

Calculation of Centrality in the Analysis of Congestion of City Roads on the Example of Petrozavodsk

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Abstract—Centrality is a numerical measure that characterizes the structural properties of a graph. In the presented work, centrality is used to analyze the load of the graph of urban roads in the city of Petrozavodsk. In the paper, we describe the method used to construct the road graph, present a modified centrality measure that takes into account the features of the transport network and the distribution of passenger traffic, and demonstrate the results of numerical simulations. For the transport graph, betweenness centralities were calculated with and without regard to the distribution of passenger traffic; a connectivity analysis was performed to identify critical, overloaded and reserve roads, and the routes that make the greatest contribution to the centrality of the most loaded roads. The results show that centrality can be used for the analysis of the structural features of the graph of urban roads, modeling sustainability and planning the development of the transport network.

Keywords: graph theory, centrality, transport graph, betweenness centrality

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1. INTRODUCTION

In many applied problems, one needs to characterize, in one way or another, a certain graph of interconnections between objects. The obtained characteristic must evaluate the structural properties of the graph, which then can be interpreted from the point of view of the original applied problem. An example is the problem of analyzing and modeling a transport network. In such a problem, the network is represented by a graph in which edges (transport routes) connect communication nodes (railway stations, airports, warehouses, route transport stops, etc.). From the point of view of the transport problem, analysis of the structural characteristics of the graph makes it possible to simulate the traffic congestion of transport routes, the occurrence of traffic jams due to road accidents, the emergence of new city districts or the introduction of new routes, as well as to calculate optimal public transport routes. One of the tools for assessing the structural properties of a graph is centrality.

Centrality is a characteristic that describes the degree of proximity of a graph vertex to its structural “center.” This characteristic is a real-valued function on a set of vertices and expresses the potential importance of a vertex in terms of the structural properties of the graph, allowing one to rank the vertices. The concept of centrality can also be defined for an edge, an arbitrary subgraph, a subset of vertices, or a subset of edges of the graph.

In applied problems, one observes that the highest centrality belongs, for example, to a vertex with the largest number of neighbors or with the largest number of possible shortest paths passing

through it (see, for example, [1]). In transport network modeling, centrality is used, for example, to numerically express the accessibility of an infrastructure point from any other part of the transport network [3].

This paper presents a centrality-based analysis of traffic congestion on urban roads in Petrozavodsk. We introduce a modified centrality measure, calculate the centrality of vertices and edges, and perform an analysis of changes in centrality characteristics when subsets of edges are removed. The work presents the results of numerical simulations; for the transport graph, betweenness centralities were constructed with and without account of population distribution; a connectivity analysis was carried out to identify critical, overloaded and backup roads, as well as the routes that make the greatest contribution to the centrality of the most congested roads. The results are used to analyze traffic congestion and plan the development of the transport network in Petrozavodsk.

2. CENTRALITY AND THE ANALYSIS OF RELATED WORKS

There are various measures of centrality. The first ones introduced in the literature are betweenness centrality [4, 5] and closeness centrality [6]. There are also a number of other centrality measures, some of which are of particular importance when modeling transport networks [7, 8].

Betweenness centrality is the number of all shortest paths in a graph that involve a considered vertex. This value expresses the number of times a vertex bridges the shortest path between two other vertices.

In applied problems, it is often required to rank not only individual graph vertices, but also subsets of vertices or edges, subgraphs. For example, in transport networks, these can be different routes of public or private transport, groups of road crossings belonging to the same microdistrict of the city, road sections that need to be blocked for repair, etc. Such variants of centrality are studied relatively rarely in the literature.

The concept of centrality was first extended to groups of vertices by the authors of [9]. Mathematical properties of centrality of arbitrary subgraphs are studied in detail in [10, 11, 13]. In mathematical game theory, the characteristics of arbitrary subsets of graph vertices as coalitions of players are analyzed, so the centrality of vertex groups [32, 33] is of great importance. In [34], the concept of centrality is extended to subsets of vertices using fuzzy measures that can be defined for both numerical and categorical characteristics of a graph. Examples of applications of centrality based on fuzzy measures are given in [35].

Various centrality measures for subsets of vertices or edges are used in the analysis of telecommunication networks [19–21], transport networks [22, 24, 25] and other applications [18, 26].

A number of works are devoted to the analysis and development of centrality measures specific to traffic flow modeling and taking into account, for example, travel delay time [15], passenger traffic volume [15, 16], road congestion [17], availability of terrain points in a natural disaster situation [22].

To analyze the transport network, it is necessary to use the characteristics of not only the topological structure of the graph (such as centrality), but also the dynamic processes occurring in it. One of the most important characteristics of such processes is the distribution of traffic in the transport graph. The number of trips made by city residents between different points can be represented in the form of a correspondence matrix. The rows and columns of such a matrix correspond to the points of departure and destination (for example, these can be public transport stops [28], metro stations, city districts), and the values of the matrix elements correspond to the number of people traveling from the point of origin to the point of destination, regardless of the intermediate route. The relationship between centrality and distribution of passenger traffic has been explored in [23, 27–29].

In this paper, we focus on betweenness centrality with account of passenger flow distribution to analyze traffic congestion and the importance of certain roads in the connectivity of the whole transport graph. A similar centrality measure is used, for example, in [29–31] and can also take into account the weighted volume of traffic between two nodes [31] or the dynamically changing volume of traffic during the day [29]. The centrality measure proposed in this work is calculated for edges and takes into account their width, or number of lanes, along with the volume of traffic, and is calculated for both the original graph and the modified ones, serving as the basis for ranking routes.

3. CENTRALITY ANALYSIS OF THE TRANSPORT NETWORK

Centrality analysis was performed basing on the transport system of Petrozavodsk. Petrozavodsk is a city located in the north-west of Russia on the shore of Onego lake and has an extensive transport infrastructure. The population of the city is about 300 thousand people, and is located quite compactly. The active housing construction has exhausted the possibilities of compaction development and leads to the emergence of new densely populated areas on the outskirts of the city. At the same time, the limitation of transport communication is the stretching along the lake, the railway passing through the city and the limited possibilities for expanding transport highways in the central part of the city. All these factors lead to the need of modeling transport communication in the city, the impact of the new residential areas and occurring accidents on the road congestion.

3.1. Transport Graph

The road network graph presented in the work [37] was taken as the basis of the model (the initial data for constructing the graph was obtained from the OpenStreetMap service¹); the vertices of the graph correspond to road intersections, and the edges correspond to sections of highways connecting them. To reduce the dimensionality, courtyard driveways were excluded from the graph. The original version of the graph is shown in Fig. 1 — it has 875 vertices and 1182 edges.

To improve the accuracy of the analysis, the original graph was enriched to include data on one-way traffic, the numbers of road lanes and road lengths. The final model of the road network is a weighted directed graph consisting of 1510 vertices and 3979 edges:

$$G = \{V, (E, D, L)\},$$

where V is the set of vertices (road intersections), E is the set of edges (road segments) connecting them, D is the set of edge lengths, and L is the edge weights (the numbers of road lanes).

The undirected version of the graph has 1510 vertices and 2041 edges; two vertices are connected by an edge if they are connected by an arc in the directed graph. The structural characteristics of the undirected graph of the Petrozavodsk transport network are presented in Table 1.

Table 1. Structural characteristics of the transport network of Petrozavodsk

Nos.	Characteristic	Value
1	Diameter (km/edges)	23.58 / 76
2	Radius (km/edges)	12.02 / 38
3	Total length of edges, L (km)	394.95
4	Average edge length (m)	193.5
5	Area, A (km ²)	100
6	Average vertex degree	2.7
7	Organic coefficient, r_N	0.83
8	Meshedness coefficient, m	0.176
9	Clustering coefficient, c	0.063

¹ OpenStreetMap contributors: Planet dump retrieved from <https://planet.osm.org>.
<https://www.openstreetmap.org> (2022)

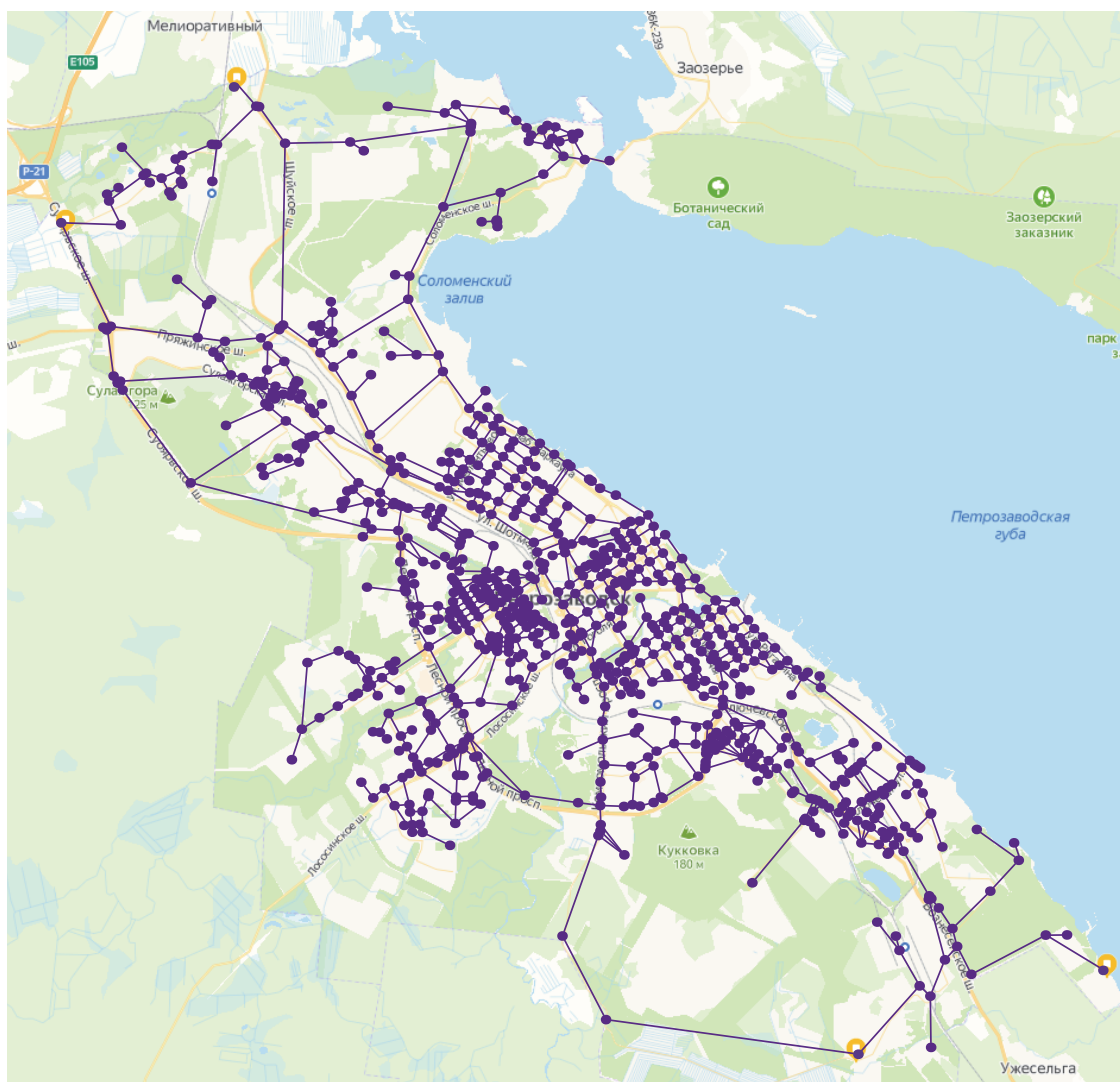


Fig. 1. The simplified transport graph of Petrozavodsk.

The first group of values (1–5) characterizes the geographical properties of the transport network. Compared to large cities, Petrozavodsk has a large share of long bypass roads, because the average distance between road intersections is 193 m, while, for example, in Moscow [40] or large German cities [42] it is 3–5 times less. The transport graph of Petrozavodsk is extended along the lake, therefore it occupies a relatively small area compared, for example, with the city of Hannover [42], where the number of vertices and edges of the transport graph, the average length of the edge are comparable to Petrozavodsk, but the occupied area is two times more. With the exception of this elongation, the natural topography does not affect the structure of the transport network as much as, for example, in Vladivostok, where the average edge length is twice as large [39].

The next group of values (6–9) characterizes the topological properties of the graph. We can conclude that in this transport network, there are quite a lot of T-shaped intersections and dead ends (organic coefficient $r_N = 0.83$ is close to 1 [38]), which is typical for “self-organizing” road networks in such cities such as Vladivostok [39], New Moscow [40], Ahmedabad, Venice and Cairo [41]. Such transport networks developed spontaneously under the influence of the needs of the population, taking into account the natural topography. This is also indicated by the distribution of vertex degrees (Table 2). The meshedness coefficient $m = 0.176$ is low, which indicates that the transport

Table 2. Distribution of vertex degrees of the transport network of Petrozavodsk

Vertex degree	1	2	3	4	5	6
Number of vertices	328	47	887	242	5	1

graph is close to a simple, tree-like structure [41] such as, for example, in New Moscow [40]. The clustering coefficient $c = 0.063$ is low, that is, the graph has few cliques and strongly interconnected subsets of vertices. This, in particular, indicates the vulnerability of the transport network to edge removal and the relevance of the research presented in this paper.

The transport graph G corresponds to the correspondence matrix $C = \{C_{st}\}_{s,t \in V}$. Element C_{st} of the matrix is the number of trips starting at vertex s and ending at vertex t . For the calculations, a correspondence matrix was used, calculated using a gravitational algorithm for the transport graph of the city of Petrozavodsk [37].

3.2. Centrality Calculation

For the transport graph, we calculate edge betweenness centralities considering multiple road lanes

$$B(e) = \frac{1}{l(e)} \sum_{\substack{s,t \in V \\ s \neq t}} \frac{\sigma_{st}(e)}{\sigma_{st}}. \quad (3.1)$$

Here, $e = (i, j)$ is an edge; $l(e)$ is the number of lanes on this edge; $\sigma_{st}(e)$ is the count of the shortest paths $s \rightarrow t$ transitioning through edge e , σ_{st} is the count of all the shortest paths $s \rightarrow t$. The shortest paths are defined considering edge weights (road lengths).

Figure 2a shows centrality values, divided into categories for clarity. To define categories, centrality values were normalized from 0 to 1. The segment $[0,1]$ was divided into $K = 10$ intervals so that the length of the k th interval was proportional to e^k , and the edges with centrality falling in this interval received the k th category.



Fig. 2. Edge centralities in the transport graph of Petrozavodsk: 1 point is the minimal centrality, 10 points is the maximal one.



Fig. 3. Traffic situation at “rush hour” (17:50, working day). Source: Yandex.Maps.

The obtained centrality values correspond to the topology of the graph, taking into account the weights of the edges and multi-lane traffic: thus, the sections of Antikainen and Marshal Meretskov streets, connecting the center with the Kukkovka district through bridges, Kalinin Street, connecting the center with the Kukkovka and Klyuchevaya districts, as well as Solomenskoye Highway, have the highest centralities. More examples of the roads with high centralities are Shotman Street, adjacent to the railway station; located in the city center and adjacent to Lenin Avenue, Kuybyshev Street, Mikhail Isserson Street and Karl Marx Avenue; an important transport interchange: the Leningradskoye Ring, Lesnoy Avenue, connecting the Kukkovka and Drevlyanka districts, as well as the Vytegorskoye Highway, connecting the Kukkovka and Klyuchevaya districts.

However, such a centrality measure uses only the graph structure in the calculation without taking into account the number of people transported. To eliminate this shortcoming, we include a correspondence matrix in our consideration.

Due to the fact that any pair of vertices in the transport graph under analysis is connected by exactly one shortest path (this has been verified numerically), one may assume that all trips are made along a single shortest path. Then, one can calculate the centrality of edge $e = (i, j)$ in a directed graph taking into account correspondences using the formula

$$BC(e) = \frac{1}{l(e)} \sum_{\substack{s, t \in V \\ s \neq t}} C_{st} \delta_{st}(e), \quad (3.2)$$

where d_{st} is the distance between vertices s and t ,

$$\delta_{st}(e) = \begin{cases} 1, & d_{st} = d_{si} + d_{ij} + d_{jt}, \\ 0, & \text{otherwise.} \end{cases}$$

Figure 2b shows the obtained centrality values. Taking into account correspondence, the sections of Antikainen and Marshal Meretskov streets, Lenin Avenue, Pravdy Street, Kalinin Street, Vysotny Proezd have the greatest centrality. Shotman Street, Karl Marx Avenue, Murmanskaya Street, and Gogol Street have high centrality. In general, the obtained estimates of the centrality of street sections, taking into account normalization, are consistent with empirical observations (see Fig. 3). The centrality distribution density is presented in Fig. 4.

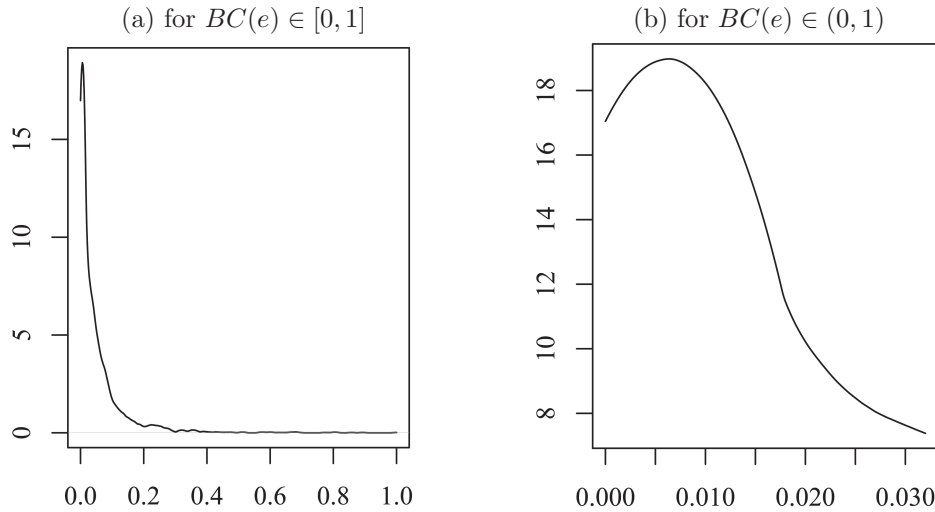


Fig. 4. Density distribution of edge centralities of the transport graph of the city of Petrozavodsk $BC(e)$.

3.3. Connectivity Analysis

Edge centrality depends on the topology of the graph. At the same time, in applied problems, it is often necessary to investigate the properties of a modified transport graph where some vertices and/or edges have been removed or added. For example, it may be necessary to predict traffic congestion when a particular street is closed for repairs, accidents occur, or a railway crossing is closed for a long time.

To study the properties of the modified transport graph, computational experiments were carried out to remove a randomly selected subset of edges. 10 000 computational experiments were conducted to remove 10 to 53 edges, selected randomly with probabilities proportional to their centralities, while maintaining connectivity. For each modified graph, the centralities of the remaining edges were calculated and normalized. Figure 5 shows the dynamics of centrality using the example of three road sections.

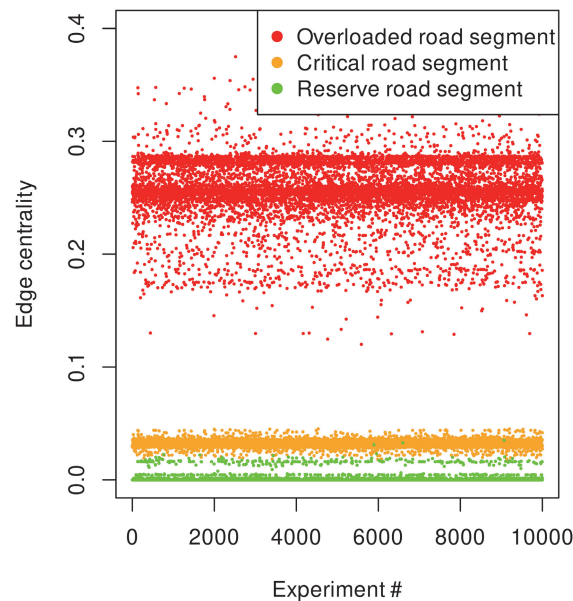


Fig. 5. Dynamics of centrality during an experiment on the example of road sections of three different types.



Fig. 6. Critical road sections: edges with high centrality in the original graph, high mean centrality, and low sample variance for random graph modifications. These sections may be heavily loaded during graph modifications.



Fig. 7. Reserve road sections: edges that have low centrality in the original graph and change it greatly with random modifications of the graph. These sections may be heavily loaded when other routes are unavailable.



Fig. 8. Overloaded road sections: edges that have high centrality in the original graph and significantly reduce it with random modifications of the graph.

The experimental results are used to analyze the transport situation during disruptions in the transport network. First, roads of interest are those that maintain high load despite graph modifications. Such roads can be called “critical” ones. Second, the roads that become highly loaded during graph modifications are important — such roads can be called “backup” ones. Finally, we call the roads that significantly reduce their load level “overloaded”. Figures 6, 7 and 8 show examples of critical, backup and overloaded roads in the central part of the city.

3.4. Loaded Routes

In practice, the load of individual routes of a transport network may be a more indicative characteristic than the load of specific edges. At the same time, it is important to highlight the routes that make the greatest contribution to the load of the “bottlenecks” of the transport network. Table 3 presents the characteristics of traffic flow passing through the most loaded road sections. Enumerating all possible routes in a graph is a very computationally intensive task, so to highlight the most important routes, it is advisable to use, for example, the method described in this section.

Table 3. Characteristics of traffic flow for the 10 most loaded edges of the transport graph of Petrozavodsk

Egde	Number of shortest paths	Traffic volume
1	29063 (0.35%)	23201.5 (8.5%)
2	11277 (0.13%)	20707.9 (7.6%)
3	26084 (0.31%)	19912.8 (7.3%)
4	25983 (0.31%)	18762.1 (6.9%)
5	19288 (0.23%)	18573.0 (6.8%)
6	26143 (0.31%)	17949.6 (6.6%)
7	10843 (0.13%)	15994.1 (5.8%)
8	10309 (0.12%)	15731.0 (5.7%)
9	30550 (0.37%)	15582.6 (5.7%)
10	20486 (0.24%)	15200.0 (5.6%)



Fig. 9. A subset of the most loaded routes in the transport graph. The route that makes the greatest contribution to the load of the edge with the highest centrality is highlighted in dark red: Pervomaisky Avenue, Antikainen Street and Marshal Meretskov Street.

Consider the edge e^* with maximal centrality in the original graph, calculated by the formula (3.2). In total, 29 063 of shortest paths pass through this edge, which corresponds to a traffic flow of 23 201.5 trips (8% of city traffic). The largest contribution comes from the 17-edge path, shown in dark red in Fig. 9, with a traffic flow of 1019.1 trips. Moreover, this path is part of the 7% shortest paths passing through the edge e^* . Also shown in red in the figure is a subset of the most loaded routes, obtained in a similar way for the ten edges with the highest centrality.

4. CONCLUSION

Centrality is a common tool for graph analysis with applications in various application domains. Betweenness centrality, one of the commonly used measures, characterizes the structural properties of a graph by listing the shortest paths between all pairs of vertices.

In the presented work, we use modified betweenness centrality (taking into account the amount of transported population) to analyze the road load based on a road graph corresponding to the transport network of Petrozavodsk. The resulting distribution of normalized centrality values

corresponds to empirical data (road congestion during rush hour), which suggests that the proposed modified betweenness centrality measure can be used to analyze the city's transport network. We also identified the passenger flow routes that make the greatest contribution to the high centrality of the most loaded roads.

Further work was related to connectivity analysis, i.e., analysis of the changes in the load of individual roads when removing edges that do not violate the connectivity of the graph. In the experiments, 10 to 53 edges were randomly removed, which simulates the blocking of individual roads when road accidents occur. At the same time, the probability of removing an edge was set proportional to its load, because with all other things being equal, accidents are more likely to occur on busier roads. As a result, we identified the roads that are consistently loaded upon such edge removals. These roads form the “backbone” of the transport system.

On the other hand, a set of roads was identified that are not loaded in the original road graph, but significantly increase their centrality when edges are randomly removed. Such roads serve as backup; they become important transport arteries in case of accidents in the city.

Finally, the third type of roads are overloaded ones, those that have high centrality but significantly reduce it when edges are randomly removed.

The obtained results allow us to conclude that the modified betweenness centrality measure is a powerful tool for analyzing the city's transport network. The proposed analysis scenarios provide important information that can be used when planning the development of urban transport infrastructure. In this case, as for other approaches to analyzing a transport problem, the initial data associated with people's mobility and the structure of the road graph are critically important.

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