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Game-Theoretic Analysis of the Interaction of Economic Agents in the Cournot Oligopoly with Consideration of the Linear Structure, the Green Effect and Fairness Concern

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Abstract—A comparative analysis of the efficiency of various ways of organization of economic agents is carried out, taking into account the structure and regulations of their interaction in the models of the Cournot oligopoly. The Cournot oligopoly models in the form of a supply chain are constructed and analytically investigated, taking into account the green effect and concern for fairness. For symmetric models of the Cournot oligopoly with different ways of organization of economic agents, the respective structures of social and individual preferences are analytically obtained. A numerical study of the Cournot oligopoly models in various forms with asymmetrical agents has been carried out, and the corresponding structures of social and individual preferences have been obtained.

Keywords: Cournot oligopoly, supply chain, fairness concern, green effect, preference structures

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1. INTRODUCTION

1.1. Problem of the Inefficiency of Equilibria

A problem of the inefficiency of equilibria plays an important role in the game theoretic modeling of social-economic systems. A detailed analysis of the problem is presented in [1, 6, 15, 22, 28]. The outcome of rational behavior by selfish players can be inferior to a centrally designed or voluntarily cooperative outcome. An important question is: by how much? To answer this question a specific payoff function is introduced. It is defined on the set of outcomes of the game, and numerically expresses “social good” of an outcome. Two main payoff functions are utilitarian and egalitarian functions, defined as the sum of the players’ payoffs and the minimum player payoff, respectively. Introducing a payoff function enables us to quantify the inefficiency of equilibria, and in particular to deem certain outcomes of a game optimal or approximately optimal. All of these measures are defined as the ratio between the payoff function value of an equilibrium of the game and that of an optimal outcome. It is supposed that all payoff functions are positive, and the ratio is also positive.

The price of anarchy, the most popular measure of the inefficiency of equilibria, resolves the issue of multiple equilibria by adopting a worst-case approach. The price of anarchy of a game is defined as the ratio between the worst payoff function value of an equilibrium of the game and that of an optimal outcome [27]. If the price of anarchy is close to 1 then all equilibria are good approximations of an optimal outcome, and a profit from the centralized planning (that can be expensive or even not implemented) is small [1].

However, the problem of inefficiency of equilibria should be formulated in a more general form. First, we should compare not only the payoffs corresponding to the basic ways of organization of economic agents (competition, cooperation, hierarchy) but also the payoffs with consideration of different additional effects (information structure of the agents' interaction). Second, it is necessary to not only the values of social welfare but also the payoffs of separate economic agents. An outcome generated by a specific way of organization that is the most effective fir the whole society is not obligatory the same for each agent. For example, the leader's payoff in a hierarchical game can exceed a share of a player under a symmetrical allocation of the cooperative payoff.

Basic ways of organization of the interaction of economic agents are their selfish behavior (competition), hierarchy and cooperation. Competing agents choose their actions simultaneously and independently, and the solution of the respective game in normal form is a Nash equilibrium. A hierarchical organization allows for two variants. In the first variant the leader chooses and reports to one or several other players (followers) her action, and they choose the best response. Then we have a Stackelberg game, and its solution is a Stackelberg equilibrium. In the second variant the leader chooses and reports to the followers her strategy as a function of their expectable actions, and they choose the best response to this strategy. Then we have an inverse Stackelberg game, and its solution is found on the base of the guaranteed result principle. At last, in the case of cooperation all players unite and jointly maximize the summary payoff function by all control variables. Then a respective game is reduced to an optimization problem, and its cooperative solution is Pareto-optimal [3, 10, 24, 26].

When accounting a structure of interaction of the economic agents a concept of supply chain management takes an important place [14]. A supply chain is an ordered set of economic agents that provides production of a good and its transfer from a manufacturer to a consumer. It is natural to model the respective interaction of agents in a supply chain as a Stackelberg game [5]. In the simplest case the game includes a manufacturer and a seller, in a more general case other value-adding mediators are considered. Performance and evaluation of efficiency of supply chains are described in [7]. In a marketing context similar models are considered in [16]. Ivanov and Dolgui [13] propose a concept of intertwined supply chains. This model considers connections (and feedback) between several supply chains. The authors pay the most attention to the issues of stability and viability of supply chains.

A very interesting stream of research in the recent years in this domain is green supply chains [35, 36, 23, 2, 11, 17, 33]. Here the elements of a supply chain make some efforts to mitigate a negative environmental impact of the production and logistics. It is assumed that such an activity increases a demand to the green products from environmentally responsible consumers. The respective reviews are presented in [4, 32, 29, 8, 12]. Note that the agents in a supply chain can invest their resources not only in environmental protection but also to the innovative potential or social utility of their production that also increases the demand.

That approach is developed by a concept of fairness concern [9, 31]. It is well known that a postulate of economic rationality which is a base of the game theory does not describe completely all behavioral incentives. Thus, there are some attempts to consider in game theoretic models some additional effects. The fairness concern means that if an agent regards a payoff allocation to be unfair then he can abandon business relationships [25]. To avoid it a leader's payoff function in a Stackelberg game is extended by a penalty for essential difference between payoffs of the leader and the followers. The fairness concern changes essentially the players' strategies [18, 19]. Sharma and Jain [30] investigate the fairness concerned behavior in a dyadic supply chain with one manufacturer and one retailer, wherein the manufacturer puts efforts for improving the product's greening level and sells it to the customers through the retailer.

1.2. Concept, Contribution and Structure of the Paper

This paper attempts to compare an efficiency of the different ways of organization of economic agents (selfish behavior, hierarchy, cooperation) with consideration of additional features of the structure and rules of their interaction. These features are supply chains, green effect and fairness concern. The comparison of efficiency is made from the point of view of both social welfare and separate agents. For a quantitative evaluation of the comparative efficiency we calculate ratios of summary or individual payoffs corresponding to the different types of organization. In symmetrical models which can be investigated analytically these indicators are equivalent. For models with asymmetrical agents these two cases are analyzed numerically. The comparison implies structures of social and private preferences (coincident or not).

A convenient tractable model for the mentioned comparative analysis is the Cournot oligopoly [20] that describes the competition of manufacturers by their output volumes of a homogeneous good. A general theory of oligopoly is exposed in [34]. A game theoretic analysis of the Cournot oligopoly is presented in [21]. In this paper we consider the models of Cournot oligopoly with constant and scale-dependent costs. The paper makes the following contribution:

— we build and analytically investigate new models of the Cournot oligopoly in the form of supply chain, with green effect and fairness concern;

— for symmetrical models of the Cournot oligopoly and different ways of organization of economic agents we receive analytically coincident structures of social and individual preferences;

— we conduct a numerical simulation of the models of Cournot oligopoly in different forms with asymmetrical agents, and receive the respective structures of social and individual preferences.

The rest of the paper is organized as follows. In Chapter 2 we build and analytically investigate symmetrical models of the Cournot oligopoly in different forms with two types of costs. In Chapter 3 we conduct an analytical comparison of the efficiency of the ways of organization of economic agents in the built models. In Chapter 4 we present the results of numerical simulation for asymmetrical models and the respective structures of social and individual preferences. Conclusive remarks are made in Chapter 5.

2. MODELS AND THEIR ANALYTICAL INVESTIGATION

2.1. A Basic Model of the Cournot Oligopoly

2.1.1. A selfish behavior of economic agents. In this case a model of the Cournot oligopoly describes a competition of n economic agents (manufacturers, firms and so on) producing a homogeneous good. Denote x_i – an output volume of the ith agent, $x = (x_1, \dots, x_n)$, $\overline{x} = \sum_{i=1}^{n} x_i$. The model includes two main elements: an inverse demand function and a cost function. The inverse demand function $Q(x)$ is the same for each agent and has the form $Q(x) = a - \overline{x}$, where a is a maximally feasible total output volume. A value $Q(x)$ shows a price for a unit of the good. The cost function $C_i(x_i)$ is considered in two forms. For constant costs $C_i(x_i) = c_i x_i$ a parameter c_i shows a cost for production of a unit of the good. For scale-dependent costs $C_i(x_i) = c_i x_i - d_i x_i^2$ a parameter d_i reflects that a cost for production of a unit of the good decreases when the number of units increases. In general, a payoff (profit) of the ith agent is equal to $u_i(x) = Q(x)x_i - C_i(x_i)$. Then for constant costs the model is

$$u_i(x) = (a - c_i - \overline{x})x_i \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n. \tag{2.1}$$

and for scale-dependent costs

$$u_i^d(x) = (a - c_i - \overline{x})x_i - d_i x_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n. \tag{2.2}$$

Each of the models (2.1) and (2.2) defines a game of n agents in normal form. Its solution is a Nash equilibrium. For an analytical investigation of the models of Cournot oligopoly we accept

Proposition 2.1. $c_i = c,\ d_i = d,\ i = 1, \dots, n.$

This assumption generates symmetrical games

$$u_i(x) = (a - c - \overline{x})x_i \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n, \tag{2.3}$$

$$u_i^d(x) = (a - c - \overline{x})x_i - d x_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n, \tag{2.4}$$

and their solution are found in an explicit form. Suppose for convenience that $a > c > 0$. Solving a system of equations $\frac{\partial u_i}{\partial x_i} = 0$, $i = 1, \dots, n$, we receive for the model (2.3)

$$x_i^{NE} = x_{NE} = \frac{a - c}{n + 1}, \quad i = 1, \dots, n. \tag{2.5}$$

As $\frac{\partial^2 u_i}{\partial x_i^2} = -2 < 0$, $|H| = \begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix} = 3 > 0$ then the formula (2.5) really determines a Nash equilibrium in the game (2.3). Each agent's payoff is equal to

$$u^{NE} = \frac{(a - c)^2}{(n + 1)^2}, \quad i = 1, \dots, n. \tag{2.6}$$

For the model (2.4) we receive similarly

$$x_d^{NE} = \frac{a - c}{n + 1 + 2d}, \quad u_d^{NE} = \frac{(1 + d)(a - c)^2}{(n + 1 + 2d)^2}.$$

2.1.2. Cooperative behavior of economic agents. In this case the economic agents of an oligopoly unite (for example, in a cartel) and jointly maximize a total payoff (a utilitarian social welfare function) $\overline{u}(x) = \sum_{i=1}^{n} u_i(x)$ by all control variables x_i, $i = 1, \dots, n$. Then the model (2.3) takes the form

$$\overline{u}(x) = (a - c - \overline{x})\overline{x} \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n, \tag{2.7}$$

and the model (2.4) takes the form

$$\overline{u}^d(x) = (a - c - \overline{x})\overline{x} - d\sum_{j=1}^{n} x_j^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n. \tag{2.8}$$

The symmetrical cooperative solution of the model (2.7) has the form

$$x^C = \frac{a - c}{2n}; \quad \overline{x}^C = \frac{a - c}{2}, \quad \text{and} \quad u^C = \frac{(a - c)^2}{4n}; \quad \overline{u}^C = \frac{(a - c)^2}{4}.$$

For the model (2.8) we receive similarly

$$x_d^C = \frac{a - c}{2(n + d)}; \quad \overline{x}_d^C = \frac{n(a - c)}{2(n + d)} u_d^C = \frac{(a - c)^2}{4(n + d)}; \quad \overline{u}_d^C = \frac{n(a - c)^2}{4(n + d)}.$$

2.1.3. Presence of a leader firm. Suppose that agent (firm) 1 is a leader. She chooses and reports to other agents her output volume x_1. Given x_1 the other agents (followers) find a Nash equilibrium in their game in normal form. We consider that equilibrium as a best response to the strategy x_1.

Then in fact the firm 1 chooses her strategy x_1 that maximizes her payoff on the set of Nash equilibria in the game of other agents. The respective outcomes form a set of Stackelberg equilibria $ST1$ in the Cournot oligopoly with leadership of the first firm. Using the first order condition $\frac{\partial u_i}{\partial x_i} = 0$, $i = 1, \ldots, n$, we receive

$$2x_i + \sum_{j=2(j\neq i)}^{n} x_j = a - c - x_1, \quad i = 2, \ldots, n,$$

and

$$x_i = x, \quad i = 2, \ldots, n,$$
$$2x + (n-2)x = a - c - x_1,$$
$$x^{NE}(x_1) = \frac{a - c - x_1}{n};$$
$$u_1(x_1, x_{-1}^{NE}(x_1)) = \frac{1}{n}(a - c - x_1)x_1; \quad \frac{\partial u_1}{\partial x_1} = \frac{1}{n}(a - c - 2x_1) = 0;$$
$$x_1^{ST1} = \frac{a-c}{2}; \quad x_i^{ST1} = \frac{a-c}{2n}, \quad i = 2, \ldots, n; \quad \overline{x}^{ST1} = \frac{(2n-1)(a-c)}{2n};$$
$$u_1^{ST1} = \frac{(a-c)^2}{4n}; \quad u_i^{ST1} = \frac{(a-c)^2}{4n^2}, \quad i = 2, \ldots, n; \quad \overline{u}^{ST1} = \frac{(2n-1)(a-c)^2}{4n^2}.$$

For the model with scale-dependent costs we receive similarly

$$x_{1d}^{ST1} = \frac{a-c}{2(1+d)}; \quad x_{id}^{ST1} = \frac{(1+2d)(a-c)}{2(1+d)(2d+n)}, \quad i = 2, \ldots, n; \quad \overline{x}_d^{ST1} = \frac{(2n-1+d(1+n))(a-c)}{2(1+d)(2d+n)};$$
$$u_{1d}^{ST1} = \frac{(a-c)^2(2d^2+4d-dn+1))}{4(1+d)^2(2d+n))}; \quad u_{id}^{ST1} = \frac{(a-c)^2(2d+1)^2}{4(1+d)(2d+n)^2}, \quad i = 2, \ldots, n;$$
$$\overline{u}_d^{ST1} = \frac{(a-c)^2(4d^3n+8d^2+8d^2n-dn^2+9dn+6d+2n+1)}{4(1+d)^2(2d+n)^2}.$$

2.2. The Cournot Oligopoly with Consideration of Green Effect

2.2.1. A selfish behavior of economic agents. A green effect in the model of Cournot oligopoly means that the agents assign additional resources to make the production environmentally friendly. Denote the respective greening efforts of the ith agent by g_i, $g = (g_1, \ldots, g_n)$, $\overline{g} = \sum_{i=1}^{n} g_i$. Denote also α — a coefficient characterizing increase in demand due to green effect, β_i — a coefficient characterizing a greening cost of the ith agent. Then the model (2.1) takes the form

$$u_i^G(x, g) = (a - c_i - \overline{x} + \alpha\overline{g})x_i - \beta_i g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n.$$

For analytical investigation we consider a symmetrical model using an additional

Proposition 2.2. $\alpha = \beta_i = 1, i = 1, \ldots, n.$

Then

$$u_i^G(x, g) = (a - c - \overline{x} + \overline{g})x_i - g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n. \tag{2.9}$$

Theorem 2.1. *In the model* (2.9) *equilibrium strategies of the agents have the form*

$$x^{GNE} = \frac{2(a-c)}{n+2}, \quad g^{GNE} = \frac{a-c}{n+2},$$

and each agent's payoff is equal to

$$u^{GNE} = \frac{3(a-c)^2}{(n+2)^2}.$$

The proof of the Theorem 2.1 is given in Appendix A.

Theorem 2.2. *In the model of Cournot oligopoly with scale-dependent costs and green effect*

$$u^G_{id}(x,g) = (a - c - \overline{x} + \overline{g})x_i - dx_i^2 - g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n.$$

equilibrium strategies of the agents and the respective payoffs have the form

$$x_d^{GNE} = \frac{2(a-c)}{n+2+4d}, \quad g_d^{GNE} = \frac{a-c}{n+2+4d}, \quad u_d^{GNE} = \frac{(3+4d)(a-c)^2}{(n+2+4d)^2}.$$

The proof is similar to the proof of Theorem 2.1.

2.2.2. Cooperative behavior of the economic agents. When agents cooperate and there are a green effect and constant costs the model takes the form

$$u_i^{GC}(x,g) = (a - c - \overline{x} + \overline{g})x_i - \sum_{j=1}^{n} g_j^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n, \tag{2.10}$$

and for scale-dependent costs

$$u_{id}^{GC}(x,g) = (a - c - \overline{x} + \overline{g})x_i - d\sum_{j=1}^{n} x_j^2 - \sum_{j=1}^{n} g_j^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n. \tag{2.11}$$

Theorem 2.3. *In the model* (2.10) *optimal cooperative strategies of the agents and their total payoff are equal respectively to*

$$x^{GC} = \begin{cases} \dfrac{2(a-c)}{n(4-n)}, & n < 4, \\ 0, & \textit{otherwise}; \end{cases} \quad g^{GC} = \begin{cases} \dfrac{a-c}{4-n}, & n < 4, \\ 0, & \textit{otherwise}; \end{cases} \quad u^{GC} = \begin{cases} \dfrac{(a-c)^2}{n(4-n)}, & n < 4, \\ 0, & \textit{otherwise}. \end{cases}$$

Theorem 2.4. *In the model* (2.11) *optimal cooperative strategies of the agents and their total payoff are equal respectively to*

$$x_d^{GC} = \frac{2(a-c)n)}{n(4-n)+4d)}, \quad g_d^{GC} = \frac{(a-c)n}{n(4-n)+4d}, \quad u_d^{GC} = \frac{(a-c)^2(4d+4n-4dn-n^2)}{(n(4-n)+4d)^2}$$

when $n \leqslant 2 + 2\sqrt{1+d}$.

The proof of the Theorem 2.4 is similar to the proof of Theorem 2.3. If then green production is not profitable.

2.2.3. Presence of a leader firm. Suppose that agent (firm) 1 is a leader. The information structure of the game is the same as in the Section 2.1.3, and all firms equally care for a green effect. Each of agents except the leader solves the problem

$$u_k(x,g) = \left(a - c - x_1 + g_1 - \sum_{j=2}^{n} x_j + \sum_{j=2}^{n} g_j\right) x_k - g_k^2 \to \max, \quad k = 2, \ldots, n.$$

The first order conditions $\frac{\partial u_k}{\partial x_k} = \frac{\partial u_k}{\partial g_k} = 0$, $k = 2, \ldots, n$ give

$$g = \frac{a - c - x_1 + g_1}{n+1}; \quad x = \frac{2(a - c - x_1 + g_1)}{n+1}.$$

A substitution of the found values in u_1 gives

$$u_1(x_1, g_1) = \frac{1}{n}(a - c - x_1 + g_1)x_1 - g_1^2.$$

The first order conditions $\frac{\partial u_1}{\partial x_1} = \frac{\partial u_1}{\partial g_1} = 0$ give

$$g_1^{ST1G} = \frac{a-c}{3}; \quad g_i^{ST1G} = \frac{2(a-c)}{3(n+1)};$$
$$x_1^{ST1G} = \frac{2(a-c)}{3}; \quad x_i^{ST1G} = \frac{4(a-c)}{3(n+1)}, \quad i = 2, \ldots, n.$$

At last, an immediate substitution gives

$$u_1^{ST1G} = \frac{8(a-c)^2}{9(n+1)}; \quad u_i^{ST1G} = \frac{16(a-c)^2}{9(n+1)^2}, \quad i = 2, \ldots, n;$$
$$\overline{u}^{ST1G} = \frac{8(3n-1)(a-c)^2}{9(n+1)^2}.$$

We omit a case of the scale-dependent costs as too cumbersome.

2.3. The Cournot Oligopoly with Green Effect in a Supply Chain

Consider the Cournot oligopoly in the form of a supply chain (Fig. 1).

$$1 \to 2 \to \ldots \to n \to$$

Fig. 1. The Cournot oligopoly in the form of a supply chain.

A connection between agents in the linear structure of a supply chain is provided by the green effect for each agent. The green effect's action is one-way and the stronger the closer are situated the respective agents. We receive the model

$$u_i(x, g) = \left(a - c_i - \overline{x} + \sum_{j=2}^{n} \alpha^{i-j} g_j\right) x_i - \beta_i g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n. \tag{2.12}$$

Accept again Propositions 2.1 and 2.2 for an arbitrary $\alpha > 0$ and $n = 2$. Then for constant costs the model (2.12) takes the form

$$u_1^{SG}(x, g) = (a - c - x_1 - x_2 + g_1)\, x_1 - g_1^2 \to \max, \quad x_1 \geqslant 0, \quad g_1 \geqslant 0, \tag{2.13}$$
$$u_2^{SG}(x, g) = (a - c - x_1 - x_2 + g_2)\, x_2 - g_2^2 \to \max, \quad x_2 \geqslant 0, \quad g_2 \geqslant 0, \tag{2.14}$$

and for scale-dependent costs

$$u_{1d}^{SG}(x, g) = (a - c - x_1 - x_2 + g_1)\, x_1 - dx_1^2 - g_1^2 \to \max, \quad x_1 \geqslant 0, \quad g_1 \geqslant 0, \tag{2.15}$$
$$u_{2d}^{SG}(x, g) = (a - c - x_1 - x_2 + \alpha g_1 + g_2)\, x_2 - dx_2^2 - g_2^2 \to \max, \quad x_2 \geqslant 0, \quad g_2 \geqslant 0. \tag{2.16}$$

Theorem 2.5. *In the model* (2.13)–(2.14) *equilibrium strategies of the agents and their payoffs have the form*

$$x_1^{SCNE} = \frac{2(a-c)}{5+2\alpha}; \quad x_2^{SCNE} = \frac{2(a-c)(1+\alpha)}{5+2\alpha};$$
$$g_1^{SCNE} = \frac{(a-c)}{5+2\alpha}; \quad g_2^{SCNE} = \frac{(a-c)(1+\alpha)}{5+2\alpha};$$
$$u_1^{SCNE} = \frac{3(a-c)^2}{(5+2\alpha)^2}; \quad u_2^{SCNE} = \frac{3(a-c)^2(1+\alpha)^2}{(5+2\alpha)^2}.$$

Theorem 2.6. *In the model* (2.13)–(2.14) *equilibrium strategies of the agents and their payoffs have the form*

$$x_{1d}^{SCNE} = \frac{2(a-c)(1+4d)}{16d^2+24d+5+2\alpha}; \quad x_{2d}^{SCNE} = \frac{2(a-c)(1+4d+\alpha)}{16d^2+24d+5+2\alpha};$$
$$g_{1d}^{SCNE} = \frac{(a-c)(1+4d)}{16d^2+24d+5+2\alpha}; \quad g_{2d}^{SCNE} = \frac{(a-c)(1+4d+\alpha)}{16d^2+24d+5+2\alpha};$$
$$u_{1d}^{SCNE} = \frac{(3+16d+16d^2)(1+4d)(a-c)^2}{(16d^2+24d+5+2\alpha)^2};$$
$$u_{2d}^{SCNE} = \frac{(3(1+\alpha)+4d(4-\alpha+4d))(a-c)^2(1+4d+\alpha)^2}{(16d^2+24d+5+2\alpha)^2}.$$

The proofs of the Theorems 2.5 and 2.6 are similar to the previous proofs.

2.4. A Hierarchical Control in the Cournot Oligopoly for Achievement of the Green Effect

2.4.1. A basic hierarchical model. Suppose now that an agent external to the Cournot oligopoly is interested to provide the green effect. Call this agent a Principal (P). To provide the green effect the Principal appoints obligatory costs of increasing the environmental features of the products g_i, $i = 1, \dots, n$. Suppose that the Principal is also interested in maximization of the total payoff of all agents (social welfare). The agents' control variables are still output volumes x_i, $i = 1, \dots, n$. The relations between the Principal and other agents are presented in Fig. 2. For constant costs

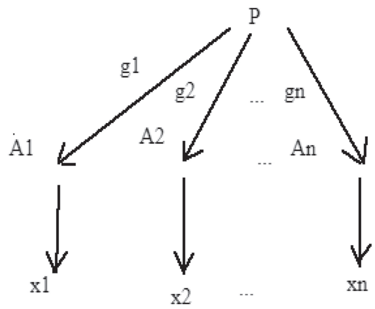

Fig. 2. A hierarchical control system providing the green effect

the model takes the form

$$U(g,x) = \overline{u}(x,g) \to \max, \quad g_i \geqslant 0, \quad i = 1, \dots, n; \tag{2.17}$$
$$u_i^{ST}(g,x) = (a - c - \overline{x} + \overline{g})x_i - g_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n. \tag{2.18}$$

A Stackelberg game (2.17)–(2.18) has the following information structure. The Principal makes the first move: she chooses and reports to other agents the values of norms g_i, $i = 1, \dots, n$. Given the values, all other agents simultaneously and independently choose the values of their control variables x_i, $i = 1, \dots, n$. We suppose that the agents' best response to a Principal's strategy is a Nash equilibrium in their game in normal form. Therefore in fact the Principal chooses a strategy that maximizes her payoff on a set of Nash equilibria. The resulting outcome is a Stackelberg equilibrium in the game (2.17)–(2.18).

Theorem 2.7. *Stackelberg equilibrium strategies and the respective payoffs of the players in the game* (2.17)–(2.18) *have the form*

$$g^{ST} = \frac{n(a-c)}{2n+1}; \quad x^{ST} = \frac{(n+1)(a-c)}{2n+1}; \quad u^{ST} = \frac{(a-c)^2}{2n+1}.$$

The proof of the Theorem 2.7 is given in Appendix A.

For constant costs the function (2.18) takes the form

$$u_{id}^{ST}(g,x) = (a - c - \overline{x} + \overline{g})x_i - dx_i^2 - g_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \dots, n. \tag{2.19}$$

Theorem 2.8. *The proof of the Theorem 2.8 is similar to the proof of the Theorem 2.7.*

$$g_d^S T = \frac{n(1+d)(a-c)}{2n+1+d(4(d+n+1)-n^2)}; \quad x_d^S T = \frac{(n+1+2d)(a-c)}{2n+1+d(4(d+n+1)-n^2)};$$

$$u_d^S T = \frac{(4d^3+8d^2+4dn+5d+2n+1+4d^2n-dn^2-d^2n^2)(a-c)^2}{(2n+1+d(4(d+n+1)-n^2))^2}.$$

The proof of the Theorem 2.8 is similar to the proof of the Theorem 2.7.

2.4.2. The Principal's fairness concern. Suppose additionally the Principal's fairness concern in the model (2.17)–(2.18). Let's limit ourselves by the case $n = 2$. If $u_1 \approx u_2$, then the fairness concern is senseless. Therefore, suppose without loss of generality that $u_1 \gg u_2$. Then the Principal's payoff function takes the form

$$U(g,x) = \overline{u}(g,x) - \delta(u_1(g,x) - u_2(g,x)), \tag{2.20}$$

where $\delta \in (0,1)$ is a parameter of fairness concern.

Theorem 2.9. *In the model* (2.18), (2.20) *for* $n = 2$ *and* $\delta < \sqrt{\frac{41}{81}}$ *a Stackelberg equilibrium has the form*

$$g_1^{STFC} = \frac{2(1+\delta)(a-c)}{5-9\delta^2}; \quad g_2^{STFC} = \frac{2(1-\delta)(a-c)}{5-9\delta^2}; \quad x_1 = x_2 = x^{STFC} = \frac{3(1-\delta)^2(a-c)^2}{(5-9\delta^2)^2},$$

and the respective agents' payoffs are equal to

$$u_1^{STFC} = \frac{(1+\delta)^2(5-18\delta+\delta^2)(a-c)^2}{(5-9\delta^2)^2}; \quad u_2^{STFC} = \frac{(1-\delta)^2(5-18\delta+\delta^2)(a-c)^2}{(5-9\delta^2)^2}.$$

If $\delta \geqslant \sqrt{\frac{41}{81}}$ then the Principal's optimal strategies are $g_1 = g_2 = 0$, then $u_1 = u_2$ and a fairness concern is not required. The proofs of the Theorems 2.1, 2.3 and 2.9 are presented in Appendix A. The solutions of all considered models are shown in Table 1. As in symmetrical models $\overline{u} = nu$, we present only the agents' payoffs u.

3. COMPARATIVE ANALYSIS OF THE SOLUTIONS OF SYMMETRICAL MODELS

To compare an efficiency of the different ways of organization of economic agents in symmetrical models it is natural to use an indicator

$$\frac{u^A}{u^B}, \tag{3.1}$$

where u^A,u^B are payoffs of the agents for the ways of organization A and B respectively. For example, A is a selfish behavior of independent agents, B is the same behavior with green effect. If A is a selfish behavior of independent agents, and B is their cooperation then the indicator (3.1) for symmetrical models shows the price of anarchy.

The values of (3.1) for considered models are collected in Tables 1 and 2. If it is possible then the results of comparison of the value (3.1) with 1, otherwise we conducted a numerical simulation in dependency of the parameters.

Table 1. Comparative analysis of efficiency in the symmetrical models with constant costs

	C	G	GC	ST
NE	$\frac{4n}{(n+1)^2} < 1$	$\frac{(n+2)^2}{3(n+1)^2} < 1$	$\frac{n(4-n)}{(n+1)^2} < 1, \quad n < 4$ $+\infty, \quad$ otherwise;	$\frac{4n^3}{(n+1)^2(2n-1)} > 1$
C		$\frac{(n+2)^2}{12n}$ $< 1, \ n \leqslant 7$ $> 1, \ n > 7;$	$\frac{4-n}{4} < 1, \quad n < 4$ $+\infty, \quad$ otherwise;	$\frac{n^2}{2n-1} > 1$
G			$\frac{3n(4-n)}{(n+2)^2} < 1, \quad n < 4,$ $+\infty, \quad$ otherwise;	$\frac{4n^3}{(n+1)^2(2n-1)} > 1$
GC				$\frac{12n^3}{(n+2)^2(2n-1)} > 1, \quad n < 4,$ $0, \quad$ otherwise;

Table 2. Comparative analysis of efficiency in the symmetrical models with scale-dependent costs

	C_d	G_d	GC_d
NE_d	$\frac{4(n+d)(1+d)}{(n+1+2d)^2} < 1$	$\frac{(n+2+4d)^2(1+d)}{(3+4d)(n+1+2d)^2} < 1$	$\frac{(4n-n^2+4d)(1+d)}{(n+1+2d)^2}, \quad n < 2+2\sqrt{1+d}$ $+\infty, \quad$ otherwise;
C_d		$\frac{(n+2+4d)^2}{4(3+4d)(n+d)} < 1$	$\frac{4n-n^2+4d}{4(n+d)}, \quad n < 2+2\sqrt{1+d}$ $+\infty, \quad$ otherwise;
G_d			$\frac{(4n-n^2+4d)(3+4d)}{(n+2+4d)^2}, \quad n < 2+2\sqrt{1+d},$ $+\infty, \quad$ otherwise;

Immediate calculations based on the presented data imply the following main conclusions.

Theorem 3.1. *For constant costs and different number of agents the preference table is the following:*

	$n = 2, 3$	$n = 4, 5, 6, 7$	$n > 7$
Independent players	$GC \succ G \succ C \succ NE \succ ST$	$C \succ G \succ NE \succ ST \succ GC$	$C \succ NE \succ ST \succ G \succ GC$
Leader	$ST \succ GC \succ C \succ G \succ NE$	$ST \succ C \succ G \succ NE \succ GC$	$ST \succ C \succ NE \succ G \succ GC$
Social welfare	$GC \succ G \succ C \succ NE \succ ST$	$C \succ G \succ NE \succ ST \succ GC$	$C \succ NE \succ ST \succ G \succ GC$

and for scale-dependent costs

	$n = 2, 3$	$n = 4, 5, 6, 7$	$n > 7$
Independent players	$G \succ C \succ NE \succ ST$	$C \succ G \succ NE \succ ST$	$C \succ NE \succ ST \succ G$
Leader	$ST \succ C \succ G \succ NE$	$ST \succ C \succ G \succ NE$	$ST \succ C \succ NE \succ G$
Social welfare	$G \succ C \succ NE \succ ST$	$C \succ G \succ NE \succ ST$	$C \succ NE \succ ST \succ G$

Thus, for the considered models with constant costs and a small number of agents (less than four) the most profitable way of their organization from the point of view of social welfare is the agents' cooperation with constraints on the green effect. Then a pure cooperation, an independent green effect, a selfish behavior, and a leadership of an agents are situated sequentially.

For $n > 4$ it is not profitable for the agents to provide the green effect.

If there is a leader agent then her payoff differs from other payoffs. A hierarchy is the most profitable way for an agent that becomes a leader.

Theorem 3.2. *In the models with scale-dependent costs for $n < 4$ and $d > \frac{1}{24}$*

$$u_d^{GC} \geqslant u_d^{GC} \geqslant u_d^{GC} \geqslant u_d^{GC}, \quad or \;\; GC_d \succ C_d \succ G_d \succ NE_d \succ ST1_d.$$

Thus, a preference structure does not depend on the payoff function.

Theorem 3.3. $C \succ C_d$, *i.e. cooperation of agents in a model with constant costs is more profitable that on the model with scale-dependent costs.*

4. MODELS WITH ASYMMETRICAL AGENTS

All previous results relate to the case of symmetrical agents that produce the same good sold by the same price and having the same costs. It is a strong simplification that is justifiable in some situations. But how will change the preference systems for separate agents and the whole society if the agents are asymmetrical?

4.1. Basic Model of the Cournot Oligopoly with Asymmetrical Agents

Building of the model is described in Section 2.1.1. Consider several formulations.

4.1.1. A selfish behavior of economic agents. For constant costs we receive the model

$$u_i(x) = (a - c_i - \overline{x})x_i \to \max, \quad x_i \geqslant 0, \quad i = 1, \ldots, n, \tag{4.1}$$

and for scale-dependent costs

$$u_i^d(x) = (a - c_i - \overline{x})x_i - dx_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \ldots, n. \tag{4.2}$$

Find a Nash equilibrium in these games.

The first order conditions $\frac{\partial u_i}{\partial x_i} = 0$ give for the model (4.1)

$$x_i^{NE} = \frac{a + \sum\limits_{j \neq i} c_j - nc}{n+1}, \quad i = 1, \ldots, n. \tag{4.3}$$

As $\frac{\partial^2 u_i}{\partial x_i^2} = -2 < 0$, $|H| = \begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix} = 3 > 0$ the formula (4.3) really determines a Nash equilibrium in the game (4.1). Each agent's payoff is equal to

$$u_i^{NE} = \frac{\left(a + \sum\limits_{j \neq i} c_j - nc\right)^2}{(n+1)^2}, \quad i = 1, \ldots, n.$$

For the model (4.2) we receive similarly

$$x_{id}^{NE} = \frac{a - c_i - \frac{\sum\limits_{j=1}^{n} \frac{a - c_j}{1 + 2d_j}}{1 + \sum\limits_{j=1}^{n} \frac{1}{1 + 2d_j}}}{1 + 2d_i}.$$

Remark 1. Note that even in a relatively simple model of the Cournot oligopoly with symmetrical agents we can find an explicit form of solution but it is cumbersome enough. That's why from here onwards the results of an analytical investigation will be described as follows.

1) We find a value $\overline{x} = \sum\limits_{i=1}^{n} x_i$ in an explicit form. It also has quite cumbersome but still tractable form.

2) We show a dependency $x_i(\overline{x})$. It is supposed that given $\overline{x}$ we can calculate x_i.

3) Given x_i and $\overline{x}$ we can calculate u_i and $\overline{u} = \sum\limits_{i=1}^{n} u_i$.

Apply this sequence of actions to the current problem formulation. An analytical investigation gives

$$\overline{x_d^{NE}} = \sum_{i=1}^{n} x_{id}^{NE} = \frac{\sum\limits_{j=1}^{n} \frac{a-c_j}{1+2d_j}}{1 + \sum\limits_{j=1}^{n} \frac{1}{1+2d_j}}$$

due to which we can find the dependency

$$x_{id}^{NE}(\overline{x_d^{NE}}) = \frac{a - c_i - \overline{x_d^{NE}}}{1 + 2d_i}.$$

4.1.2. Cooperation of asymmetrical economic agents. The model (2.3) takes the form

$$\overline{u}(x) = \sum_{i=1}^{n}(a - c_i - \overline{x})x_i = a\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} c_i x_i - \left(\sum_{i=1}^{n} x_i\right)^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \ldots, n, \tag{4.4}$$

and the model (2.4) takes the form

$$\overline{u}^d(x) = \sum_{i=1}^{n}(a - c_i - \overline{x})x_i - d\sum_{i=1}^{n} x_i^2 = a\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} c_i x_i - \left(\sum_{i=1}^{n} x_i\right)^2 - d\sum_{i=1}^{n} x_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1, \ldots, n. \tag{4.5}$$

The cooperative solution of (4.4) has the form

$$\overline{x}^C = \frac{a - c_k}{2},$$

where k is an index of the agent with a minimal production cost. Then we find

$$x_i^C = \begin{cases} \dfrac{a - c_k}{2}, & i = k, \\ 0, & i \neq k. \end{cases}$$

For the model (4.5) we receive similarly

$$\overline{x_d^C} = \frac{a\sum\limits_{j=1}^{n} \frac{1}{d_j} - \sum\limits_{j=1}^{n} \frac{c_j}{d_j}}{2\left(1 + \sum\limits_{j=1}^{n} \frac{1}{d_j}\right)}$$

due to which we find the dependency

$$x_{id}^C(\overline{x_d^C}) = \frac{a - c_i - \overline{x_d^C}}{2d_i}.$$

4.1.3. Presence of a leader firm in the model with asymmetrical players. Assume that agent 1 is a leader. Find a Stackelberg equilibrium. The first order conditions $\frac{\partial u_i}{\partial x_i} = 0$, $i = 2, \ldots, n$,

$$x_i^{ST} = \frac{a - c_1 n + \sum\limits_{j=2}^{n} c_j}{2}, \quad \sum_{i=2}^{n} x_i^{ST} = \frac{(n-1)a - \sum\limits_{j=2}^{n} c_j - x_1^{ST}(n-1)}{n},$$

due to which we find the dependency

$$x_{i>2}^{ST}\left(\sum_{i=2}^{n} x_i^{ST}\right) = a - c_i - x_1^{ST} - \sum_{i=2}^{n} x_i^{ST}.$$

For the model with scale-dependent costs we receive similarly

$$x_{1d}^{ST} = \frac{a - c_1 - c_1 \sum\limits_{j=2}^{n} \frac{1}{1+2d_j} + \sum\limits_{j=2}^{n} \frac{c_j}{1+2d_j}}{2 + 2d_1 + (2d_1 - 1)\sum\limits_{j=2}^{n} \frac{1}{1+2d_j}}, \quad \sum_{j=2}^{n} x_{id}^{ST} = \frac{a \sum\limits_{j=2}^{n} \frac{1}{1+2d_j} - \sum\limits_{j=2}^{n} \frac{c_j}{1+2d_j} - x_{1d}^{ST} \sum\limits_{j=2}^{n} \frac{1}{1+2d_j}}{1 + \sum\limits_{j=2}^{n} \frac{1}{1+2d_j}},$$

due to which we find the dependency

$$x_{i>2,d}^{ST}\left(\sum_{i=2}^{n} x_i^{ST}\right) = \frac{a - c_i - x_1 - \sum\limits_{i=2}^{n} x_{1d}^{ST}}{1 + 2d_i}.$$

4.2. The Cournot Oligopoly with Green Effect

4.2.1. A selfish behavior of asymmetrical agents. Find a Stackelberg equilibrium in the Cournot oligopoly model with green effect and asymmetrical agents. Denote as earlier α — a coefficient characterizing increase in demand due to green effect, β_i — a coefficient characterizing a greening cost of the ith agent. Then the model (2.1) takes the form

$$u_i^G(x, g) = (a - c_i - \overline{x} + \alpha\overline{g})x_i - \beta_i g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \dots, n.$$

First, find a Nash equilibrium. In the case of green effect the analytical results are described as follows.

1) We find a value $\overline{x} = \sum\limits_{i=1}^{n} x_i$ or some other value that can serve as a base for calculation of the value enumerated below. The value is also cumbersome enough but still tractable.

2) Then we present a dependency $x_i(\overline{x})$. It is supposed that given $\overline{x}$ we can calculate x_i using this dependency.

3) Then we find the dependencies $g_i(x_i)$ and, probably, $\overline{g} = \sum\limits_{i=1}^{n} g_i$.

4) Given x_i, $\overline{x}$, g_i and $\overline{g}$, we can calculate u_i and $\overline{u} = \sum\limits_{i=1}^{n} u_i$.

Theorem 4.1. *Instead of $\overline{x}$ it is sufficient to find a value*

$$z = \sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1\right) x_i^{GNE} = \frac{\sum\limits_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1\right)(a - c_i)}{1 - \sum\limits_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1\right)},$$

and to reveal a dependency

$$x_i^{GNE}(z) = a - c_i + z, \quad g_i^{GNE} = \frac{x_i^{GNE}}{2\beta_i}.$$

Theorem 4.2. *In the model of Cournot oligopoly with scale-dependent costs and green effect*

$$u_{id}^G(x, g) = (a - c_i - \overline{x} + \overline{g})x_i - dx_i^2 - \beta_i g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \dots, n$$

we have:

$$z = \sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right) x_{id}^{GNE} = \frac{\sum\limits_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right) \frac{a-c_i}{1+2d_i}}{1 - \sum\limits_{i=1}^{n} \frac{\left(\frac{\alpha^2}{2\beta_i} - 1 \right)}{1+2d_i}},$$

and can reveal a dependency

$$x_{id}^{GNE}(z) = \frac{a - c_i + z}{1 + 2d_i}, \quad g_{id}^{GNE} = \frac{\alpha x_{id}^{GNE}}{2\beta_i}.$$

The proof of this Theorem is presented in Appendix D.

4.2.2. Cooperative of economic agents. In the case of cooperation the model with green effect and constant costs has the form

$$\overline{u}^{GC}(x, g) = a \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} c_i x_i - \left(\sum_{i=1}^{n} x_i \right)^2 + \alpha \sum_{i=1}^{n} g_i \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \beta_j g_j^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n, \tag{4.6}$$

and for scale-dependent costs — the form

$$\overline{u}_d^{GC}(x, g) = a \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} c_i x_i - \left(\sum_{i=1}^{n} x_i \right)^2 + \alpha \sum_{i=1}^{n} g_i \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \beta_j g_j^2 - \sum_{i=1}^{n} d_j x_j^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1, \ldots, n. \tag{4.7}$$

Theorem 4.3. *In the model* (4.6) *we have*

$$\overline{x}^{GC} = \frac{a - c_k}{2 - \frac{\alpha^2}{2} \sum\limits_{j=1}^{n} \frac{1}{\beta_j}},$$

where k is an index of the agent with a minimal production cost. Then we find

$$x_i^{GC} = \begin{cases} \dfrac{a - c_k}{2 - \frac{\alpha^2}{2} \sum\limits_{j=1}^{n} \frac{1}{\beta_j}}, & i = k, \\ 0, & i \neq k, \end{cases} \qquad g_i^{GC} = \frac{\alpha \overline{x}^{GC}}{2\beta_i}.$$

Theorem 4.4. *In the model* (4.7) *we have*

$$\overline{x_d^{GC}} = \frac{\sum\limits_{j=1}^{n} \frac{a-c_j}{d_j}}{2\left(1 + \sum\limits_{j=1}^{n} \frac{1}{d_j} - \frac{\alpha^2}{4} \sum\limits_{i=1}^{n} \frac{1}{d_j} \sum\limits_{i=1}^{n} \frac{1}{\beta_j} \right)},$$

and can reveal a dependency

$$x_{id}^{GC}(\overline{x_d^{GC}}) = \frac{a - c_i - 2\overline{x_d^{GC}} + \frac{\alpha^2 \overline{x_d^{GC}} \sum\limits_{j=1}^{n} \frac{1}{\beta_j}}{2}}{2di}, \quad g_{id}^{GC} = \frac{\alpha \overline{x_d GC}}{2\beta_i}.$$

The proof of the Theorem 4.4 is similar to the proof of the Theorem 4.3.

4.2.3. The Cournot oligopoly with green effect in a supply chain. Consider the Cournot oligopoly in the form of a supply chain

$$u_i(x,g) = \Big(a - c_i - \overline{x} + \sum_{j\leqslant i} \alpha^{i-j} g_j\Big) x_i - \beta_i g_i^2 \to \max, \quad x_i \geqslant 0, \quad g_i \geqslant 0, \quad i = 1,\dots,n. \tag{4.8}$$

For an analytical investigation take $n = 2$. Then for constant costs the model (4.8) takes the form

$$u_1^{SC}(x,g) = (a - c_1 - x_1 - x_2 + g_1)x_1 - \beta_1 g_1^2 \to \max, \quad x_1 \geqslant 0, \quad g_1 \geqslant 0, \tag{4.9}$$

$$u_2^{SC}(x,g) = (a - c_2 - x_1 - x_2 + \alpha g_1 + g_2)x_2 - \beta_2 g_2^2 \to \max, \quad x_2 \geqslant 0, \quad g_2 \geqslant 0, \tag{4.10}$$

and for scale-dependent costs

$$u_{1d}^{SC}(x,g) = (a - c_1 - x_1 - x_2 + g_1)x_1 - \beta_1 g_1^2 - d_1 x_1^2 \to \max, \quad x_1 \geqslant 0, \quad g_1 \geqslant 0, \tag{4.11}$$

$$u_{2d}^{SC}(x,g) = (a - c_2 - x_1 - x_2 + \alpha g_1 + g_2)x_2 - \beta_2 g_2^2 - d_2 x_2^2 \to \max, \quad x_2 \geqslant 0, \quad g_2 \geqslant 0. \tag{4.12}$$

Theorem 4.5. *In the model* (4.9)–(4.10) *equilibrium strategies of the agents have the form*

$$x_1^{SCNE} = \frac{a - c_2 + (\frac{1}{2\beta_2} - 2)(a - c_1)}{(1 - \frac{1}{2\beta_1}) - (\frac{1}{2\beta_2} - 2)(\frac{1}{2\beta_1} - 2)}; \quad x_2^{SCNE}(x_1^{SCNE}) = a - c_1 + x_1^{SCNE}\left(\frac{1}{2\beta_1} - 2\right);$$

$$g_i^{SCNE} = \frac{x_i^{SCNE}}{2\beta_i}.$$

Theorem 4.6. *In the model* (4.11)–(4.12) *equilibrium strategies of the agents have the form*

$$x_{1d}^{SCNE} = \frac{a - c_2 + \left(\frac{1}{2\beta_2} - 2 - 2d_2\right)(a - c_1)}{\left(1 - \frac{1}{2\beta_1}\right) - \left(\frac{1}{2\beta_2} - 2 - 2d_2\right)\left(\frac{1}{2\beta_1} - 2 - 2d_1\right)};$$

$$x_{2d}^{SCNE}(x_{1d}^{SCNE}) = a - c_1 + x_{1d}^{SCNE}\left(\frac{1}{2\beta_1} - 2 - 2d_1\right);$$

$$g_{id}^{SCNE} = \frac{x_{id}^{SCNE}}{2\beta_i}.$$

The proofs of the Theorems 4.5 and 4.6 are similar to the previous proofs.

4.3. Hierarchical Control in the Cournot Oligopoly Providing the Green Effect

4.3.1. Basic hierarchical model. The model has the same information structure as in Section 2.4.1. For constant costs the model takes the form

$$U(g,x) = \overline{u}(x,g) \to \max, \quad g_i \geqslant 0, \quad i = 1,\dots,n; \tag{4.13}$$

$$u_i^{ST}(g,x) = (a - c_i - \overline{x} + \alpha\overline{g})x_i - \beta_i g_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1,\dots,n, \tag{4.14}$$

or

$$u_{id}^{ST}(g,x) = (a - c_i - \overline{x} + \alpha\overline{g})x_i - \beta_i g_i^2 - d_i x_i^2 \to \max, \quad x_i \geqslant 0, \quad i = 1,\dots,n. \tag{4.15}$$

Theorem 4.7. *Stackelberg equilibrium strategies of the agents in the game* (4.13)–(4.14) *have the form*

$$\overline{g^{ST}} = \frac{\alpha\left(an - \sum\limits_{i=1}^n c_i\right)\sum\limits_{i=1}^n \frac{1}{\beta_i}}{(n+1)^2 - n\alpha^2 \sum\limits_{i=1}^n \frac{1}{\beta_i}}; \quad g_i^{ST} = \frac{\alpha\left(an - \sum\limits_{i=1}^n c_i + n\alpha\overline{g^{ST}}\right.}{\beta_i(n+1)^2},$$

$$\overline{x^{ST}} = \frac{an - \sum\limits_{i=1}^n c_i + n\alpha\overline{g^{ST}}}{n+1}; \quad x_i^{ST} = a - c_i - \overline{x^{ST}} + \alpha\overline{g^{ST}}.$$

The proof of the Theorem 4.7 is given in Appendix D.

Theorem 4.8. *Stackelberg equilibrium strategies of the agents in the game* (4.13), (4.15) *have the form*

$$\overline{g_d^{ST}} = \frac{\alpha \sum\limits_{i=1}^n \frac{1}{\beta_i}\left(a\sum\limits_{i=1}^n \frac{1+d_i}{(1+2d_i)^2} - \sum\limits_{i=1}^n \frac{c_i(1+d_i)}{(1+2d_i)^2} + \sum\limits_{i=1}^n \frac{1}{1+2d_i}\sum\limits_{i=1}^n \frac{c_i d_i}{(1+2d_i)^2} - \sum\limits_{i=1}^n \frac{c_i}{1+2d_i}\sum\limits_{i=1}^n \frac{d_i}{(1+2d_i)^2}\right)}{\left(1+\sum\limits_{i=1}^n \frac{1}{1+2d_i}\right)^2 - \alpha^2 \sum\limits_{i=1}^n \frac{1}{\beta_i}\sum\limits_{i=1}^n \frac{1+d_i}{(1+2d_i)^2}};$$

$$g_{id}^{ST} = \frac{\alpha}{\beta_i\left(1+\sum\limits_{i=1}^n \frac{1}{1+2d_i}\right)^2}$$

$$\times\left(a\sum_{i=1}^n \frac{1+d_i}{(1+2d_i)^2} - \sum_{i=1}^n \frac{c_i(1+d_i)}{(1+2d_i)^2} + \sum_{i=1}^n \frac{1}{1+2d_i}\sum_{i=1}^n \frac{c_i d_i}{(1+2d_i)^2} - \sum_{i=1}^n \frac{c_i}{1+2d_i}\sum_{i=1}^n \frac{d_i}{(1+2d_i)^2} + \alpha\overline{g_d^{ST}}\sum_{i=1}^n \frac{1+d_i}{(1+2d_i)^2}\right),$$

$$\overline{x_d^{ST}} = \frac{a\sum\limits_{i=1}^n \frac{1}{1+2d_i} - \sum\limits_{i=1}^n \frac{c_i}{1+2d_i} + \alpha\overline{g_d^{ST}}\sum\limits_{i=1}^n \frac{1}{1+2d_i}}{1+\sum\limits_{i=1}^n \frac{1}{1+2d_i}}; \quad x_{id}^{ST} = \frac{a - c_i - \overline{x_d^{ST}} + \alpha\overline{g_d^{ST}}}{1+2d_i}.$$

The proof of the Theorem 4.8 is similar to the proof of the Theorem 4.7.

4.3.2. The Principal's fairness concern. Suppose additionally the Principal's fairness concern in the model (4.13)–(4.14) . Let's limit ourselves by the case $n = 2$. If $u_1 \approx u_2$, then the fairness concern is senseless. Therefore, suppose without loss of generality that $u_1 \gg u_2$. Then the Principal's payoff function takes the form

$$U(g, x) = \overline{u}(g, x) - \delta(u_1(g, x) - u_2(g, x)), \tag{4.16}$$

where $\delta \in (0, 1)$ is a parameter of fairness concern.

Theorem 4.9. *In the model* (4.14), (4.16) *for* $n = 2$ *a Stackelberg equilibrium has the form*

$$g_1^{STFC} = \frac{2a - (1-3\delta)c_1 - (1+3\delta)c_2}{\frac{9(1-\delta)\beta_1}{\alpha} - 2\alpha\left(\frac{(1-\delta)\beta_1}{(1+\delta)\beta_2} + 1\right)}; \quad g_2^{STFC}(g_1^{STFC}) = \frac{(1-\delta)\beta_1}{(1+\delta)\beta_2} g_1^{STFC};$$

$$x_i^{STFC} = \frac{a - 2c_i + c_{3-i} + \alpha(g_1^{STFC} + g_2^{STFC})}{3},$$

The proof of Theorem 4.9 is given in Appendix D.

5. NUMERICAL SIMULATION

Consider an example of production of eatable disposable tableware, say, waffle glasses. The cases of constant and scale-dependent costs are grounded by purchase of raw materials with constant price or with a wholesale price that decreases when the purchase volume increases.

For numerical investigation of the model we took 27 agents with non-repeatable ratios of the parameters c_i, d_i and β_i. A value of each parameter can be characterized as small, medium or big. Thus, we receive $3^3 = 27$ possible combinations. However, when the number of agents is big, a green production is not profitable and may be omitted. Also, an influence of the parameter β_i may be ignored. So, it is sufficient to combine only two values, and we receive $3^2 = 9$ cases. For investigation of the green effect we differentiated additionally the cases with two and three agents.

The results for nine agents are presented in Appendix C and give the following conclusions.

A production of waffle glasses is not profitable for both strong and weak agents.

The preferences of any agent (not a leader) with any combination of input values for constant costs are:

$$NE \succ ST \succ C,$$

and for scale-dependent costs are

$$C_d \succ NE_d \succ ST_d.$$

So, for the case of scale-dependent costs the preferences are the same as in the model with symmetrical agents. For the case of constant costs only the conclusion about profitability of cooperation does not hold. It can be explained as follows. In the case of asymmetrical agents only the agents having a minimal cost receive a payoff. Other agents produce nothing and receive nothing. In the case of symmetrical agents all prices are equal, and cooperative payoffs of all agents are greater than for other ways of organization.

Preferences of the society for constant costs are:

$$C \succ NE \succ ST,$$

and for scale-dependent costs are

$$C_d \succ NE_d \succ ST_d.$$

So, the results are the same as for the model with symmetrical agents.

Preferences of the leader are:

$$ST \succ NE \succ C,$$

that are the same as for the model with symmetrical agents.

The results of numerical investigations for two agents (strong and weak) are presented in Appendix D.

Preferences of the strong agent for constant costs are:

$$GNE \succ GC \succ ST \succ C \succ SCST \succ NE \succ STFC \succ SCNE.$$

and for scale-dependent costs are

$$GNE_d \succ GC_d \succ C_d \succ ST_d \succ ST1_d \succ NE_d \succ SCNE_d \succ SCST_d.$$

Preferences of the weak agent for constant costs are:

$$GNE \succ STFC \succ NE \succ ST \succ SCST \succ C \succ CNE \succ SCNE.$$

and for scale-dependent costs are

$$SCST_d \succ GNE_d \succ NE_d \succ ST1_d \succ GC_d \succ C_d \succ CNE_d \succ GCd.$$

Preferences of the whole society for constant costs are:

$$GNE \succ (C = GC) \succ (NE \sim SCST) \succ ST \succ STFC \succ SCNE.$$

and for scale-dependent costs are

$$GNE_d \succ (C_d = GC_d) \succ NE_d \succ ST_d \succ SCNE_d \succ SCST_d,$$

So, for the whole society and for each agent it is profitable to produce an eatable tableware.

The results of numerical investigations for three agents are also presented in Appendix D.

The preferences of any agent (not a leader) for constant costs are:

$$GNE \succ SCST \succ NE \succ C \succ ST$$

and for scale-dependent costs are

$$SCST_d \succ GNE_d \succ C_d \succ NE_d \succ ST_d \succ GC_d.$$

For small d_i and big and medium c_i (an agent has small cost but big profit) in the green production it is not profitable for the agents to create coalitions:

$$GNE_d \succ GC_d,$$

but for other combinations in the green production coalitions are profitable:

$$GC_d \succ C_d.$$

Preferences of the whole society for constant costs are

$$GNE \succ C \succ NE \succ ST$$

and for scale-dependent costs are

$$SCST_d \succ GC_d \succ NE_d \succ ST_d.$$

Preferences of the leader for constant costs are:

$$ST \succ GC \succ C \succ NE$$

and for scale-dependent costs are

$$GC_d \succ NE_d \succ SCST_d.$$

A situation when cooperation is less profitable for the whole society than other ways of organization must be commented. A greater degree of the environmental friendly production (a greater value of the parameter g_i) is more profitable for the whole society. However, for separate agents this high degree is not profitable, and they refuse to produce at all that impacts the summary payoff function. The less agents are in a society, the more essential is the behavior of each agent.

6. CONCLUSION

A problem of comparative analysis of the ways of organization of economic (and other active) agents seems to be very important. Really, if a decision is profitable for the whole society (and for the respective centralized control body) but not profitable for separate agents who are responsible for its implementation then the decision will not be implemented. The respective examples are well known.

Meanwhile, the received results show that the preference systems for the whole society and for separate agents not always coincide. There are also many nuances concerned with a difference between leaders and followers, strong and weak economic agents, with consideration of additional issues of the structure, environmental and innovative expenses, social responsibility of business, arguments of justice in payoffs allocation.

The presented results are rather an empirical material because it is still difficult to make some conclusions about a comparative efficiency in a general form. Nevertheless, for any specific case in the frame of a corresponding models the results seem to be useful for analysis and decision support.

APPENDIX A

Proof of Theorem 2.1.

$$\frac{\partial u_i^G}{\partial x_i} = a - c - 2x_i - \sum_{j \neq i} x_j + \sum_{i=1}^{n} g_j = 0, \quad i = 1, \ldots, n;$$

$$\frac{\partial u_i^G}{\partial g_i} = x_i - 2g_i = 0, \quad i = 1, \ldots, n;$$

$$x_i = 2g_i, \quad i = 1, \ldots, n;$$

$$a - c - 4g_i - 2\sum_{j \neq i} g_j + \sum_{j=1}^{n} g_j = 0; \quad a - c - 3g_i - \sum_{j \neq i} g_j = 0;$$

$$3g_i + \sum_{j \neq i} g_j = a - c, \quad i = 1, \ldots, n;$$

$$g_i = g, \quad i = 1, \ldots, n; \quad 3g + (n-1)g = a - c;$$

$$g = \frac{a-c}{n+2}, \quad x = \frac{2(a-c)}{n+2}$$

the value u is found by an immediate substitution.

Proof of Theorem 2.3.

$$\frac{\partial \overline{u}}{\partial x_i} = -\overline{x} + a - c - \overline{x} + \overline{g} = a - c + \overline{g} - 2\overline{x} = 0, \quad i = 1, \ldots, n; \quad \frac{\partial \overline{u}}{\partial g_i} = \overline{x} - 2g_i = 0, \quad i = 1, \ldots, n;$$

$$g_i = g, \quad x_i = x, \quad i = 1, \ldots, n; \quad \overline{x} = nx, \quad \overline{g} = ng,$$

$$\overline{x} = 2g; \quad a - c + ng - 4g = 0;$$

$$g = \begin{cases} \dfrac{a-c}{4-n}, & n < 4, \\ 0, & \text{otherwise.} \end{cases}$$

If $g = 0$ then $x = x^C$ is an optimal solution of the cooperative problem without "green" effect. Then

$$x = \begin{cases} \dfrac{2(a-c)}{n(4-n)}, & n < 4, \\ x^C, & \text{otherwise,} \end{cases} \quad \overline{u} = \begin{cases} \dfrac{(a-c)^2}{4-n}, & n < 4, \\ u^C, & \text{otherwise,} \end{cases}$$

Proof of Theorem 2.7. A best response of the agents to the Principal's strategy $(g_1, \ldots, g_n)$, or a Nash equilibrium in the game of agents, is determined by solution of the system

$$\frac{\partial u_i^{ST}}{\partial x_i} = a - c - 2x_i - \sum_{j \neq i} x_j + \overline{g} = 0, \quad i = 1, \ldots, n;$$

and then

$$2x_i + \sum_{j \neq i} x_j = a - c + \overline{g}, \quad i = 1, \ldots, n;$$

$$x_i = x^{ST}, \quad i = 1, \ldots, n;$$

$$x^{ST} = \frac{a - c + \overline{g}}{n+1}; \quad \overline{x}^{ST} = \frac{n(a - c + \overline{g})}{n+1}.$$

A substitution of the found best response in the Principal's payoff function gives

$$\overline{u}(g, x^{ST}) = (a - c - \overline{x}^{ST} + \overline{g})\overline{x}^{ST} - \sum_{i=1}^{n} g_i^2$$

$$= \left(a - c + \overline{g} - \frac{n(a-c+\overline{g})}{n+1}\right)\frac{n(a-c+\overline{g})}{n+1} - \sum_{i=1}^{n} g_i^2 = \frac{n(a-c+\overline{g})^2}{(n+1)^2} - \sum_{i=1}^{n} g_i^2.$$

Then

$$\frac{\partial \overline{u}}{\partial g_i} = \frac{2n(a-c+\overline{g})}{(n+1)^2} - 2g_i = 0, \quad i = 1, \ldots, n;$$
$$g_i = g^{ST}, \quad i = 1, \ldots, n; \quad \overline{g}^{ST} = ng^{ST};$$
$$n(a - c + ng^{ST}) - (n+1)^2 g^{ST} = 0;$$
$$g^{ST} = \frac{n(a-c)}{2n+1}$$

and we find $x^{ST} = \frac{(n+1)(a-c)}{2n+1}$, $u^{ST} = \frac{(a-c)^2}{2n+1}$.

Proof of Theorem 2.9. The best response of the agents is still determined by the expression (when $n = 2$)

$$x^{STFC} = \frac{a-c+\overline{g}}{3}; \quad \overline{x}^{STFC} = \frac{2(a-c+\overline{g})}{3}.$$
$$x^{ST} = \frac{a-c+\overline{g}}{n+1}; \quad \overline{x}^{ST} = \frac{n(a-c+\overline{g})}{n+1}.$$

The Principal's payoff function is

$$U^{FC}(g, x) = (a - c + \overline{g} - \overline{x})\overline{x} - \delta(u_1 - u_2) - g_1^2 - g_2^2.$$

Notice that

$$u_1(g, x^{STFC}) - u_2(g, x^{STFC}) = Q(g, x^{STFC})(x^{STFC} - x^{STFC}) - g_1^2 + g_2^2 = g_2^2 - g_1^2.$$

Then a substitution of x^{STFC} in U^{FC} gives

$$U^{FC}(g, x^{STFC}) = \left(a - c + \overline{g} - \frac{2}{3}(a - c + \overline{g})\right)\frac{2}{3}(a - c + \overline{g}) - \delta(g_2^2 - g_1^2) - g_1^2 - g_2^2$$
$$= \frac{2}{9}(a - c + \overline{g})^2 - (1-\delta)g_1^2 - (1+\delta)g_2^2.$$

Then we solve the system of equations

$$\frac{\partial U^{FC}}{\partial g_1} = \frac{4}{9}(a - c + \overline{g}) - 2(1-\delta)g_1 = 0,$$
$$\frac{\partial U^{FC}}{\partial g_2} = \frac{4}{9}(a - c + \overline{g}) - 2(1+\delta)g_2 = 0,$$

and

$$g_1 = \frac{2(1+\delta)(a-c)}{5-9\delta^2}, \quad g_2 = \frac{2(1-\delta)(a-c)}{5-9\delta^2}.$$

These are positive when $\delta < \frac{\sqrt{5}}{3}$. The pair (g_1^{STFC}, g_2^{STFC}) really maximizes U^{FC} under the conditions $\frac{\partial^2 U^{FC}}{\partial g_1^2} = \frac{4}{9} - 2(1-\delta) < 0$, or $\delta < \frac{7}{9}$ and $|H| = \begin{vmatrix} \frac{4}{9} - 2(1-\delta) & \frac{4}{9} \\ \frac{4}{9} & \frac{4}{9} - 2(1+\delta) \end{vmatrix} > 0$, or $\delta < \sqrt{\frac{41}{81}}$. A comparison of the three received inequalities gives the final condition $\delta < \sqrt{\frac{41}{81}}$. Immediate calculations give

$$x_1 = x_2 = x^{STFC} = \frac{3(1-\delta^2)(a-c)}{(5-9\delta^2)^2};$$

$$u_1^{STFC} = \frac{(1+\delta)^2(5-18\delta+\delta^2)(a-c)^2}{(5-9\delta^2)^2}, \quad u_2^{STFC} = \frac{(1-\delta)^2(5-18\delta+\delta^2)(a-c)^2}{(5-9\delta^2)^2}.$$

APPENDIX B

Proof of Theorem 4.1.

$$\frac{\partial u_i^G}{\partial x_i} = -x_i + a - c_i - \sum_{j=1}^{n} x_j + \alpha \sum_{i=1}^{n} g_j = 0, \quad i = 1, \ldots, n;$$

$$\frac{\partial u_i^G}{\partial g_i} = \alpha x_i - 2\beta_i g_i = 0, \quad i = 1, \ldots, n.$$

From the latter expression

$$g_i = \frac{\alpha x_i}{2\beta_i}, \quad i = 1, \ldots, n;$$

A substitution of the received dependency into the first equality gives

$$x_i = a - c_i + \sum_{j=1}^{n} \left(\frac{\alpha^2}{2\beta_j} - 1 \right) x_j, \quad i = 1, \ldots, n.$$

Multiply both sides of the equality by the expression in brackets:

$$\left(\frac{\alpha^2}{2\beta_i} - 1 \right) x_i = \left(\frac{\alpha^2}{2\beta_i} - 1 \right)(a - c_i) + \left(\frac{\alpha^2}{2\beta_i} - 1 \right) \sum_{j=1}^{n} \left(\frac{\alpha^2}{2\beta_j} - 1 \right) x_j.$$

The summation of the received equality by i gives

$$\sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right) x_i = \sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right)(a - c_i) + \sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right) \sum_{j=1}^{n} \left(\frac{\alpha^2}{2\beta_j} - 1 \right) x_j.$$

Now calculate $\sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right) x_i$:

$$\sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right) x_i^{GNE} = \frac{\sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right)(a - c_i)}{1 - \sum_{i=1}^{n} \left(\frac{\alpha^2}{2\beta_i} - 1 \right)}.$$

Proof of Theorem 4.3.

$$\frac{\partial \overline{u}}{\partial x_i} = -2\overline{x} + a - c_i + \alpha \overline{g} = 0, \quad i = 1, \ldots, n; \quad \frac{\partial \overline{u}}{\partial g_i} = \alpha \overline{x} - 2\beta_i g_i = 0, \quad i = 1, \ldots, n.$$

From the latter relation

$$g_i = \frac{\alpha \sum\limits_{i=1}^{n} x_i}{2\beta_i}, \quad i = 1, \ldots, n;$$

A substitution of the received dependency into the first equality gives

$$\sum_{i=1}^{n} x_i = \frac{a - c_i}{2 - \frac{\alpha^2}{2} \sum\limits_{j=1}^{n} \frac{1}{\beta_j}}, \quad i = 1, \ldots, n.$$

A substitution of the received result into the objective function of the cooperative problem shows that it is increases when c_i decreases, and then

$$\sum_{i=1}^{n} x_i = \frac{a - c_k}{2 - \frac{\alpha^2}{2} \sum\limits_{j=1}^{n} \frac{1}{\beta_j}},$$

where k is an index of the agent with a minimal c_k. Then

$$x_k = \frac{a - c_k}{2 - \frac{\alpha^2}{2} \sum\limits_{j=1}^{n} \frac{1}{\beta_j}},$$

$$x_{i \neq k} = 0.$$

Proof of Theorem 4.7. A best response of the agents to the Principal's strategy $(g_1, \ldots, g_n)$, or a Nash equilibrium in the game of agents, is determined by the solution of the system

$$\frac{\partial u_i^{ST}}{\partial x_i} = a - c_i - x_i - \sum_{j=1}^{n} x_j + \alpha \overline{g},$$

and

$$x_i = a - c_i - \sum_{j=1}^{n} x_j + \alpha \sum_{j=1}^{n} g_j.$$

Sum both sides of the inequality:

$$\sum_{i=1}^{n} x_i = na - \sum_{i=1}^{n} c_i - n \sum_{j=1}^{n} x_j + \alpha n \sum_{j=1}^{n} g_j.$$

The calculation of $\sum\limits_{i=1}^{n} x_i$ gives:

$$\sum_{i=1}^{n} x_i = \frac{na - \sum\limits_{i=1}^{n} c_i + \alpha n \sum\limits_{j=1}^{n} g_j}{1 + n}.$$

A substitution of the received sum into the expression for x_i gives

$$x_i^{ST} = \frac{a - (1 + n)c_i + n\alpha \overline{g}}{n + 1}.$$

A substitution of the found best response into the Principal's payoff function gives

$$\overline{u}(g, x^{ST}) = \frac{na^2 - a\sum_{i=1}^{n} c_i + \alpha a n \sum_{j=1}^{n} g_j}{1+n}$$
$$- \frac{\sum_{i=1}^{n}\left(c_i a - (1-n)c_i^2 + c_i \sum_{i=1}^{n} c_i + \alpha c_i \sum_{j=1}^{n} g_j\right)}{1+n} + \alpha \sum_{i=1}^{n} g_j \left(\frac{na - \sum_{i=1}^{n} c_i + \alpha n \sum_{j=1}^{n} g_j}{1+n}\right)$$
$$- \left(\frac{na - \sum_{i=1}^{n} c_i + \alpha n \sum_{j=1}^{n} g_j}{1+n}\right)^2 - 2\sum_{i=1}^{n} \beta_i g_i^2.$$

Then

$$\frac{\partial \overline{u}}{\partial g_i} = \frac{a\alpha n}{1+n} - \frac{\alpha \sum_{i=1}^{n} c_i}{1+n} + \alpha \frac{na - \sum_{i=1}^{n} c_i + \alpha n \sum_{j=1}^{n} g_j}{1+n} + \alpha^2 \frac{n \sum_{i=1}^{n} g_j}{1+n}$$
$$- \frac{2n\alpha}{(n+1)^2}\left(na - \sum_{i=1}^{n} c_i + \alpha n \sum_{j=1}^{n} g_j\right) - 2\beta_i g_i = 0, i = 1, \ldots, n;$$

and

$$g_i^{ST} = \frac{\alpha\left(na - \sum_{i=1}^{n} c_i + \alpha n \sum_{j=1}^{n} g_j\right)}{\beta_i (n+1)^2},$$

and the summation by all indices gives

$$\sum_{i=1}^{n} g_i^{ST} = \frac{\alpha\left(na - \sum_{i=1}^{n} c_i\right)\sum_{i=1}^{n}\frac{1}{\beta_i}}{(n+1)^2 - n\alpha^2 \sum_{i=1}^{n}\frac{1}{\beta_i}},$$

Proof of Theorem 4.9. The best response of the agents is still determined by the expression (when $n = 2$)

$$x_1^{STFC} = \frac{a - 2c_1 + c_2 + \alpha\overline{g}}{3}; \quad x_2^{STFC} = \frac{a - 2c_2 + c_1 + \alpha\overline{g}}{3}.$$

The Principal's payoff function is

$$U^{FC}(g, x) = (1-\delta)(a - c_1 - x_1 - x_2 + \alpha\overline{g})x_1 - (1-\delta)\beta_1 g_1^2$$
$$+ (1+\delta)(a - c_2 - x_1 - x_2 + \alpha\overline{g})x_2 - (1+\delta)\beta_2 g_2^2.$$

A substitution of x^{STFC} into U^{FC} gives

$$U^{FC}(g, x^{STFC}) = \frac{1-\delta}{9}(a - 2c_1 + c_2 + \alpha\overline{g})^2 - (1-\delta)\beta_1 g_1^2$$
$$+ \frac{1+\delta}{9}(a - 2c_2 + c_1 + \alpha\overline{g})^2 - (1+\delta)\beta_2 g_2^2.$$

The first order conditions determine the system

$$\frac{\partial U^{FC}}{\partial g_1} = \frac{2\alpha(1-\delta)}{9}(a - 2c_1 + c_2 + \alpha\overline{g})x_2 - 2(1-\delta)\beta_1 g_1$$
$$+\frac{2\alpha(1+\delta)}{9}(a - 2c_2 + c_1 + \alpha\overline{g})x_2 = 0,$$
$$g_2 = \frac{(1-\delta)\beta_1}{(1+\delta)\beta_2}g_1$$

and

$$g_1^{STFC} = g_2^{STFC} = \frac{2a - (1-3\delta)c_1 - (1+3\delta)c_2}{\frac{9(1-\delta)\beta_1}{\alpha} - 2\alpha\left(\frac{(1-\delta)\beta_1}{(1+\delta)\beta_2} + 1\right)},$$

and all other values are found by substitution of the found ones.

APPENDIX C

Table C.1. Solution of the Cournot oligopoly with nine asymmetrical agents (when a=3000)

no.	c_i	d_i	u^{NE}	u_d^{NE}	u^C	u_d^C	u^{ST}	u_d^{ST}
1	1	1	129 000	598 000	25 000 000	1 690 000	936 000	451 000
2	1	3	129 000	220 000	0	565 000	34 700	193 000
3	1	5	129 000	133 000	0	339 000	34 700	117 000
4	3	1	127 000	596 000	0	1 690 000	33 900	523 000
5	3	3	127 000	219 000	0	565 000	33 900	192 000
6	3	5	127 000	133 000	0	339 000	33 900	117 000
7	5	1	126 000	595 000	0	1 690 000	33 200	522 000
8	5	3	126 000	218 000	0	565 000	33 200	192 000
9	5	5	126 000	133 000	0	339 000	33 200	116 000
Total			1 146 000	2 845 000	25 000 000	7 782 000	1 206 700	2 423 000

APPENDIX D

Table D.1. Solution of the Cournot oligopoly with two asymmetrical agents (when $a = 10$), $c_1 = 1$, $d_1 = 1$, $\beta_1 = 100$, $c_2 = 5$, $d_2 = 1$, $\beta_2 = 100$

	x_1	u_1	g_1	x_2	u_2	g_2	x	u	g
NE	4.33	18.8	–	0.33	0.11	–	4.66	18.91	–
NEd	2.07	8.54	–	0.73	1.08	–	2.8	9.62	–
C	4.5	20.2	–	0	0	–	4.5	20.2	–
Cd	2.17	9.8	–	0.17	0.4	–	2.34	10.2	–
ST (leader 1)	6.5	21.1	–	0.75	0.6	–	7.25	21.7	–
ST (leader 2)	0.5	0.125	–	4.25	18.1	–	4.75	18.225	–
STd (leader 3)	2.38	8.53	–	0.654	0.855	–	3.034	9.38	–
GNE	6.51	75	0.0325	2.51	18.9	0.0125	9.02	93.8	0.0451
$GNEd$	2.58	19.8	0.0125	1.25	6.27	0.00625	3.84	26.1	0.0192
GC	4.51	20.3	0.226	0	0	0	4.51	20.3	0.226
GCd	2.17	0.0117	9.8	0.171	0.0117	0.4	2.34	10.2	0.0234
$SCNE$	0.00563	0.000028	0.0000316	0	0	0	0.00563	0.000028	0.0000316
$SCNEd$	0.0146	0.000428	0	0	0	0	0.0146	0.000428	0
$STFC$	4.34	18.8	0.0156	0.34	0.0938	0	4.69	18.8938	0.0156
STd	0	0	0	29.8	1.52	1580	29.8	1.52	1580
STd	0	0	0	0	0	0	0	0	0

Table D.2. Solution of the Cournot oligopoly with three asymmetrical agents and small cost coefficients and $c_1 = 1,\ d_1 = 1,\ c_2 = 3,\ d_2 = 1,\ c_3 = 5,\ d_3 = 1$ without consideration of the “green” effect (when $a = 100$)

	x_1	u_1	x_2	u_2	x_3	u_3	x	u
NE	26.2	689	24.2	588	22.2	495	72.8	1770
NEd	16.8	567	16.2	523	15.5	480	48.5	1570
C	49.5	2450	0	0	0	0	49.5	2450
Cd	13.1	650	12.1	588	11.1	528	36.4	1770
ST (leader $i = 1$)	52.5	919	15.5	240	13.5	182	81.5	1341
ST (leader $i = 2$)	18.2	330	48.5	784	14.2	201	80.8	1310
ST (leader $i = 3$)	18.8	355	16.8	283	44.5	660	80.2	1300
STd (leader $i = 1$)	21.6	562	15.2	462	14.5	423	51.4	1450

Table D.3. Solution of the Cournot oligopoly with three asymmetrical agents and medium cost coefficients and $c_1 = 1$, $d_1 = 3$, $c_2 = 3$, $d_2 = 3$, $c_3 = 5$, $d_3 = 3$ (when $a = 100$)

	x_1	u_1	x_2	u_2	x_3	u_3	x	u
NE	26.2	689	24.2	588	22.2	495	72.8	1770
NEd	9.99	399	9.7	376	9.41	355	29.1	1130
C	49.5	2450	0	0	0	0	49.5	2450
Cd	8.42	417	8.08	392	7.75	368	24.2	1180
ST (leader $i = 1$)	52.5	919	15.5	240	13.5	182	81.5	1341
ST (leader $i = 2$)	18.2	330	48.5	784	14.2	201	80.8	1310
ST (leader $i = 3$)	18.8	355	16.8	283	44.5	660	80.2	1300
STd (leader $i = 1$)	10.6	399	9.63	371	9.35	349	29.6	1120

Table D.4. Solution of the Cournot oligopoly with three asymmetrical agents and big cost coefficients and $c_1 = 1$, $d_1 = 5$, $c_2 = 3$, $d_2 = 5$, $c_3 = 5$, $d_3 = 50$

	x_1	u_1	x_2	u_2	x_3	u_3	x	u
NE	26.2	689	24.2	588	22.2	495	72.8	1770
NEd	7.11	303	6.93	288	6.75	273	20.8	864
C	49.5	2450	0	0	0	0	49.5	2450
Cd	6.26	310	6.06	294	5.86	278	18.2	882
ST (leader $i = 1$)	52.5	919	15.5	240	13.5	182	81.5	1341
ST (leader $i = 2$)	18.2	330	48.5	784	14.2	201	80.8	1310
ST (leader $i = 3$)	18.8	355	16.8	283	44.5	660	80.2	1300
STd (leader $i = 1$)	7.3	303	6.91	287	6.73	272	20.9	862

Table D.5. Solution of the Cournot oligopoly with three asymmetrical agents (when $a = 100$) and $n = 3$, $c_1 = 1$, $d_1 = 1$, $\beta_1 = 100$, $c_2 = 3$, $d_2 = 1$, $\beta_2 = 100$, $c_3 = 5$, $d_3 = 5$, $\beta_3 = 100$

	x_1	u_1	g_1	x_2	u_2	g_2	x_3	u_3	g_3	x	u	g
GNE	35.1	5730	0.176	33.1	5340	0.166	31.1	4960	0.156	99.3	16400	0.497
$GNEd$	20.2	2370	0.102	19.6	2270	0.0979	18.9	2170	0.0945	58.7	6810	0.294
GC	49.7	2460	0.249	0	0	0	0	0	0	49.7	2460	0.249
$GCdE$	13.2	653	0.183	12.2	591	0.183	11.2	531	0.183	36.6	1720	0.549
$SCST$	26.4	693	0.183	24.4	591	0.183	22.2	498	0.183	73.2	1782	0.549
$SCSTd$	0	0	0	24.8	649	2.66	25	722	2.43	49.8	1371	5.09

Table D.6. Solution of the GNE Cournot oligopoly with three asymmetrical agents (when $a = 100$) and $n = 3$, $c_1 = 1$, $d_1 = 1$, $c_2 = 3$, $d_2 = 1$, $c_3 = 5$, $d_3 = 5$ and different coefficients β_i

	x_1	u_1	g_1	x_2	u_2	g_2	x_3	u_3	g_3	x	u	g
$\beta_i = 100$	35.1	5730	0.176	33.1	5340	0.166	31.1	4960	0.156	99.3	16 400	0.497
$\beta_i = 1$	51	10 600	25.5	49	10 100	24.5	47	9620	23.5	147	30 300	73.5
$\beta_i = 0.08$	57.9	13 100	36.2	55.9	12 600	34.9	53.9	12 100	33.7	168	37 800	105

REFERENCES

1. *Algorithmic Game Theory*, Nisan, N., Roughgarden, T., Tardos, E., and Vazirany, V., Eds., Cambridge: UniversityPress, 2007.

2. Azevedo, S.G., Carvalho, H., and Machado, V.C., The Influence of Green Practices on Supply Chain Performance: A Case Study Approach, *Transp. Res. Part E: Logist. Transp. Rev.*, 2011, vol. 47, no. 6, pp. 850–871.

3. Basar, T. and Olsder, G.Y., *Dynamic Non-Cooperative Game Theory*, SIAM, 1999.

4. Burgess, K., Singh, P.J., and Koroglu, R., Supply Chain Management: A Structured Literature Review and Implications for Future Research, *Int. J. Operat. Product. Manag.*, 2006, vol. 26, no. 7, pp. 703–729.

5. Cachon, G.P. and Netessine, S., Game Theory in Supply Chain Analysis, in *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, Simchi-Levi, D., Wu, S.D., and Shen, Z.M., Eds., Kluwer Academic Publishers, 2004.

6. Dubey, P., Inefficiency of Nash Equilibria, *Math. Operations Research*, 1986, vol. 11, no. 1, pp. 1–8.

7. Estampe, D., *Supply Chains: Performance and Evaluation*, Wiley, 2018.

8. Fahimnia, B., Sarkis, J., and Davarzani, H., Green Supply Chain Management: A Review and Bibliometric Analysis, *Int. J. Product. Econ.*, 2015, vol. 162, pp. 101–114.

9. Fehr, E. and Schmidt, K.M., A Theory of Fairness, Competition, and Cooperation, *Quart. J. Econ.*, 1999, vol. 114, no. 3, pp. 817–868.

10. Fudenberg, D. and Tirole, J., *Game Theory*, MIT Press, 2002.

11. Ivanov, D. and Dolgui, A., Viability of Intertwined Supply Networks: Extending the Supply Chain Resilience Angles towards Survivability. A Position Paper Motivated by COVID-19 Outbreak, *Int. J. of Production Research*, 2020, vol. 58, no. 10, pp. 2904–2915.

12. Ivanov, D. and Sokolov, B., *Adaptive Supply Chain Management*, Springer, 2010.

13. Johari, R. and Tsitsiklis, J.N., Efficiency Loss in a Network Resource Allocation Game, *Math. Oper. Res.*, 2004, vol. 29, no. 3, pp. 407–435.

14. Jorgensen, S. and Zaccour, G., *Differential Games in Marketing*, Kluwer Academic Publishers, 2004.

15. Govindan, K., Kaliyan, M., Kannan, D., and Haq, A.N., Barriers Analysis for Green Supply Chain Management Implementation in Indian Industries Using Analytic Hierarchy Process, *Int. J. Product. Econ.*, 2014, vol. 147, pp. 555–568.

16. Gunasekaran, A., Subramanian, N., and Rahman, S., Green Supply Chain Collaboration and Incentives: Current Trends and Future Directions, *Transp. Res. E: Logist. Transp. Rev.*, 2015,vol. 74, pp. 1–10.

17. Kannan, D., Govindan, K., and Rajendran, S., Fuzzy Axiomatic Design Approach Based Green Supplier Selection: A Case Study from Singapore, *J. Clean. Product.*, 2015, vol. 96, pp. 194–208.

18. Katok, E., Olsen, T., and Pavlov, V., Wholesale Pricing under Mild and Privately Known Concerns for Fairness, *Product. Operat. Manag.*, 2014, vol. 23, no. 2, pp. 285–302.

19. Katok, E. and Pavlov, V., Fairness in Supply Chain Contracts: A Laboratory Study, *J. Operat. Manag.*, 2013, vol. 31, no. 3, pp. 129–137.

20. Mas-Colell, A., Whinston, M.D., and Green, J.R., *Microeconomic Theory*, Oxford University Press, 1995.

21. Moulin, H., *Theorie des jeux pour l'economie et la politique*, Paris: Hermann, 1981.

22. Moulin, H. and Shenker, S., Strategy Proof Sharing of Submodular Costs: Budget Balance Versus Efficiency, *Econ. Theory*, 2001, vol. 18, no. 3, pp. 511–533.

23. Nagarajan, M. and Sosic, G., Game-Theoretic Analysis of Cooperation among Supply Chain Agents: Review and Extensions, *Europ. J. Operat. Res.*, 2008, vol. 187, no. 3, pp. 719–745.

24. Narahari, Y., *Game Theory and Mechanism Design*, World Scientific, 2014.

25. Nie, T. and Du, S., Dual-Fairness Supply Chain with Quantity Discount Contracts, *Europ. J. Oper. Res.*, 2017, vol. 258, no. 2, pp. 491–500.

26. Osborne, M.J. and Rubinstein, A., *A Course in Game Theory*, MIT Press, 1994.

27. Papadimitriou, C.H., Algorithms, Games, and the Internet, *Proc. 33rd Symp. Theory of Computing*, 2001, pp. 749–753.

28. Roughgarden, T., *Selfish Routing and the Price of Anarchy*, MIT Press, 2005.

29. Sarkis, J., Zhu, Q., and Lai, K.H., An Organizational Theoretic Review of Green Supply Chain Management Literature, *Int. J. Product. Econ.*, 2011, vol. 130, no. 1, pp. 1–15.

30. Sharma, A. and Jain, D., Game-Theoretic Analysis of Green Supply Chain Under Cost-Sharing Contract with Fairness Concern, *Int. Game Theory Review*, 2021, vol. 23, no. 2, p. 2050017.

31. Sharma, A. and Nandi, S., A Review of Behavioral Decision Making in the Newsvendor Problem, *Operat. Supply Chain Management Int. J.*, 2018, vol. 11, no. 4, pp. 200–213.

32. Srivastava, S.K., Green Supply-Chain Management: A State-of-the-Art Literature Review, *Int. J. Manag. Rev.*, 2007, vol. 9, no. 1, pp. 53–80.

33. Tian, Y., Govindan, K., and Zhu, Q. A System Dynamics Model Based on Evolutionary Game Theory for Green Supply Chain Management Diffusion among Chinesemanufacturers, *J. Clean. Product.*, 2014, vol. 80, pp. 96–105.

34. Vives, X., *Oligopoly Pricing: Old Ideas and New Tools*, MIT Press, 1999.

35. Zhu, Q. and Cote, R.P., Integrating Green Supply Chain Management into an Embryonic Eco-Industrial Development: A Case Study of the Guitang Group, *J. Clean. Product.*, 2004, vol. 12, nos. 8–10, pp. 1025–1035.

36. Zhu, Q., Geng, Y., Fujita, T., and Hashimoto, S., Green Supply Chain Management in Leading Manufacturers: Case Studies in Japanese Large Companies, *Management Res. Rev.*, 2010, vol. 33, no. 4, pp. 380–392.