \otimes \alpha_2 \otimes \alpha_3$; $\otimes - t$-norm.

To compute this index, it is proposed to use any of the following two well-known formulas for the similarity of sets sim_Y:

$$sim_{SMC}(B_1, B_2) = \frac{|B_1 \cap B_2| + |Y - (B_1 \cup B_2)|}{|Y|}, \tag{2}$$

$$sim_J(B_1, B_2) = \frac{|B_1 \cap B_2|}{|B_1 \cup B_2|}. \tag{3}$$

Next, two formulas for computing the cohesion of a formal concept are introduced:

$$coh^{\varnothing}(A, B) = \frac{\sum_{\{x_1,x_2\} \subseteq A, x_1 \neq x_2} sim(x_1, x_2)}{|A| \cdot (|A| - 1)/2} \tag{4}$$

— the average similarity of the two objects included into the extent of this concept.

On the other hand,

$$coh^{m}(A, B) = \min_{x_1, x_2 \,\in\, A} sim(x_1, x_2) \tag{5}$$

— the smallest degree of similarity of two objects included in the extent of this concept.

Since in [7] the authors conclude that the index based on the cohesion formula $coh^{\varnothing}(A, B)$ gives the best results in selecting the concepts, in this paper only two types of *basic level* index will be used: BL_{ees}, using sim_{SMC}, and BL_{eeJ}, using sim_J, where

$$\alpha_1^{\varnothing} = coh^{\varnothing}(A, B), \tag{6}$$

$$\alpha_2^{\varnothing\varnothing} = 1 - \frac{\sum_{c \,\in\, \mathcal{UN}(A,B)} coh^{\varnothing}(c)/coh^{\varnothing}(A, B)}{|\mathcal{UN}(A, B)|}, \tag{7}$$

$$\alpha_3^{\varnothing\varnothing} = \frac{\sum_{c \,\in\, \mathcal{LN}(A,B)} coh^{\varnothing}(A, B)/coh^{\varnothing}(c)}{|\mathcal{LN}(A, B)|}. \tag{8}$$

3.2. Target Entropy

Target entropy of the formal concept is defined as the variance of the target attribute values of the formal concept.

3.3. Δ-Stability

The stability of a formal concept is its widely used characteristic. However, the complexity of the algorithm for computing it grows exponentially in the number of attributes. As a criterion of a formal concept convenient for calculation, in [8] the stability estimate, Δ-*stability*, was introduced.

$$\Delta(p) = \min(\Delta(p, q)), \quad q < p, \tag{9}$$

$\Delta(p, q)$ is a stability estimate from above.

This value is the minimum difference in the sizes of the extents of the concept and its closest concept from below.

3.4. Lift

According to [9], *lift* is defined as the ratio of the observed joint probability of X and Y to their expected joint probability if they were statistically independent.

In [10] the formula for computing the *lift* interestingness index for a formal concept is given, while it is noteworthy that only the intent of the formal concept and the general set of attributes can be considered:

$$lift(A, B) = \frac{\prod_{b \in B} Pr(b)}{Pr(B)}, \quad \text{where} \quad Pr(\cdot) = \frac{|(\cdot)'|}{|G|}. \tag{10}$$

4. PROBLEM SETTING

Above, four interestingness indices of formal concepts were considered:

1) *Basic Level* (BL_{ees} and BL_{eeJ} were used in this work);

2) Δ-*stability*;

3) *target entropy*;

4) *lift*.

The task was set to study the effect of the choice of indices for criteria for selecting formal concepts (when the set of all formal concepts has been already obtained).

Therefore, this work was carried out according to the following algorithm:

— binarization and preparation of the dataset for processing;

— building a formal context based on a dataset;

— computing the set of formal concepts of a formal context;

— computing each interestingness index for each formal concept;

— sorting formal concepts based on the value of the studied index;

— selecting the k-best formal concepts for building a neural network.

5. NEURAL NETWORK ARCHITECTURE

After selecting the “best” formal concepts, the neural network is built as the covering relation on chosen concepts. The architecture of a neural network based on a concept lattice is organized as follows [2] (Fig. 1):

— the *Input Layer* consists of neurons associated with the attributes of the $m \in M$ context $K = (G, M, I)$;

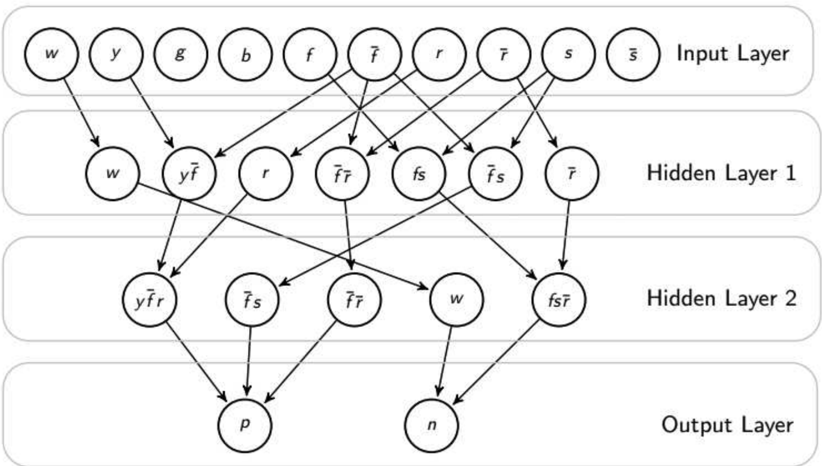

Fig. 1. Architecture of neural network based on the concept lattice.

— *Hidden Layer$_i$*. Each formal concept can be uniquely represented by its intent. Attributes from M are iteratively connected in hidden layers in such a way that neurons corresponding to the selected formal concept are obtained in the last hidden layer;

— *Output layer*. The number of neurons in this layer corresponds to the number of target classes.

6. EXPERIMENTS

FCApy library tools were used to construct formal concepts from a formal context. The functions for computing indices BL_{ees} and BL_{eeJ}, *lift* were written according to the definitions and formulas from the part 3. The built-in capabilities of the FCApy library were used to compute the indices *target entropy* and Δ-*stability*.

When choosing the number of formal concepts, the following criterion was used: the smallest subset of formal concepts covering the entire set of objects:

$\{(A_1, B_1), (A_2, B_2), \ldots, (A_n, B_n)\} : A_1 \cup A_2 \cup \ldots \cup A_n = G$

After computing the interestingness indices, k concepts with the highest value of this index were selected for each index. Further, a neural network was built on the basis of this set of concepts (the capabilities of the neural_lib library, built on the basis of the description from the work [2], were used. This library is based on the PyTorch package).

Its main parameters are: initialization method – ReLU; optimizer: Adam.

Previously, datasets were divided in relation to 70% and 30% into training and test samples. Experiments were conducted with a different number of generations, the best results are presented in the tables.

6.1. Data Description

Four data sets from the UCI library were taken for the analysis (http://archive.ics.uci.edu/ml/) and are pre-binarized. The names and main characteristics of the datasets used are given in Table 1.

Table 1. Characteristics of Datasets

Dataset name	Number of objects	Number of attributes	Number of classes
Heart Disease	303	33	2
House Votes	232	16	2
Car Evaluation	1727	21	4
Iris	150	16	3

All used datasets are balanced, except for the Car Evaluation dataset.

6.2. Experiments with Other ML Methods

Before conducting the main experiments, a number of baseline models were used to analyze the datasets taken (Table 2). As can be seen from the table, the best results of the model are shown on the House Votes and Iris datasets, while the XGBoost model and the Random Forest get the best quality for all datasets.

Table 2. Results of Baseline Models (metric – Accuracy)

Dataset name	Nearest Neighbor Method	Random forest	Naiive Bayes	XGBoost	SVM
Heart Disease	0.83	0.85	0.81	0.81	0.79
House Votes	0.96	0.96	0.94	0.96	0.97
Car Evaluation	0.88	0.95	0.81	0.96	0.91
Iris	0.94	0.94	0.94	0.94	0.92

6.3. Comparison of the Results of the Neural Network for Different Interestingness Indices

Tables 3, 4, 5, and 6 show the results of experiments with interestingness indices. The highlighted color shows results comparable to the quality obtained using baseline models for the same datasets.

— It can be noted that the quality results using the Δ-*stability* index as a criterion for selecting concepts in all four datasets turned out to be comparable with reference models (Nearest Neighbor Method, Random Forest, Naiive Bayes, XGBoost, SVM), whereas the *target entropy* index showed comparable results only in the House Votes set (Table 4).

— The lift index was successful in all experiments except for the Car Evaluation set (Table 5).

— Indices BL_{ees} and BL_{eeJ} showed similar results, but for the Heart Disease set (Table 3), the BL_{ees} index turned out to be more successful and comparable to reference models, unlike BL_{eeJ}.

— The lowest results were obtained for the Car Evaluation dataset (Table 5), which can be explained by its imbalance in the presence of four values of the target attribute.

Table 3. The results of the application of interestingness indices for the selection of concepts of the Heart Disease dataset

	BL_{ees}	BL_{eeJ}	Target entropy	Δ-Stability	Lift
# epochs	8000	6000	8000	6000	7000
Recall	0.88	0.91	0.89	0.96	0.85
F1	0.84	0.80	0.88	0.95	0.84
Accuracy	***0.82***	0.76	0.72	***0.94***	***0.83***
# concepts	7	7	20	7	7

Table 4. The results of the application of interestingness indices for the selection of concepts of the House Votes dataset

	BL_{ees}	BL_{eeJ}	Target entropy	Δ-Stability	Lift
# epochs	5000	2000	3000	2000	3000
Recall	0.85	0.94	0.94	0.97	0.94
F1	0.88	0.91	0.95	0.95	0.95
Accuracy	0.88	0.91	***0.95***	***0.95***	***0.95***
# concepts	7	7	20	7	7

Table 5. The results of the application of interestingness indices for the selection of concepts of the Car Evaluation dataset

	BL_{ees}	BL_{eeJ}	Target entropy	Δ-Stability	Lift
# epochs	5000	5000	5000	5000	5000
Recall	0.44	0.45	0.25	0.47	0.25
F1	0.40	0.41	0.20	0.43	0.20
Accuracy	***0.82***	***0.84***	0.68	***0.87***	0.68
# concepts	7	7	20	7	7

Table 6. The results of the application of interestingness indices for the selection of concepts of the Iris dataset

	BL_{ees}	BL_{eeJ}	Target entropy	Δ-Stability	Lift
# epochs	5000	3000	7000	5000	3000
Recall	0.95	0.95	0.87	0.95	0.95
F1	0.95	0.95	0.86	0.95	0.95
Accuracy	***0.95***	***0.95***	0.86	***0.95***	***0.95***
# concepts	7	7	20	7	7

— The highest quality indicators came from the sets House Votes (Table 4) and Iris (Table 6). These are balanced datasets with a relatively small number of attributes, unlike the rest of the datasets used.

— It is also worth noting that the Δ-*stability* index in all cases showed higher indicators than those shown by other interestingness indices for the same datasets.

7. CONCLUSION

Based on the results obtained, the following conclusions can be drawn:

1) by using interestingness indices, it is possible to obtain a classification quality comparable to the work of reference models;

2) the *target entropy* interestingness index showed the lowest results as compared to other interestingness indices;

3) the *lift* index showed good results, but it failed to classify an unbalanced dataset with several target attributes;

4) interestingness indices of the Basic Level category BL_{ees} and BL_{eeJ} coped with the classification of datasets with a small number of attributes;

5) it can be concluded that Δ-*stability* as a criterion for the selection of formal concepts showed good results both on datasets with a binary target attribute and in classification with several target classes, unlike other indices studied in the work. Its quality indicators are superior to those obtained using other indices.

As a further work, it is planned to study other interestingness indices as criteria for selecting interesting concepts for building neural networks based on them.

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