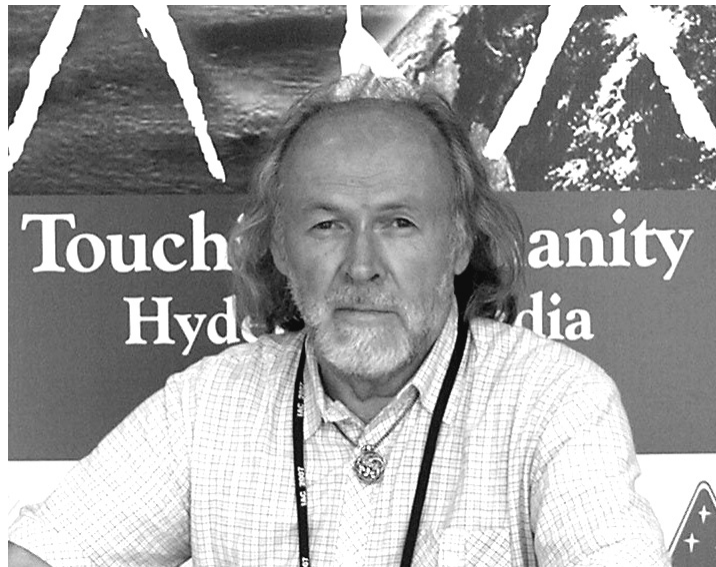

OBITUARY



Mikhail M. Khrustalev
(1938–2023)

Doctor of Physics and Mathematics, Professor Mikhail Khrustalev passed away on August 11, 2023, at the age of 86. He was an outstanding researcher, a wise teacher, and a reliable friend.

Mikhail Khrustalev, in full Mikhail Mikhailovich Khrustalev, was born on May 2, 1938, in Vologda. In 1963, he graduated from Kazan Aviation Institute with a degree in the dynamics of aircraft. Mikhail began his career in the ballistics department of V.N. Chelomei’s Design Bureau (nowadays Military-Industrial Corporation “Research and Industrial Association of Machine Building” (MIC NPO Mashinostroyeniya), Reutov). In 1970, Khrustalev defended his candidate’s dissertation at Moscow State University. Mikhail’s doctoral dissertation was defended in a specialized council of Moscow Aviation Institute (MAI) in 1984. Three years later, he was conferred the title of professor.

A lecturer of the highest level and a scientist in the field of optimal control, Khrustalev taught MAI’s students to mathematical analysis, differential equations, and the theory of functions of a complex variable as well as a course on the modern theory of optimal control. He was able to explain the most complex mathematical constructs simply and understandably, which was appreciated by both students and colleagues: Mikhail thought in terms of the world known to him and was able to explain it to all his friends. He was a lifelong learner of new things and perfectly selfless in sharing his knowledge. Many young researchers became candidates of physics and mathematics under Khrustalev’s supervision. It was a good school of attitude to life.

In addition to teaching, Mikhail devoted much time to organizational and administrative work. In different years, he served as Deputy Director for Science at the Moscow branch of the Institute of Transport Problems, the Russian Academy of Sciences (RAS), and then at the Research Center for Stability and Nonlinear Dynamics, Mechanical Engineering Research Institute RAS. Simultaneously, he worked at the Institute of Applied Mechanics and Electrodynamics (MAI) to create mathematical models of working fluid flows in plasma engines. Numerous colleagues and friends have recognized Mikhail as “one of the few who equally well understands both mathematics and the way aircraft moves.”

Khrustalev's scientific accomplishments are quite extensive. He proposed and rigorously justified sufficient and necessary conditions of global optimality for systems described by ordinary differential equations. In particular, these conditions are applicable to optimal control problems with state constraints. His works formulated global optimality conditions for stochastic diffusion systems with incomplete information about the state, as well as conditions of Nash equilibrium in stochastic differential n -person games.

Mikhail considered necessary and sufficient conditions of terminal invariance to be one of his most striking results. Even when working in the ballistics department, he noticed that this problem has a much higher applied significance compared to the classical invariance problem: it admits a solution much more often and, as a rule, essentially nonunique ones. Khrustalev proposed to use the available freedom of choice in terminally invariant control for the parallel solution of additional problems regularly encountered in practice. In particular, he introduced the concept of absolute invariance as the property where the terminal criterion is independent of both current disturbances and the initial state of the system. He formulated sufficient conditions for this problem.

Khrustalev always maintained close ties with the Institute of Control Sciences (ICS), especially with the Laboratory of Optimal Controlled Systems headed by V.F. Krotov, at whose invitation he came to the Institute in 2014. After Krotov's decease, Mikhail headed the Laboratory from 2015 to 2019. Led by him, employees of the Laboratory obtained necessary optimality conditions and effective numerical optimization algorithms for control processes of nonlinear stochastic diffusion systems and jump diffusion systems on finite and infinite horizons.

During his work at ICS RAS, Khrustalev returned to terminal invariance and advanced brilliantly in this area of research. In particular, he posed a new problem of terminal invariance for stochastic diffusion systems and jump diffusion systems and established sufficient conditions of terminal invariance for both classes of systems.

Khrustalev's last scientific results were connected with the development of Krotov's theory of space-time continuum, a generalization extending the well-known Einstein's general relativity theory and the Poincaré gauge theory of gravity. He investigated the elastic properties of the space continuum and proposed a space-time analog of the Hubble redshift theory and the hypothesis of the distribution of dark matter across the universe. In Khrustalev's theory, the effects attributed to dark matter and dark energy arise by themselves due to time deformation.

Mikhail actively practiced yoga and was well-versed in Eastern philosophy and religion. As destined in his well-studied Buddhism, Mikhail will stay with us, passing into the next form of existence of an enlightened person. He was interesting, non-conflicted, able to defend his point of view, persistent, and hardworking; appreciated simplicity and beauty, and understood and accepted the complexity of our unimaginable world. A pleasant, warm, and peaceful kindness always emanated from that marvelous man. He was as pure as rock crystal, *khrustal'* in Russian. The brightest feelings about him will remain in our memory.

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