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Angular Motion Control of a Large Space Structure with Elastic Elements

V. Yu. Rutkovskii[*,a], **V. M. Glumov**[*,a], **and A. S. Ermilov**[*,b]

[*] *Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia*
e-mail: [a]*vglum@ipu.ru,* [b]*44eas@mail.ru*

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Abstract—The task of angular orientation and stabilization of a space structure during its assembly in orbit is solved. The structure includes elastic elements that are installed during the assembly process. The elastic elements of the structure have no sensors to obtain information about their deformation parameters. Control algorithms are proposed to ensure the stability of the angular motion of the structure. A nonlinear extended Kalman filter is used to obtain the necessary information. A joint estimation algorithm for the coordinates of the angular motion of the considered mechanical system and the coordinates of the elastic vibration tones, as well as an algorithm for the identification of their unobservable parameters are developed. The results of mathematical modeling of a variant of the mechanical system of a space structure are presented, which confirm the operability and efficiency of the developed algorithms for estimating coordinates and parameters.

Keywords: mathematical model, control algorithm, space structure, gyroscopic drive, vibration damping, coordinate estimation

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1. INTRODUCTION

Modern spacecraft are dynamic control objects with mechanical structures containing elastic elements. It is noted in [1] that as the size and complexity of the mechanical structure of such vehicles grows, the influence of the elastic properties of the structure on the dynamics of the orientation mode increases. In addition, there is a tendency to complexity of modern spacecraft structure itself, for example, the use of extended elastic elements. Perturbation in the dynamics of spacecraft is also brought by the transformation of elements of the design during operation [2]. With the development of space technology, large-size space structures have emerged, called "large space structures" (LSS), which can be created in space in various ways. LSS—a multidimensional multi-frequency mechanical system with varying parameters [3, 4]. Some of the first LSSs were considered to be large-sized umbrella reflectors, whose structures were envisioned to be built by assembly in space [5]. The development of space robotics makes it possible to solve LSS assembly problems using various robotic devices [6]. In [7] it is noted that the development of space robotics is characterized by two trends. On the one hand, elements of future space infrastructure such as large, multi-modular spacecraft, for example, orbital stations, are expected to be improved, with robotics as an integral component. On the other hand, more and more attention is being paid to robotic servicing, interpreted in a broad sense, which also includes robotic assembly operations for a very broad class of objects [8]. In the robotic environment, space manipulation robots [9], including free-flying [10] robots, are expected to be used extensively.

This paper considers an umbrella-type LSS assembled in space, which is a dynamic control object with variable parameters and a large and discretely time-varying number of degrees of freedom. As a mechanical system, such LSS can be considered as a sequence of intermediate mechanical structures formed in the assembly process. The structure contains elastic elements installed in the assembly process using a space manipulator or a free-flying space manipulation robot. A variant of LSS is considered, in which elastic elements have no sensors of information about coordinates and vibration parameters. One of the main LSS control tasks is orientation control and stabilization of the structure hull axes. The solution of this problem is traditionally obtained on the basis of relay or discrete algorithms [11]. The breaking character of control actions on the hull and shock effects during installation of new structural elements are the causes of elastic oscillations of LSS. When controlling the angular motion of the LSS, a contradiction arises between the main goal of controlling an elastic dynamic object as a rigid body and the need to damp the appearing elastic oscillations. Absence of atmospheric resistance forces leads to accumulation of energy of elastic vibrations in the process of controlling “rigid” motion of LSS. Exceeding the critical amplitude of elastic oscillations and the proximity of their frequencies to the frequencies of controlling the “rigid” motion lead to instability of the system [12]. The lack of accurate determination of the mathematical model (MM) variables in ground conditions leads to the necessity to solve the problem of stable and accurate control of angular motion at all stages of LSS assembly using robust or adaptive control methods of dynamic objects [13].

In [14] an algorithm for LSS orientation control is proposed with low frequencies of elastic vibrations, which significantly affect the quality of transients due to the proximity of natural frequencies of the structure to the frequency of control of its “rigid” motion. In [15] the problem of providing robust stability of elastic vibrations of spacecraft with a nonlinear orientation control system using flywheel motors is solved. The solution is based on the purposeful change of stability region boundaries in the space of object and regulator parameters to maximize the number of robustly stable elastic components of the spacecraft structure. It should be noted that the algorithms providing robust control are effective for the final assembled spacecraft structure (SS). The approach proposed in [14, 15] is limited by the need to obtain current information on the state of the system and its MM parameters. Adaptive control algorithms allow to ensure stability and damping vibration in a wide enough range of LSS elastic vibration natural frequencies with a minimum value of structural damping. In [16] three types of adaptive control strategy for SS on the sequence of stages of its change during assembly in space were defined. The first type: control using analysis and prediction of LSS elastic vibration state. The second type: control with estimation of the phase of the dominant vibrational component in the frequency spectrum of elastic vibrations at the moment of control switching. The third type: control based on fuzzy logic [17]. In [18], an adaptive control algorithm with a reference model for the angular motion of the assembled LSS is proposed. Its functioning does not depend on the intensity and spectral composition of the input influences and does not require the estimation of the elastic vibrations of the LSS. However, the algorithm provides high control accuracy at high energy costs. Currently, attention is paid to the realization of the first type of LSS adaptive control strategy, which uses methods of identification and estimation of the state of the mechanical system of the structure. In [11], active damping of elastic vibrations of the International Space Station structure by the orientation motors using identification algorithms is proposed. To obtain the necessary information for controlling the angular motion of a space structure with an elastic mechanical system, it is reasonable to use estimation algorithms based on the Kalman–Bucy [19] filtering theory. In [20], the task of estimating the coordinates of elastic vibrations of SS using a nonlinear extended Kalman filter is solved. In the present work (as a follow-up to [20]), an algorithm for joint estimation of the coordinates of angular motion of a mechanical system and unmeasured coordinates of elastic vibrations tones, as well as an algorithm for identification of their unobservable parameters, is developed. The problem of forming algorithms for controlling angular stabilization of SS at the assembly stages is solved. It is assumed that at each stage of assembly there is a connection of a structural element causing elastic vibrations that need to be damped within a given time interval using gyroscopic power drive of the LSS angular stabilization system.

2. MATHEMATICAL MODEL OF ANGULAR MOTION OF LSS

The structure of the umbrella-type LSS mechanical system will be considered as a set of solid bodies, one of which is a carrying body. The other (carried) bodies are building elements attached in one or another order to the carrying body using the spiral scheme of the umbrella-type frame assembly. Such a mechanical system contains non-rigid elements and is characterized by a discretely varying number of degrees of freedom [21]. At the connection points of the structural elements, the rotational degree of freedom in the considered plane of motion and elastic coupling that limits the possible displacements of the elements to the area of small deviations relative to the equilibrium state are taken into account [22]. Using a gyroscopic power drive for LSS assembly containing three identical control moment gyroscope (CMG) installed in a three-beam star pattern, gyrostabilization channel interconnections arise due to inertial and gyroscopic influences [23]. A simplified MM of the spatial angular motion of the mechanical system of the considered LSS type, obtained from the full MM, is presented in detail in [23]. To solve the problem of analytical synthesis of the structure of CMG control algorithms, the model of gyro-power-driven LSS motion, neglecting the cross-effects of CMG motions, can be simplified to three single-type control and gyrostabilization channels of the following form

$$\begin{gathered} I_x\ddot{\chi} + \sum_{i=1}^{n_x} \tilde{I}_{i,x}\ddot{q}_{i,x} - H\dot{\beta} + aI_\beta\ddot{\beta}_s + F(\dot{\chi}) = M_x, \\ a_{i,x}\ddot{\chi} + \ddot{q}_{i,x} + b_{i,x}\dot{q}_{i,x} + c_{i,x}q_{i,x} = 0, \quad i = \overline{1, n_x}, \\ I_\beta\ddot{\beta} + k_d\dot{\beta} + H\dot{\chi} + aI_\beta\ddot{\beta}_s = M_u(u_x), \end{gathered} \tag{1}$$

where $\chi = (\psi, \varphi, \vartheta)^{\mathrm{T}}$ is a vector of hull orientation angles, $\beta = (\beta_\psi, \beta_\varphi, \beta_\vartheta)^{\mathrm{T}}$ is a vector of precession angles of CMG frames, $q = (q_k)^{\mathrm{T}}$ is a composite vector of coordinates characterizing the elastic vibrations of the structure elements along each of the three channels of orientation angles such that $q_k = (q_{i,k})^{\mathrm{T}}$, $i = \overline{1, n_x}$, where n_x is a number of elastic coordinates considered in channel χ_k, $(k = \overline{1,3})$; $\beta_s = [(\beta_\varphi + \beta_\vartheta), (\beta_\psi + \beta_\vartheta), (\beta_\varphi + \beta_\psi)]^{\mathrm{T}}$; $a = \cos(\pi/4 = 0.707$ (for installation of CMGs of "star" type), I_β are moments of inertia of CMG frames; $H = \operatorname{diag}(h_1, h_2, h_3)$ is a diagonal matrix of CMG kinetic moments; k_d is a damping coefficient along the CMG suspension axis; $a_{i,x}, b_{i,x}, c_{i,x}$ are parameters of the equations of vibrations of elastic elements; $I_x = \bar{I}_x + \sum\limits_{i=1}^{n_x} \tilde{I}_{i,x}$, where $\bar{I}_x$ is a diagonal matrix of axial moments of inertia of the hull, $\tilde{I}_{i,x}$ is a matrix of inertial influence of ith elastic element on the dynamics of the structure; $F(\dot{\chi})$ is a vector of nonlinear functions containing products $\chi_i\chi_j$, $i, j = \overline{1,3}$, $i \neq j$; M_x is a vector of disturbing moments of external forces acting on the hull; $M_u(u_x)$ is a vector of control moments applied with respect to the CMG frame axes; u_x is a vector of control voltages, whose components are fed to the inputs of the corresponding CMG momentum drives.

In the mode of angular orientation and stabilization of the LSS at the assembly stage, the values of velocities $\dot{\chi}_k$ are small enough to allow neglecting in $F(\dot{\chi})$ the products $\chi_i\chi_j$, $i, j = \overline{1,3}$, $i \neq j$. In the analytical study of gyro-power-driven control with three identical CMGs, it is reasonable to neglect the interchannel cross-couplings and take $aI_\beta\ddot{\beta}_s = 0$ in (1) [22]. Then the system (1) has the form

$$
\begin{gathered}
I_x\ddot{\chi} + \sum_{i=1}^{n_x} \tilde{I}_{i,x}\ddot{q}_{i,x} - H\dot{\beta} = M_x, \\
a_{i,x}\ddot{\chi} + \ddot{q}_{i,x} + b_{i,x}\dot{q}_{i,x} + c_{i,x}q_{i,x} = 0, \quad i = \overline{1, n_x}, \\
I_\beta\ddot{\beta} + k_d\dot{\beta} + H\dot{\chi} + aI_\beta\ddot{\beta}_s = M_u(u_x).
\end{gathered} \tag{2}
$$

MM (2) is the basis for its decomposition into three subsystems, which correspond to isolated gyrostabilization channels [22].

3. LSS ANGULAR MOTION CONTROL ALGORITHMS

Synthesis of control algorithms for dynamic objects with MM of the form (1) or (2) is traditionally carried out sequentially by two steps [22]. In the first step, the type and parameters of the algorithms forming the values of the components of the vector $u_x(t)$ are determined before the start of the assembly without taking into account elastic vibrations ($q = 0$). Such algorithms are called basic algorithms, during the synthesis of which the MM (2) is transformed to the form of

$$
\begin{gathered}
I_x\ddot{\chi} - H\dot{\beta} = M_x, \\
I_\beta\ddot{\beta} + k_d\dot{\beta} + H\dot{\chi} + aI_\beta\ddot{\beta}_s = M_u(u_x).
\end{gathered} \tag{3}
$$

At the second step of synthesis for stabilization and damping of elastic vibrations it is proposed to form a control algorithm in addition to the basic algorithm, which uses information about elastic vibrations of elements and their parameters.

It is reasonable to apply basic algorithms for controlling CMG in the LSS stabilization mode at the stage of assembling PD-algorithms in each kth channel in the form of

$$
u_{x,k}(t) = p_{1,k}\chi_k(t) + p_{2,k}\dot{\chi}_k(t), \quad k = \overline{1, 3},
$$

where $p_{1,k}, p_{2,k}$ are coefficients, which are chosen taking into account the parameters of the equations (3) and without taking into account the elasticity of the structure from the conditions of ensuring stability and the required quality of control.

The control moments applied relative to the CMG precession axes are formed as [22]

$$
M_{u,k}(u_{x,k}) = p_{0,k}(p_{1,k}\chi_k(t) + p_{2,k}\dot{\chi}_k(t)), \quad k = \overline{1, 3}, \tag{4}
$$

where the coefficients $p_{0,k}$ are determined by the structure characteristics of the hull and are set depending on the moments of inertia I_x at the assembly stage. It should be noted that MM (3) with algorithms (4) describe a linear dynamic system with constant parameters at the assembly stage, whose stability condition on the angular velocity vector $\dot{\chi}$ is determined from the analysis of its characteristic equations in each kth channel in the form of [22]

$$
k_d(h_k + p_{0,k}p_{2,k}) > I_\beta p_{0,k}p_{1,k}, \quad k = \overline{1, 3}. \tag{5}
$$

Based on the same characteristic equations, the problem of determining the values of the coefficients $p_{1,k}, p_{2,k}$ of the algorithms (4) that provide the required regulation time $t_{r,k} \approx 3/\eta_k^*$, $k = \overline{1, 3}$ at the coordinates of the vector χ. Here η_k^* are the given values of the stability degrees of the characteristic equations of [22].

Studies of the dynamics of the umbrella-type LSS have shown that, when the number of elastic elements increases, lower frequencies of elastic vibrations appear in the frequency spectrum.

It should be noted that the gyro-power-driven system with the basic algorithm (4) provides the necessary damping of high-frequency elastic vibrations. However, in the low-frequency region, the processes of elastic vibration damping by means of basic control (4) under the condition (5) appear to be overly delayed [22]. Such dynamics of the processes of orientation and stabilization of the angular position of the LSS is unsatisfactory. In addition, the increase in the elastic vibration damping time creates known difficulties when a free-flying space manipulation robot is used in the LSS assembly process [24]. The mentioned disadvantages require complication of the initial basic control algorithm (4). A possible way to correct the basic algorithm is to organize a subsystem of additional gyro-power-driven stabilization of low-frequency elastic vibrations of the LSS, using estimates $\hat{q}_{i,x}, \hat{\dot{q}}_{i,x}$ of the corresponding elastic coordinates. The additional subsystem is connected after the reorientation maneuver is completed and the structural element is installed at the assembly stage. To accelerate the vibration damping, the subsystem generates additional influences of the following type at the CMG inputs

$$M_{d,k}(u_{q,k}) = \sum_{i=1}^{n_k} \tilde{p}_{1,k,i}\hat{q}_{k,i} + \sum_{i=1}^{n_k} \tilde{p}_{2,k,i}\hat{\dot{q}}_{k,i}, \quad k = \overline{1,3}, \tag{6}$$

where $\hat{q}_k,\ \hat{\dot{q}}_k$ are estimation vectors of elastic coordinates and their derivatives, $\tilde{p}_{1,k,i},\ \tilde{p}_{2,k,i}$ are constant coefficients at the assembly stage.

In choosing the values of the coefficients in (6), it is necessary to take into account the values of the parameter estimates in the equations of MM elastic vibrations (2). Estimates of partial frequency values $\omega_{i,x} = \sqrt{c_{i,x}}$ from the low-frequency spectrum of elastic vibrations allow us to choose the coefficients $\tilde{p}_{1,k,i}, \tilde{p}_{2,k,i}$ that ensure stability and minimum damping time of the elastic component [22]. Using the estimates $\hat{\chi}, \hat{\dot{\chi}}$ taking into account (6), the control moments are formed in the form of

$$M_{u,k}(u_{x,k}) = p_{0,k}\left[p_{1,k}\left(\hat{\chi}_k - I_x^{-1}\sum_{i=1}^{n_x}\tilde{I}_{i,k}\hat{q}_{i,k}\right) + p_{2,k}\left(\hat{\dot{\chi}}_k - I_x^{-1}\sum_{i=1}^{n_x}\tilde{I}_{i,k}\hat{\dot{q}}_{i,k}\right)\right], \quad k = \overline{1,3}. \tag{7}$$

The gain coefficients in (7) at estimates $\hat{q}_{i,k}, \hat{\dot{q}}_{i,k}$ depend on the values of $\tilde{I}_{i,k}$, which can be less than the values of I_x by an order of magnitude or more. For accelerated active compensation of the effect of elastic vibrations on the angular orientation of the LSS, it is reasonable to introduce reconfigurable coefficients $\tilde{p}_{1,k,i},\ \tilde{p}_{2,k,i}$ in (7). Then the algorithms (7) take the following form

$$M_{u,k}(u_{x,k}) = p_{0,k}\left[p_{1,k}\left(\hat{\chi}_k - \sum_{i=1}^{n_x}\tilde{p}_{1,k,i}\hat{q}_{i,k}\right) + p_{2,k}\left(\hat{\dot{\chi}}_k - \sum_{i=1}^{n_x}\tilde{p}_{2,k,i}\hat{\dot{q}}_{i,k}\right)\right], \quad k = \overline{1,3}, \tag{8}$$

where $\tilde{p}_{1,k,i} \gg p_{1,k}I_x^{-1}\tilde{I}_i,\ \tilde{p}_{2,k,i} \gg p_{2,k}I_x^{-1}\tilde{I}_i$.

If the elastic elements do not have information sensors, it is necessary to solve the problem of obtaining estimates of $\hat{q}$ and elastic vibrations parameters at each stage of LSS assembly after its completion. To solve this problem, a modified version of the Kalman filter-based estimation algorithm proposed in [20] is used.

4. SYNTHESIS OF AN ALGORITHM FOR JOINT ESTIMATION OF COORDINATES ELASTIC VIBRATIONS AND THEIR PARAMETERS

The synthesis of the algorithm for joint estimation of the coordinates of angular motion and coordinates of vibrations (tones) of elastic elements of the structure will be carried out on the example of an isolated channel $\chi_2 = \varphi$, which is obtained from MM (2) in the form of

$$\begin{aligned} I_\varphi \ddot{\varphi} + \sum_{i=1}^{n} \tilde{I}_i \ddot{q}_i - h_2 \dot{\beta} = M_\varphi, \\ a_i \ddot{\varphi} + \ddot{q}_i + b_i \dot{q}_i + c_i q_i = 0, \quad i = \overline{1, n_x}, \\ I_\beta \ddot{\beta} + k_d \dot{\beta} + h_2 \dot{\varphi} = M_u(u_\varphi), \end{aligned} \tag{9}$$

where $M_u(u_\varphi) = p_\varphi u_\varphi$, $p_\varphi = (p_{1,\varphi}, p_{2,\varphi})$ is a vector of coefficients, $u_\varphi = (\varphi, \dot{\varphi})^{\mathrm{T}}$.

During the synthesis of the estimation algorithm, assume $M_\varphi = 0$. Then the system (9) is transformed to the form [20]:

$$\begin{aligned} \ddot{\bar{\varphi}} - I_\varphi^{-1} h_2 \dot{\beta} = 0, \\ \left(1 - I_\varphi^{-1} \sum_{i=1}^{n} a_i \tilde{I}_i\right) \ddot{q}_i + \left(1 - I_\varphi^{-1} \sum_{i=1, j \neq i}^{n} a_j \tilde{I}_j\right) (b_i \dot{q}_i + c_i q_i) \\ + a_i \sum_{i=1, j \neq i}^{n} \tilde{I}_j (b_j \dot{q}_j + c_j q_j) + a_i h_2 \dot{\beta} = 0, \\ I_\beta \ddot{\beta} + k_d \dot{\beta} + h_2 \left(\dot{\varphi} - I_\phi^{-1} \sum_{i=1}^{n} \tilde{I}_i \dot{q}_j\right) = p_\varphi u_\varphi. \end{aligned} \tag{10}$$

and the angle φ is defined by the expression

$$\varphi = \bar{\varphi} - I_\varphi^{-1} \sum_{i=1}^{n} \tilde{I}_i q_i, \tag{11}$$

where $\bar{\varphi}$ is an angle of rotation of the hull caused by the rotation of the LSS as a rigid object.

The representation of the φ coordinate in the form of (11) allows to apply filtering algorithms for joint estimation of the coordinates of angular motion of the considered mechanical system of LSS with CMG, unmeasured coordinates q_i of elastic vibration tones, and identification of elastic vibration parameters in real time. It should be noted that unlike the [20] system (10) is nonlinear because it contains unknown parameters. A nonlinear extended Kalman filter is used to obtain the estimates. During the synthesis of the estimation algorithm, let represent the MM equations (10) and (11) in the Cauchy form

$$\dot{x}(t) = f(x(t)) + d u_\varphi + C w(t), \tag{12}$$

where $x \in R^{5n+4}$ is a state vector, $x = (\bar{\varphi}, \dot{\bar{\varphi}}, \beta, \dot{\beta}, q_i, \dot{q}_i, a_i, b_i, c_i)^{\mathrm{T}}$, $i = \overline{1, n}$, $b \in R^{5n+4}$ with non-zero element $d_4 = 1$, $f(x)$ is a nonlinear vector-function defined from (10) and (11),

$$\begin{aligned} & f_1 = x_2, \quad f_2 = I_\varphi^{-1} h_2 x_{2n+4}, \quad f_{2i+1} = x_{2i+2}, \quad f_{2n+3} = x_{2n+4}, \\ & f_{2n+4} = I_\beta^{-1} \left[d_4 u_\varphi - k_d x_{2n+4} - h_2 \left(x_2 - I_\varphi^{-1} \sum_{i=1}^{n} \tilde{I}_i x_{2i+2} \right) \right], \\ & f_{2i+2} = (\cdot)^{-1} \Bigg[x_{2n+4+i} h_2 x_{2n+4} - (\cdot)_j (x_{3n+4+i} x_{2i+2} + x_{4n+4+i} x_{2i+1}) \\ & \qquad - x_{2n+4+i} \sum_{j=1, j \neq i}^{n} \tilde{I}_j (x_{3n+4+j} x_{2j+2} + x_{4n+4+j} x_{2j+1}) \Bigg], \end{aligned}$$

where

$$(\cdot) = 1 - I_\varphi^{-1} \sum_{i=1}^{n} a_i \tilde{I}_i, \quad (\cdot)_j = 1 - I_\varphi^{-1} \sum_{j=1, j\neq i}^{n} a_j \tilde{I}_j, \quad j = \overline{1, n}, \quad j \neq i,$$

$$f_{2n+4+i} = f_{3n+4+i} = f_{4n+4+i} = 0;$$

$w \in R^{4n+2}$ is a noise vector, $C = \mathrm{diag}(C_0 \cdots C_i \cdots)$ is a block-diagonal matrix of object noise, containing blocks $C_0 \in R^{4\times 2}$, $C_i \in R^{5\times 4}$. The elements of matrix C_0 are zero except $c_{21} = c_{42} = 1$, matrices C_i also have zero elements except $c_{21,i} = c_{32,i} = c_{43,i} = c_{64,i} = 1$.

It is assumed that in (10) the unknown parameters of elastic vibrations are assumed constant at the assembly stage. If necessary, any parameters can be included in the state vector χ, which leads to cumbersome mathematical expressions.

If only coordinates φ and $\dot{\varphi}$ are measured on board the LSS, the measurement equation has the form

$$z(t) = Gx(t) + v(t), \tag{13}$$

where the measurement vector $z \in R^2$ has coordinates

$$z_1 = x_1 - I_\varphi^{-1} \sum_{i=1}^{n} \tilde{I}_i x_{4+i} + v_1, \quad z_2 = x_2 - I_\varphi^{-1} \sum_{i=1}^{n} \tilde{I}_i x_{4+n+i} + v_2;$$

v is a noise vector of the meters.

The structure of the measurement matrix $G \in R^{2\times(5n+4)}$ has the form [20]

$$G = [C_1 G_2 \cdots G_{i+2}],$$

where C_1, G_2, G_{i+2} are adjoint matrices, $i = \overline{1, n}$; G_1 is a square unit matrix, G_2 is a square zero matrix; the matrix $G_{i+2} \in R^{2\times 5}$ consists of the following non-zero elements: $g_{11,i} = g_{22,i} = -I_\varphi^{-1} \tilde{I}_\varphi$.

It is assumed that the initial values of $x(t_0), w, v$ are independent of each other, w and v are Gaussian white noise with zero mathematical expectations and correlation functions:

$$\mathrm{M}\langle w(t) w^{\mathrm{T}}(\tau)\rangle = Q_w(t)\delta(t-\tau), \quad \mathrm{M}\langle v(t) v^{\mathrm{T}}(\tau)\rangle = Q_v(t)\delta(t-\tau).$$

Here δ is the Dirac delta function, the diagonal noise intensity matrices $Q_w(t)$ and $Q_v(t)$ are continuous and positively defined for $t \geqslant t_0$. Then the problem of synthesizing an algorithm for estimating the coordinates $x(t)$ from the measurements $z(t)$ reduces to a special case of a continuous nonlinear extended Kalman filter [20] with constant matrices C and G:

$$\begin{gathered} \dot{\hat{x}}(t) = f(\hat{x}) + du(t) + P(t) G^{\mathrm{T}} Q_v^{-1} [z(t) - G\hat{x}(t)], \\ \dot{P}(t) = D(\hat{x}) P(t) + P(t) D^{-1}(\hat{x}) - P(t) G^{\mathrm{T}} Q_v^{-1} G P(t) + C Q_w(t) C^{\mathrm{T}}, \end{gathered} \tag{14}$$

where $\hat{x}(t)$ is a vector of estimates of the coordinates of vector $x(t)$, $P(t)$ is a covariance matrix, $D(\hat{x}) = \partial f(\hat{x})/\partial \hat{x}$ is a Jacobi matrix.

5. MATHEMATICAL MODELING

The investigation of the capabilities of the control algorithm (8) for active compensation of elastic vibrations at the angular orientation of the LSS along φ coordinate was carried out by means of mathematical modeling using in (8) estimates derived from the (14) algorithm. The number of tones and the values of their parameters were assumed to be known and the same in both MM (9) and the estimation algorithm (14), except for those parameters that are assumed to be unknown in (14). To reduce the simulation time in (9), only two tones $n = 2$ were investigated, and the $\hat{\bar{\varphi}}$, $\dot{\hat{\bar{\varphi}}}$ and $\hat{q}_i$, $\dot{\hat{q}}_i$ LSS estimates were used to form the control moment. The constant parameters c_1 and c_2 were chosen as unknowns, and their estimates $\hat{c}_1$ and $\hat{c}_2$ were used in (14).

The control signal is generated on the basis of (8) as follows

$$u_\phi = p_1\hat{\bar{\varphi}} - \sum_{i=1}^{2} \tilde{p}_{1,i}\hat{q}_{1,i} + p_2\dot{\hat{\bar{\varphi}}} - \sum_{i=1}^{2} \tilde{p}_{2,i}\dot{\hat{q}}_{2,i}, \tag{15}$$

where the coefficients $\tilde{p}_{1,i}$, $\tilde{p}_{2,i}$ have the same order as p_1 and p_2, respectively.

In the modeling of angular orientation dynamics to obtain measurements, a variant of the system (9) with the algorithm (15) was used as MM in the form of [20]

$$\dot{y} = Ay + \bar{d}u_\varphi, \tag{16}$$

where $y \in R^8$ is a state vector, $y = (\bar{\varphi}, \dot{\bar{\varphi}}, \beta, \dot{\beta}, q_1, q_2, \dot{q}_1, \dot{q}_2)^{\mathrm{T}}$, $\bar{d} \in R^8$ is a vector with one non-zero element $\bar{d}_4 = 1$.

Based on (16), a vector of measured coordinates $z = (\varphi^*, \dot{\varphi}^*)^{\mathrm{T}}$ was generated using the expression $z = \bar{G}y + v$, where the matrix $\bar{G} \in R^{2\times 8}$ has non-zero elements $\bar{g}_{1,1} = \bar{g}_{2,2} = 1$, $\bar{g}_{1,5} = \bar{g}_{2,7} = -I_\varphi^{-1}\tilde{I}_1$, $\bar{g}_{2,6} = \bar{g}_{2,8} = -I_\varphi^{-1}\tilde{I}_2$, $v = (v_1, v_2)^{\mathrm{T}}$ is a vector of measurement noise.

In (14), MM (12) was used with vector $x \in R^{10}$, which includes identifiable unknown parameters c_1 and c_2, $x = (\bar{\varphi}, \dot{\bar{\varphi}}, \beta, \dot{\beta}, q_i, \dot{q}_i, c_1, c_2)^{\mathrm{T}}$, $i = \overline{1,2}$, $d \in R^{10}$ is a vector with one non-zero element $d_4 = 1$. $C \in R^{10\times 6}$ is a noise matrix with non-zero elements $c_{2,1} = c_{4,2} = c_{6,3} = c_{8,4} = c_{9,5} = c_{10,6} = 1$. The measurement model for the algorithm (14) is formed as $\hat{z} = G\hat{x}$, where the matrix $G \in R^{2\times 10}$ differs from the matrix $\bar{G}$ by the presence of the ninth and tenth zero columns. Matrices $Q_w \in R^{6\times 6}$ and $Q_v \in R^{2\times 2}$ (14) are assumed constant.

The initial values at $t_0 = 0$ of the coordinates and parameters, as well as the vectors y, $\hat{x}$, and the elements of the diagonal covariance matrix $P(0)$ are assumed to be [20] as follows:

$$\begin{gathered}
y_1(0) = 0.017;\ \ y_2(0) = 0.016\ \mathrm{s}^{-1};\ \ y_3(0) = 0.18 \times 10^{-3};\\
y_4(0) = 0.7 \times 10^{-4}\ \mathrm{s}^{-1};\ \ y_5(0) = 0.017;\ \ y_6(0) = 0.19 \times 10^{-4}\ \mathrm{s}^{-1};\\
y_7(0) = 0.37 \times 10^{-2};\ \ y_8(0) = 0.13 \times 10^{-4}\ \mathrm{s}^{-1};\\
a_1 = 1.2;\ \ a_2 = 2.32;\ \ b_1 = 0.24\ \mathrm{s}^{-1};\ \ b_2 = 0.12\ \mathrm{s}^{-1};\\
c_1 = (0.34)^2\ \mathrm{s}^{-2};\ \ c_2 = (0.47)^2\ \mathrm{s}^{-2};\\
I_\varphi = 69\,200\ \mathrm{Nms}^2;\ \ \tilde{I}_1 = 1270\ \mathrm{Nms}^2;\ \ \tilde{I}_2 = 2500\ \mathrm{Nms}^2;\\
I_\beta = 1.1\ \mathrm{Nms}^2;\ \ k_d = 2.5\ \mathrm{Nms};\ \ h = 240\ \mathrm{Nms};\\
p_{1,1} = 3.9 \times 10^{-6};\ \ p_{2,2} = 3.8 \times 10^{-6}\ \mathrm{s}^{-2};\ \ p_{3,3} = 0.49 \times 10^{-2};\\
p_{4,4} = 6.1 \times 10^{-4}\ \mathrm{s}^{-2};\ \ p_{5,5} = 4.7 \times 10^{-4};\ \ p_{6,6} = 0.54 \times 10^{-2}\ \mathrm{s}^{-2};\\
p_{7,7} = 0.11 \times 10^{-4};\ \ p_{8,8} = 0.11 \times 10^{-6}\ \mathrm{s}^{-2};\\
p_{9,9} = 1.1 \times 10^{-3}\ \mathrm{s}^{-4};\ \ p_{10,10} = 1.8 \times 10^{-3}\ \mathrm{s}^{-4}.
\end{gathered}$$

The initial values of the estimates were used:

$$\hat{x}_1(0) = \varphi^*,\ \ \hat{x}_2(0) = \dot{\varphi}^*,\ \ \hat{x}_j(0) = 0\ \forall j = \overline{3,8}.$$

Given that the parameters c_1 and c_2 can only be positive, then $\hat{x}_9(0) = 0.002\ \mathrm{s}^{-2}$; $\hat{x}_{10}(0) = 0.005\ \mathrm{s}^{-2}$.

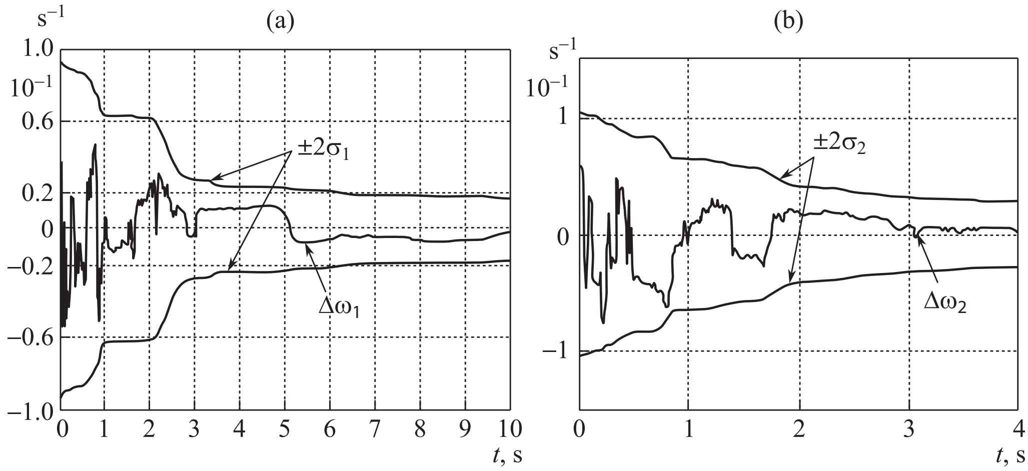

Fig. 1. Identification errors of partial frequencies.

The following standard deviations were assumed for modeling discrete analogs of the continuous white noise of the object and meters:

$$\sigma_{w,1} = 1.5 \times 10^{-5}\ \text{s}^{-2};\quad \sigma_{w,2} = 2 \times 10^{-5}\ \text{s}^{-2};\quad \sigma_{w,3} = 2.2 \times 10^{-6}\ \text{s}^{-2};$$
$$\sigma_{w,4} = 1.8 \times 10^{-6}\ \text{s}^{-2};\quad \sigma_{w,5} = 4.8 \times 10^{-2}\ \text{s}^{-1};\quad \sigma_{w,6} = 3.6 \times 10^{-2}\ \text{s}^{-1};$$
$$\sigma_{v,1} = 2.6 \times 10^{-4};\quad \sigma_{v,2} = 1.34 \times 10^{-5}\ \text{s}^{-1}.$$

The white noise intensity matrices Q_w and Q_v are assumed to be diagonal due to the lack of correlation between the object noise and the noise in the measurement channels. The elements of these matrices are calculated using the expressions

$$q_{w,kk} = 2\sigma^2_{w,k}\tau,\quad k = \overline{1,6} \quad \text{and} \quad q_{w,jj} = 2\sigma^2_{v,j}\tau,\quad j = \overline{1,2},$$

where τ is a correlation time, $\tau \leqslant \Delta t$, Δt is an integration step. The following values are adopted:

$$q_{w,11} = 2.3 \times 10^{-12}\ \text{s}^{-3};\quad q_{w,22} = 0.49 \times 10^{-14}\ \text{s}^{-3};\quad q_{w,33} = 4.8 \times 10^{-14}\ \text{s}^{-3};$$
$$q_{w,44} = 3.2 \times 10^{-14}\ \text{s}^{-3};\quad q_{w,55} = 2.3 \times 10^{-6}\ \text{s}^{-3};\quad q_{w,66} = 1.3 \times 10^{-6}\ \text{s}^{-3};$$
$$q_{v,11} = 4.7 \times 10^{-12}\ \text{s};\quad q_{v,22} = 2.9 \times 10^{-13}\ \text{s}^{-1}.$$

In statistical modeling, discretization of the equations (14) was performed using the fourth-order Runge–Kutta method with Δt, which was chosen to range from 0.002 to 0.005 s.

Figure 1 presents the identification error plots of unmeasured partial frequencies $\Delta\omega_i(t) = \sqrt{c_i} - \sqrt{\hat{c}_i(t)},\ i = \overline{1,2}$ with doubled standard deviations calculated as the corresponding diagonal elements of the matrix $P(t)$: $\sigma_{w,1} = p^{-4}_{9,9}$, $\sigma_{w,2} = p^{-4}_{10,10}$. From the results of statistical modeling, it follows that the convergence time of the parameter estimates c_1 to 2% of the maximum value of the initial value is on average from 3 to 6 s. At the same time, the convergence time of coordinate estimates $\hat{\varphi}$ and $\dot{\hat{\varphi}}$ to 2% of their maximum values averages 20–25 s.

In order to verify the possibility of using the algorithm (15) to actively compensate for the influence of vibrations of elastic parts of the LSS on its angular dynamics, mathematical simulations in the angular stabilization mode have been carried out. The results of comparative modeling of the angular motion of the LSS along the coordinate φ with the control algorithm (7) and the algorithm (15) are presented in Fig. 2.

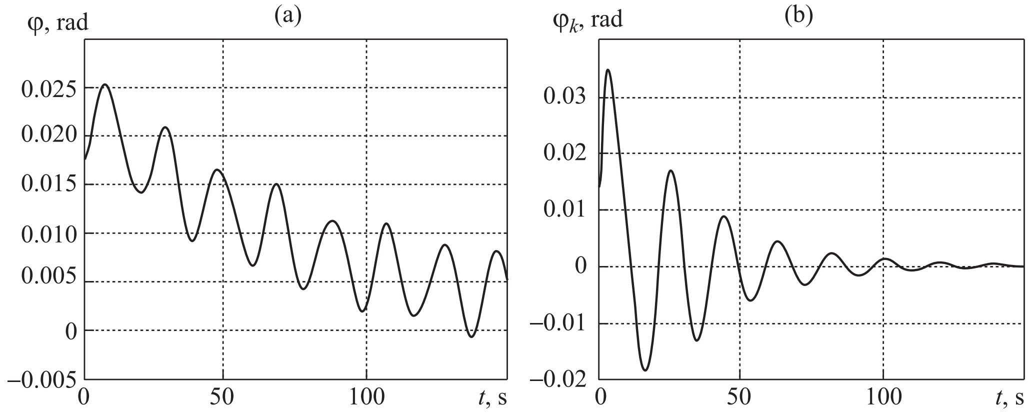

Fig. 2. Stabilization processes for the rotation angle of a structure.

Figure 2a shows plots of the real values of φ (11) obtained when using the algorithm (7) with $\tilde{p}_{1,i} = 0.017$, $\tilde{p}_{2,i} = 0.01$, $i = \overline{1,2}$, in Fig. 2b shows the graphs when using the algorithm (15) in which $\tilde{p}_{1,i} = 3.6$, $\tilde{p}_{2,i} = 2.3$. In the first case, the elastic vibrations decay to 2% of the maximum value of the initial amplitude in ~ 6000 s, while in the second case, with active compensation of the effect of elastic vibrations occurs for ~ 80 s.

In simulations of the stabilization process of the steering angle up to 150 s (see Fig. 2b), errors in the identification of partial frequencies ranged from 0.7 to 1.6% of the true values of the ω_i parameters.

6. CONCLUSION

The problem of vibration damping arises in controlling the angular motion of an LSS assembled in orbit, containing elastic elements, in the absence of information about new mechanical parameters of the assembled structure and initial characteristics arising at each stage of assembly of new elastic components. This requires ensuring not only a timely change of the estimation strategy and, accordingly, of the control during the transition of the structure from one class of mechanical systems to another, but also the application of the principles of adaptive control on the interval of the structure development within each stage of assembly above the first one. The task of optimizing the coefficients in the algorithm (8) at the coordinates of elastic vibration tones from the point of view of rapidity should be solved at the assembly stage, if necessary.

The synthesized algorithm of joint estimation of LSS angular motion coordinates, tones of elastic vibrations of the structure and their parameters allows to obtain with high accuracy estimates of their unmeasured coordinates and parameters in real time based only on the readings of LSS angular motion meters in the absence of any objective information on elastic vibrations.

It should be noted that the construction of an extended Kalman filter for estimating the motion coordinates and their parameters of such a complex mechanical system as the umbrella-type LSS considered in this paper requires using a full MM of a much higher order, in which the mutual influence of vibrational components is taken into account. It is advisable to solve such a problem when developing the system for a particular variant of the assembled structure using an appropriate amount of computational means. This paper considers the principal possibility of using the proposed approach to solve the problem of estimation of such complex dynamic objects.

The use of the synthesized algorithms (8) and (14) in LSS assembly has a number of advantages. Thus, when the first elastic element is installed, the dimensionality of the state vector in the estimation algorithm is increased by five unmeasured coordinates: two coordinates of the elastic vibration tone and three vibration parameters. Since these parameters remain constant for a long time, after their identification they become known and further identification of them is not reasonable, then these three parameters can be excluded from the state vector. After installation of the next elastic element, the above change of dimensionality is repeated and the state vector is increased by five coordinates. After the identification procedure of the next constant parameters, the state vector is also decreased by the next three coordinates and so on. Thus, the state vector after identification of vibration parameters of all elastic elements of the LSS increases only by $2n$ coordinates, where n—the number of elastic elements installed on the LSS.

The results of statistical mathematical modeling prove the possibility of active compensation of the influence of vibrations of elastic parts of the LSS on the dynamics of angular orientation and stabilization of the LSS itself using control of the form (15). When modeling the algorithm of identification of parameters a, b, c, three variants of estimation were tested, in which one of the three parameters was chosen and the other two were considered to be known.

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