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Synthesis of Test Control for Identification of Aerodynamic Characteristics of Aircraft

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Abstract—The synthesis of a control law for tracking a target informative path as a new approach to solving the problem of planning a flight experiment for identifying the aerodynamic characteristics of automatically controlled aircraft is proposed. The mathematical statement and the method for solving the synthesis problem are obtained. In the numerical experiment, it is shown that the identification accuracy on the synthesized control can be significantly improved compared to the identification accuracy on the optimal program test signal.

Keywords: aerodynamic characteristics, planning of test signals, parametric identification, automatic control

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1. INTRODUCTION

The task of planning test signals for identifying the aerodynamic characteristics (ADC) of an aircraft is to generate a specially perturbed motion of the aircraft in order to increase the accuracy of ADC identification. The disturbed movement of the aircraft (test maneuver) is formed by applying so-called test input signals (test signals) to the aircraft's controls. As a rule, criteria adopted in the theory of optimal design of experiments, which characterize to one degree or another the expected identification accuracy, are used as criteria for selecting a test signal.

Problems of active identification of ADC of aircraft are characterized by a wide variety of mathematical formulations. Already solved problems differ in their mathematical formulations in: test signal dimension (scalar [1–9], vector [1, 4, 9–16]), class of functions in which the test signal is optimized (continuous functions [9], discrete functions [1, 2, 4, 8, 17], polyharmonic functions [2, 10, 12, 15, 16, 18], type "bang-zero-bang" controls and similar controls [2, 6, 8, 12, 14, 15], parameterized controls [2, 4, 7, 11], functions of simple form [5]), by type of restrictions (only for test signal [2, 3, 5, 9] on the components of the state vector of the aircraft in perturbed motion [1–4, 6, 7, 11, 14, 17]), criterion (Turing number [1], L-, D-criterion [1–7, 9, 11, 12, 14, 15, 17, 18], peak factor [10, 12, 15, 16]). It is usually assumed that the choice of test signal is made before the experiment, but the possibility of step-by-step optimization of the test signal during the experiment is also considered [3]. Optimization of test signals is performed most often in the time domain [1–12, 14, 16], but can also occur in the frequency domain [18] or in the time and frequency domain simultaneously [13, 17]. For further presentation, it is important to note that in the known formulations of the problem of active identification of aircraft ADCs, restrictions on the components of the aircraft state vector in perturbed motion do not take into account (except [6]) possible differences between unknown ADCs and their a priori estimates, and the choice of test signals is made in the class of program controls, i.e. adaptive control for the purpose of active identification of aircraft ADCs is practically not considered [2, 19].

The safety conditions of the flight experiment, various physical and methodological restrictions determine the restrictions on the disturbances of the components of the aircraft state vector in the test maneuver. In a number of important applications, taking into account these restrictions is a necessary condition for performing a test maneuver [11]. If the restrictions are violated, the test maneuver is not performed (interrupted by the aircraft automatic control system). The fulfillment of the restrictions must be ensured whenever the ADC and the initial conditions of the test maneuver are known approximately when selecting a test signal.

In [6] a method for optimizing a test signal is proposed taking into account the specified restrictions in the class "bang-zero-bang" controls. In [7], a method for optimizing a test signal is proposed taking into account the specified restrictions in the class of parameterized controls, in particular, a solution was obtained in the class of piecewise constant functions with a short persistence time, which differs significantly from "bang-zero-bang" management. The program test signal obtained in [7] ensures that the specified restrictions are met for all a priori possible values of the ADC. But a consequence of this positive property of the test signal is its optimality "on average" on the set of all restrictions, determined by the set of possible values of the ADC. This means that in each specific case (in particular, with true ADC values), such a test signal will obviously be suboptimal. In the class of program test signals, it is impossible to select a test signal that will be optimal for all possible values of the ADC. However, it is possible to improve the informative properties of the selected test signal directly during the flight experiment due to the information obtained about the aircraft state vector. In [20] a method for approximate solution of this problem was proposed. Below we propose a method for finding its optimal solution.

2. FORMULATION OF THE PROBLEM

The proposed mathematical formulation of the control synthesis problem for identifying the ADC contains a model of the dynamics of the object in a test mode lasting T seconds, described by a linear (linearized with respect to the reference motion of the aircraft) differential equation

$$\frac{dx}{dt} = A(b)x + Gu,\ t[0,T], \quad x(0) = x_0, \tag{1}$$

and discrete measurement model

$$z_i = z(x(t_i)) = Hx(t_i) + v_i, \quad i = \overline{1,N}, \tag{2}$$

where: x is n-dimensional vector of the aircraft state; $u = u(t,x)$ is optimized dimension control vector m; z_i is p is dimensional vector of measurements; v_i is vector of "white" Gaussian noise of measurements, $E(v_i) = 0$, $E(v_i v_j^T) = 0$, $i \neq j$, $E(v_i v_i^T) = R$, $i = \overline{1,N}$, $j = \overline{1,N}$ (E is mathematical expectation); $A(b)$, G, H is matrices of corresponding dimensions; t_i is timepoints at which measurements are taken, $t_i = h(i-1)$, $h = T/(N-1)$; N is number of measurements. The matrix $A(b)$ depends on the identified vector of unknown parameters b (the desired ADCs) of dimension k. The true values of the b^{true} parameters b are not known. The a priori estimate b^{pri} of the vector b^{true} contains an error Δb, $b^{\text{pri}} = b^{\text{true}} + \Delta b$, with respect to which it is known that the components Δb_i of the vector Δb belong to the intervals $[-\Delta_i, \Delta_i]$: $\Delta b_i \in [-\Delta_i, \Delta_i]$, $i = \overline{1,k}$. We denote the set of possible values of b by the symbol B.

We will assume that the movement of the aircraft before the start of the test maneuver is quasi-stationary. This means that the components of the vector x_0 in (1) are close to zero, but may be different from zero. We will assume that x_0^{true} belongs to the closed bounded set X^0 containing the zero vector. We will consider the possible values of the components of the vectors x_0 and b to be independent of each other.

It's required by choosing on the interval $[0, T]$ a vector function $u = u(t, x)$ from a certain class of functions U (defined below):

1) ensure the fulfillment of scalar linear restrictions on the state vector of the aircraft for all a priori possible values of b and x_0

$$|x_s(t, b, x_0, u)| \leqslant q_s(t), \quad b \in B, \quad x_0 \in X^0, \quad s = \overline{1, r}, \tag{3}$$

where: x_s is components of the vector x on which restrictions are imposed; $q_s(t)$ is specified functions; r is number of restrictions;

2) minimize the control u functional

$$J = \operatorname{tr}\left(WM^{-1}(b^{\text{pri}}, x_0, u)\right), \tag{4}$$

where tr is matrix trace notation, W is non-negative definite weight matrix (usually diagonal), M is information matrix:

$$M(b, x_0, u) = \sum_{i=1}^{N} (\partial x(t_i, b, x_0, u)/\partial b)^T \; Q(\partial x(t_i, b, x_0, u)/\partial b). \tag{5}$$

In (5) matrix $Q = H^T R^{-1} H$; $x_0 = 0$; derivatives $S_j = \dfrac{\partial x(t, b, x_0, u)}{\partial b_j}$, $j = \overline{1, k}$ are determined from a system of differential equations for sensitivity functions:

$$\begin{cases} \dfrac{dS_j}{dt} = A(b)S_j + \dfrac{\partial A(b)}{\partial b_j} x(t, b, x_0, u), \\ S_j(0) = 0, \quad j = \overline{1, k}. \end{cases} \tag{6}$$

Equations (6) and (1) are solved jointly.

The mathematical formulation of the test signal planning problem in the class of program controls differs from the above formulation of the problem only in that the desired control is sought in a given class of time functions, i.e., $u = u(t)$ (usually in the class of continuous or piecewise continuous functions of time [1–18]).

The solution $u = u(t, x)$ to problem (1)–(4) is proposed to be sought among controls that ensure tracking of a certain trajectory of system (1), which has good information content about the identified parameters and satisfies restrictions (3), and precisely in the class of functions representable in the form

$$u(t, x) = \mu u^{\text{pri}}(t) + L\left(\mu x^{\text{pri}}(t) - x(t)\right), \tag{7}$$

where $x^{\text{pri}}(t) = x(t, b^{\text{pri}}, 0, u^{\text{pri}})$—trajectory of system (1) for optimal program test signal $u^{\text{pri}}(t)$ at restrictions $B = b^{\text{pri}}$, $X^0 = 0$; coefficient μ, $0 \leqslant \mu \leqslant 1$, and elements L_{ij} of matrix L

$$|L_{i,j}| \leqslant C, \quad i = \overline{1, l}, \quad j = \overline{1, n} \tag{8}$$

subject to determination from the minimum condition of criterion (4) under restrictions (3). The constant C reflects restrictions on the feedback coefficients of the automatic control system (ACS). For the convenience of further references, we will call the given problem the problem of selecting a test control, and the desired function $u(t, x(t))$ will be called a test control.

System (1) under control (7) can be written in the form

$$\frac{dx}{dt} = (A(b) - GL)x + \mu G(u^{\text{pri}}(t) + Lx^{\text{pri}}(t)), \quad x(0) = x_0, \tag{9}$$

therefore, for sufficiently small values of the coefficient μ, restrictions (3) will certainly be satisfied. In addition, from (4)–(6) and (9) it follows that for an arbitrary function $u = u(t)$ the equality $J(\mu u) = J(u)/\mu^2$ is true, therefore, to minimize functional (4), the value of μ should be chosen as maximum as possible, subject to the fulfillment of restrictions (3).

The equations for the sensitivity functions S_j, $j = \overline{1, k}$ are written in the form (6), since it is assumed that in the procedure for post-flight estimation of the vector b the technique of artificially disconnecting the system will be used (9), when a signal $u_\Sigma(t)$ known from a flight experiment is applied to the input of a customized motion model with an excluded ACS circuit $u_\Sigma(t) = \mu u^{\text{pri}}(t) + L(\mu x^{\text{pri}}(t) - x(t))$. If the customized model includes an ACS model, then the matrix A in (6) must be replaced by the matrix $A - GL$.

A fairly complete characteristic of the solution to problem (1)–(5) is the distribution density of the function values $J(b, x_0) = \operatorname{tr} M^{-1}(b, x_0, u)$. The function $J(b, x_0)$ characterizes the expected identification error (the lower bound of the sum of variances of parameter estimates) on the test control $u\,(t, x(t))$ (or on the test signal $u(t)$) if $b^{\text{true}} = b$, $x(0) = x_0$. To construct an estimate of a given distribution density (polygon), it is sufficient to calculate the values of the function $J(b, x_0)$ for a plenty large number N_P of pairs of vectors b and x_0, $b \in B$, $x_0 \in X^0$, selected randomly. If the distribution densities of the components of the vectors b and x_0 on the intervals of their possible values are unknown, then according to the recommendations [21] they should be assumed to be uniform. The number N_P is chosen so that when it increases, the position and shape of the polygon do not change. With little computational time spent, the polygon of expected identification error values represents an integral characteristic of test management quality that is convenient for analysis, allowing one to estimate the probability of obtaining certain values of the expected identification error parameters.

3. SOLUTION METHOD

In the case of $B = b^{\text{pri}}$, $X^0 = 0$, the optimal program test signal $u^{\text{pri}}(t)$ and the corresponding trajectory $x^{\text{pri}}(t) = x(t, b^{\text{pri}}, 0, u^{\text{pri}})$ can be found, for example, by one of the methods described in [2, 7]. Further, we present a method for optimizing the coefficient μ and matrix L in (7).

We combine the elements of the matrix L and the coefficient μ into one vector $v \in V$, where V is a hypercube defined by inequalities (8) and the inequality $0 \leqslant \mu \leqslant 1$. The dimension of the vector v is equal to $N_v \leqslant nl + 1$ (some elements of the matrix L can be set equal to zero to eliminate feedback on the corresponding components and reduce the number of adjustable coefficients). We set $v_{N_v} = \mu$. We denote by $x(t, b, x_0, u^v)$ the solution of system (1) on control (7) for a given vector v.

Let N_C be a positive integer. We divide the optimization interval $[0, T]$ with points $t_i = \Delta_C(i-1)$, $i = \overline{1, N_C}$ into subintervals of equal length $\Delta_C = T/(N_C - 1)$. We choose N_C so large that when the restrictions are satisfied

$$\begin{gathered} |x_s(t_i, b, x_0, u^v)| \leqslant q_s(t_i), \quad t_i = \Delta_C(i-1), \quad i = \overline{1, N_C}, \\ b \in B, \quad x_0 \in X^0, \quad s = \overline{1, r} \end{gathered} \tag{10}$$

restrictions (3) can be considered fulfilled for all $t \in [0, T]$ with sufficient accuracy. Thus, to solve the problem posed, it is sufficient to solve the problem of minimizing criterion (4) on the set S of vectors v satisfying the set of restrictions (10).

We define the following auxiliary problem. Minimize by $v \in V$ the criterion

$$J = \mathrm{tr}\left(WM^{-1}(b^{\mathrm{pri}}, 0, u^v)\right) \tag{11}$$

on some closed, bounded set $\check{S}$ of vectors v, defined by a finite number of restrictions

$$\begin{gathered} |x_s(t_i, b^j, x_0^j, u^v)| \leqslant q_s(t_i), \quad i = \overline{1, N_C}, \\ b^j \in B, \quad x_0^j \in X^0, \quad s = \overline{1, r}, \quad j = \overline{1, K}, \quad v \in V. \end{gathered} \tag{12}$$

The solution to this typical nonlinear programming problem can be found by various methods, for example, the linearization method [22]. The gradients of restrictions (12) over the components of the vector v are equal

$$S_{v_j} = \frac{\partial x(t, b, x_0, u^v)}{\partial v_j}, \quad j = \overline{1, N_v}.$$

The gradient of functional (11) can be calculated if the functions are known

$$S_{v_j}^{b_i} = S_{v_j}^{b_i}(t, b, x_0, u^v) = \frac{\partial}{\partial v_j} S_i, \quad i = \overline{1, k}, \quad j = \overline{1, N_v}.$$

The functions $S_{v_j}, S_{v_j}^{b_i}$ can be determined from solving the following systems of equations, which must be solved together with equations (1) and (6):

$$\begin{cases} \dfrac{dS_{v_j}}{dt} = (A(b) - GL)S_{v_j} - G\dfrac{\partial L}{\partial v_j} x(t, b, x_0, u) + \mu G \dfrac{\partial L}{\partial v_j} x^{\mathrm{pri}}, \text{ if } j = \overline{1, N_v - 1}, \\ \dfrac{dS_{v_{N_v}}}{dt} = (A(b) - GL)S_{v_{N_v}} + G(u^{\mathrm{pri}} + Lx^{\mathrm{pri}}), \\ S_{v_j}(0) = 0, \quad j = \overline{1, N_v}; \end{cases}$$

$$\begin{cases} \dfrac{dS_{v_j}^{b_i}}{dt} = A(b) S_{v_j}^{b_i} + \dfrac{\partial A(b)}{\partial b_i} S_{v_j}, \\ S_{v_j}^{b_i}(0) = 0, \quad j = \overline{1, N_v}, \quad i = \overline{1, k}. \end{cases}$$

The solution to the original minimization problem with respect to the vector v of criterion (11) under restrictions (8) and (10) can be obtained by the following iterative algorithm:

Step 0. We set the counter for the number of iterations: $iter = 0$. We define arbitrary $b^j \in B$, $x_0^j \in X^0$, $j = \overline{1, K}$ and define the set S^{iter} as set of vectors v satisfying inequalities and conditions (12).

Step 1. We solve an auxiliary problem in which $\check{S} = S^{iter}$. We denote the solution by v^{iter}, the corresponding test control (7)—by $u^{v^{iter}}$.

Step 2. To check the fulfillment of restrictions (10) on the found control $u^{v^{iter}}$ for each $s = \overline{1, r}$ and $i = \overline{1, N_C}$ we define $\max\limits_{b \in B, x_0 \in X^0} \left| x_s(t_i, b, x_0, u^{v^{iter}}) \right|$.

Step 3. If for all $s = \overline{1, r}$, $i = \overline{1, N_C}$

$$\max_{b \in B, x_0 \in X^0} \left| x_s(t_i, b, x_0, u^{v^{iter}}) \right| \leqslant q_s(t_i)$$

is true, then problem (11)–(10) is solved—a test control that satisfies restrictions (10) and minimizes functional (11) is found. Next go to Step 5.

Step 4. If for some s^*, i^*

$$\max_{b\in B, x_0\in X^0} \left| x_{s^*}(t_{i^*}, b, x_0, u^{v^{iter}}) \right| = \left| x_{s^*}(t_{i^*}, b^*, x_0^*, u^{v^{iter}}) \right| > q_{s^*}(t_{i^*})$$

is true, that is, restrictions (10) are violated, then we supplement the set S^{iter} with restrictions $\left| x_{s^*}(t_{i^*}, b^*, x_0^*, u^{v^{iter}}) \right| \leqslant q_{s^*}(t_{i^*})$. We again denote the obtained set by S^{iter}, having previously set $iter = iter + 1$. Next go to Step 1.

Step 5. Constructing a polygon of function values $J(b, x^0) = \operatorname{tr}\left(WM^{-1}(b, x_0, u^{opt})\right)$, where $u^{opt} = u^{v^{iter}}$. The method for constructing the polygon was described in Section 2.

We explain: each subsequent set S^{i+1} of vectors v is already contained in the previous set S^i due to the fact that each restriction added at Step 4 narrows the set on which criterion (11) is minimized. Thus $S^0 \supset S^1 \supset \cdots \supset S^i \supset \cdots \supset S$, where S is the set of vectors v defined by the formulas (10). Consequently, the minimum of criterion (11) on the set S is not less than the minimum on the set S^i. Therefore, if at the ith iteration the conditions of Step 3 of the algorithm are met, then restrictions (10) are satisfied, and the minimum found on the set S^i is the minimum on the set S.

Thus, the solution to problem (4), (10) is reduced to solving a sequence of standard nonlinear programming problems that "approximate" the original problem in the vicinity of the desired minimum with the approximation accuracy increasing during iterations. This approach seems preferable to optimization of test signals using the dynamic programming method [6, 14] due to the "curse of dimensionality."

The presented method for solving the problem can be generalized to the case of dependence of the matrices G and H on the identified parameters b.

4. NUMERICAL MODELING

We consider the problem of constructing a two-component ($m = 2$) test control $u(t, x(t))$ on a time interval of eight seconds ($T = 8$) in order to identify the coefficients b_i, $i = \overline{1,5}$ models of aircraft lateral movement [9]

$$\begin{cases} \dot{\beta} = b_1\beta + w_y + 0.0565\gamma + 0.0289\delta_N, \\ \dot{w}_x = b_2\beta - 0.935w_x - 0.124w_y + 1.4\delta_N + 2.88\delta_e, \\ \dot{w}_y = b_3\beta + 0.119w_x + b_4w_y + b_5\delta_N, \\ \dot{\gamma} = w_x, \end{cases} \quad (13)$$

supplemented with the simplest models of the rudder and aileron drive:

$$\begin{cases} \dot{\delta}_N = \omega_N, \\ \dot{\omega}_N = k(\delta_N^{\text{set}} - \delta_N) - k_2\omega_N, \quad \delta_N^{\text{set}} = u_1(t, x(t)), \\ \dot{\delta}_e = \omega_e, \\ \dot{\omega}_e = k(\delta_e^{\text{set}} - \delta_e) - k_2\omega_e, \quad \delta_e^{\text{set}} = u_2(t, x(t)), \\ k = \dfrac{0.456}{\tau^2}, \quad k_2 = \dfrac{0.8}{\tau}, \quad \tau = 0.02. \end{cases} \quad (14)$$

In (13) and (14): β is gliding angle of the aircraft, w_x, w_y is angular velocities of roll and yaw, γ is roll angle, δ_N, δ_e are rudder and aileron deflection angles, ω_N, ω_e are rudder and aileron deflection angular rates, k, k_2, τ are parameters of the rudder and aileron drives, coefficients b_1, b_2, b_3, b_4, b_5 are derivatives of the lateral aerodynamic force and aerodynamic moments of roll and yaw to be identified corresponding components of the aircraft state vector: β, w_x, w_y, δ_N. The dimension of angular velocities is—degrees per second, angles are—degrees. The variables β, w_x, w_y, γ, δ_N, δ_e are measured independently at a frequency of 25 hertz.

We have the state vector of the aircraft $x = (\beta, w_x, w_y, \gamma, \delta_N, \delta_e, \omega_N, \omega_e)^T$, vector of identifiable parameters $b = (b_1, b_2, b_3, b_4, b_5)^T$, measurement vector $z_i = z(t_i) = Hx(t_i) + v_i$, $t_i = h(i-1)$,

$i = \overline{1, N}$, where H is matrix with elements $H_{ii} = 1$ when $i = \overline{1, 6}$, $H_{ij} = 0$ when $i = \overline{1, 6}$, $j = \overline{1, 8}$, $i \neq j$; v_i is vector of "white" Gaussian noise of measurements, $E(v_i) = 0$, $E(v_i v_j^T) = 0$, $i \neq j$, $E(v_i v_i^T) = R$, $i = \overline{1, N}$, $j = \overline{1, N}$, $h = 0.04$ s, $N = 201$. The root-mean-square measurement errors ($\sqrt{R_{ii}}$, $i = \overline{1, 6}$) are: for β—$1°$, for w_x, w_y—$0.71^{\circ/\mathrm{s}}$, for δ_N, δ_e—$0.5°$.

A priori estimate of the true values b^{true} of the vector b:

$$b^{\mathrm{pri}} = (-0.119,\ -4.43,\ -2.99,\ 0.178,\ 1.55)^T.$$

The boundaries of the tolerance intervals $[-\Delta_i, \Delta_i]$, such that $\Delta b_i \in [-\Delta_i, \Delta_i]$, have the form: $\Delta_i = \pm 0.5 \left| b_i^{\mathrm{pri}} \right|$, $i = \overline{1, 4}$, $\Delta_5 = \pm 0.2 \left| b_5^{\mathrm{pri}} \right|$. Thus, the a priori uncertainty of the first four components of the vector b is $\pm 50\%$ of the nominal values. The set of possible values of the vector b defines a parallelepiped with center at the point b^{pri}—set B. The test maneuver should start from a quasi-stationary state:

$$\begin{aligned} &|\omega_x(0)| \leqslant 0.25^{\circ/\mathrm{s}}, \quad |\beta(0)| \leqslant 0.5°, \quad |\omega_N(0)| \leqslant 0.025^{\circ/\mathrm{s}}, \quad |\delta_N(0)| \leqslant 0.25°, \\ &|\omega_y(0)| \leqslant 0.25^{\circ/\mathrm{s}}, \quad |\gamma(0)| \leqslant 0.25°, \quad |\omega_e(0)| \leqslant 0.025^{\circ/\mathrm{s}}, \quad |\delta_e(0)| \leqslant 0.25°. \end{aligned} \tag{15}$$

The set of possible values of the initial conditions of the test maneuver $x_0 = x(0)$ defines the polyhedron—set X^0. Intervals $I_6 = \pm 0.25^{\circ/\mathrm{s}}$, $I_7 = \pm 0.5°$, $I_8 = \pm 0.025^{\circ/\mathrm{s}}$, $I_9 = \pm 0.25°$, $I_{10} = \pm 0.25^{\circ/\mathrm{s}}$, $I_{11} = \pm 0.25°$, $I_{12} = \pm 0.025^{\circ/\mathrm{s}}$, $I_{13} = \pm 0.25°$, defining possible values x_0, as well as tolerance intervals $I_i = [-\Delta_i, \Delta_i]$, $i = \overline{1, 5}$ will be further called intervals of a priori uncertainty.

When constructing polygons of values $J(b, x_0)$, we will assume that the components of the a priori estimate of the vector b and the components of the vector x_0 are uniformly distributed over the intervals of a priori uncertainty $I_i = [-\Delta_i, \Delta_i]$, $i = \overline{1, 13}$ and are independent of each other. The matrix W in (4) was assumed to be unit.

We will impose restrictions on the permissible disturbances of each of the components of the vector x in the test maneuver:

$$\begin{gathered} |\omega_N(t, b, x_0, u)| \leqslant 30^{\circ/\mathrm{s}}, \quad |\omega_e(t, b, x_0, u)| \leqslant 30^{\circ/\mathrm{s}}, \quad |\beta(t, b, x_0, u)| \leqslant 3°, \\ |w_x(t, b, x_0, u)| \leqslant 5^{\circ/\mathrm{s}}, \quad |w_y(t, b, x_0, u)| \leqslant 5^{\circ/\mathrm{s}}, \quad |\gamma(t, b, x_0, u)| \leqslant 5°, \\ b \in B, \quad x_0 \in X^0, \quad t \in [0, 8]. \end{gathered} \tag{16}$$

The first two restrictions in (16) reflect physical restrictions on the speed of movement of the drives, and the remaining restrictions are intended to ensure the safety of the test maneuver. Time discretization (see (10)) of restrictions (16) was carried out with the parameter $\Delta_C = h$.

The task is to determine such a test control $u^A(t, x(t))$:

$$u_i^A(t, x(t)) = \mu u_i^{\mathrm{pri}}(t) + \sum_{j=1}^{4} L_{i,j}(\mu x_i^{\mathrm{pri}}(t) - x_i(t)), \quad i = 1, 2, \tag{17}$$

on which the functional (4) reaches its minimum value. The restrictions on the elements of the matrix $L_{i,j}$ in test control (17) were taken in the form (8) with $C = 0.5, 1, 2$. The optimal test control $u^A(t, x(t))$ was determined in accordance with the algorithm of Section 3. Optimization of the program test signal $u^{\mathrm{apr}}(t)$ with $B = b^{\mathrm{apr}}$, $x_0 = 0$ and replacing restrictions (16) with restrictions

$$\begin{gathered} |\omega_N(t, b^{\mathrm{pri}}, 0, u)| \leqslant 30^{\circ/\mathrm{s}}, \quad |\omega_e(t, b^{\mathrm{pri}}, 0, u)| \leqslant 30^{\circ/\mathrm{s}}, \quad |\beta(t, b^{\mathrm{pri}}, 0, u)| \leqslant 3°, \\ |w_x(t, b^{\mathrm{pri}}, 0, u)| \leqslant 5^{\circ/\mathrm{s}}, \quad |w_y(t, b^{\mathrm{pri}}, 0, u)| \leqslant 5^{\circ/\mathrm{s}}, \quad |\gamma(t, b^{\mathrm{pri}}, 0, u)| \leqslant 5°, \quad t \in [0, 8] \end{gathered}$$

was performed by the method described in [7], in the class of parameterized controls, presented in the form

$$u_j^{\mathrm{pri}}(t) = \sum_{i=1}^{50} d_{i+50(j-1)} \sin(\pi i t / T), \quad j = 1, 2,$$

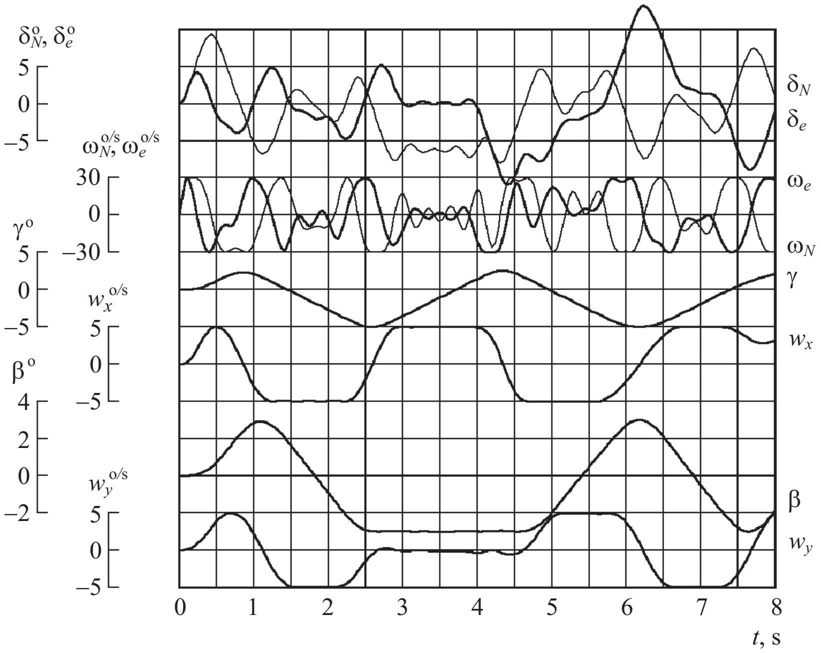

Fig. 1. Optimal solution to the problem in the class of program controls for $B = b^{\rm pri}$, $X^0 = 0$.

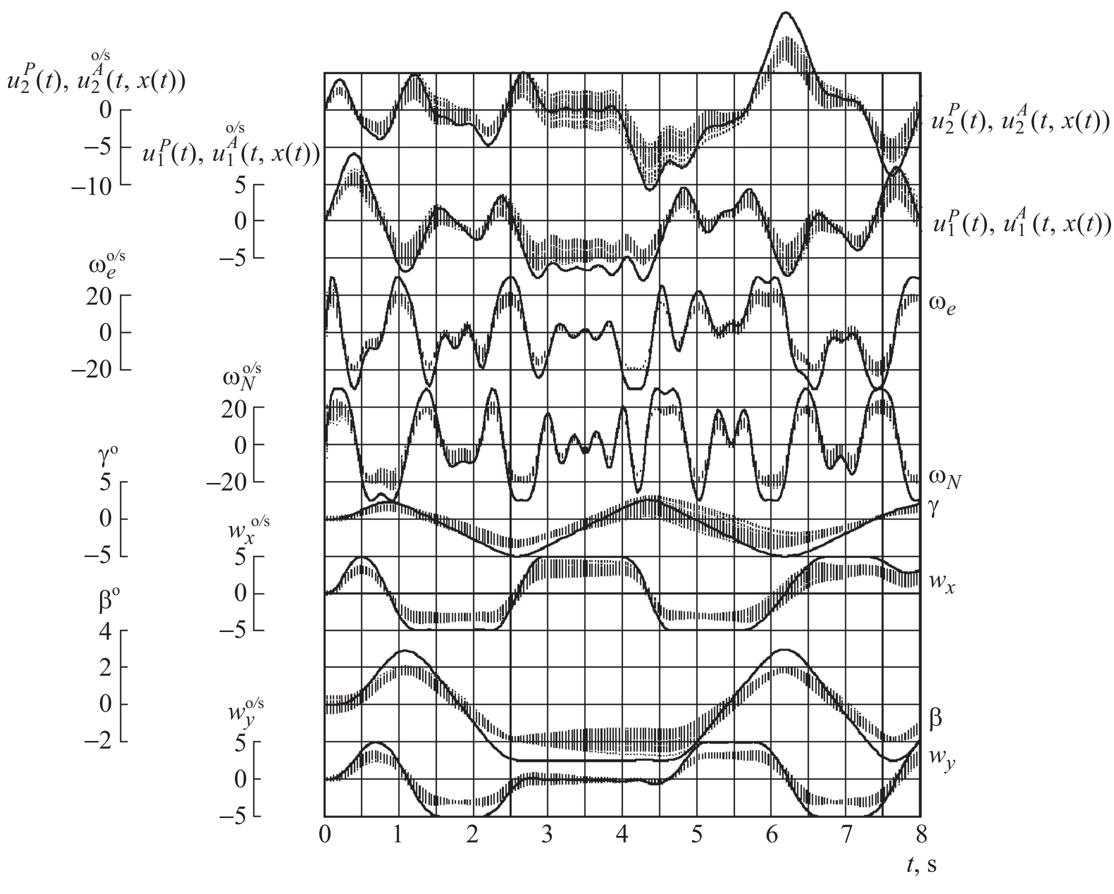

Fig. 2. Trajectory fields of system (1) under optimal test control for $C = 2$. Trajectory components $x^{\rm pri}(t) = x(t, b^{\rm pri}, 0, u^{\rm pri})$ are shown by thick lines.

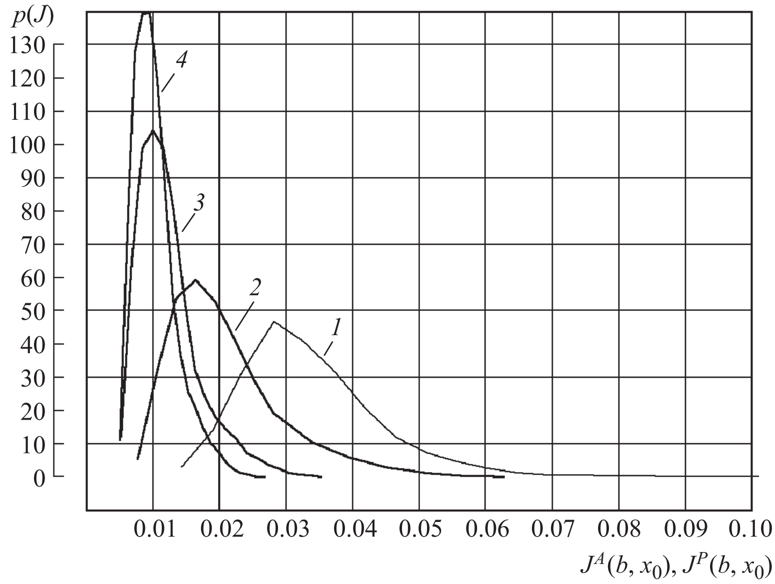

Fig. 3. Polygons of the expected identification error on (*1*) the optimal program test signal and on test controls with C are equal to: (*2*) 0.5, (*3*) 1, (*4*) 2.

where d_i, $i = \overline{1, 100}$ is optimized parameters. Figure 1 shows the trajectory $x(t, b^{\rm pri}, 0, u^{\rm pri})$ of system (13)–(14), corresponding to the optimal program test signal $u^{\rm pri}(t)$ for this task. The components of the optimal program test signal $u^{\rm pri}(t)$ practically coincide with the dependences $\delta_N(t)$, $\delta_e(t)$ shown on the graph. The value of the criterion on the optimal test signal is equal to $\mathrm{tr}\left(M^{-1}(b^{\rm pri}, 0, u^{\rm pri})\right) = 0.0036$.

Next, in accordance with the algorithm of Section 3, we found the optimal values of μ and $L_{i,j}$, $i = 1, 2$, $j = \overline{1, 4}$ in (17). All corner points of cube B were taken as the initial sample of values $b^j \in B$, $x_0^j \in X^0$, $j = \overline{1, 32}$ for $x_0^j = 0$. Finding test controls for each $C = 0.5, 1, 2$ required five to eight iterations of the algorithm. The values of the criterion $\mathrm{tr}\, M^{-1}\left(b^{\rm pri}, 0, u^A(t, x(t))\right)$ on optimal test controls are equal to: 0.0089 at $C = 2$; 0.011 for $C = 1$ and 0.018 for $C = 0.5$.

Figure 2 shows the fields of values of the components of the vector x, calculated on the test control $u^A(t, x(t))$ with $C = 2$ for 60 different pairs b^j, x_0^j from a priori possible ones (i.e. for 60 possible solutions of system (13)–(14)). At $C = 1$ and 0.5, the fields of the components of the vector x differed mainly in the larger width of the "tracks" values. The figure shows that all specified restrictions (16) are satisfied. Numerical verification of the fulfillment of restrictions (16) was carried out for 20 000 different pairs b^j, x_0^j for each value of $C = 0.5, 1, 2$. The optimal value of μ for $C = 2$ was equal to $\mu = 0.75$. We note that on the program test signal $u(t) = \mu u^{\rm pri}(t)$ restrictions (15) would be violated already at $\mu = 0.1$.

At the same time, the limitations and stability of the system were tested (9). In all these cases, all eigenvalues of the matrices $A(b^j) - GL$ had negative real parts.

Figure 3 shows the polygons of expected identification errors $J^A(b, x_0) = \mathrm{tr}\, M^{-1}\left(b, x_0, u^A(t, x(t))\right)$ on optimal test controls in comparison with the polygon of expected identification errors $J^P(b, x_0) = \mathrm{tr}\, M^{-1}\left(b, x_0, u^P(t)\right)$ on the optimal program test signal $u^P(t)$. The program test signal $u^P(t)$ for problem (13)–(16) was found using the method described in [7]. The value of the criterion on the optimal program test signal is $\mathrm{tr}\left(M^{-1}(b^{\rm pri}, 0, u^P(t))\right) = 0.031$.

The expected identification errors $J^P(b, x_0)$ and $J^A(b, x_0)$ were calculated using solutions to the same systems of equations (13)–(14) and (6), differing only in input signals $u = u(t) = u^P(t)$ and $u = u_\Sigma(t) = u^A(t, x(t))$ respectively. The number of points to construct the polygon was $N_P =$ 20 000.

Figure 3 shows that the test control is significantly better than the program test signal. The polygons of expected identification errors on test controls are located to the left of the polygon of expected identification errors on the program test signal in the region of lower values of expected identification errors. The spread of possible values of the expected identification error in test controls is significantly smaller. The right "tails" of the polygons, corresponding to large values of the expected error, are noticeably shorter on the test controls than on the polygon on the program test signal. When $C = 2$ the average value (standard deviation) of the expected identification error on the test control is more than 3.2 (3.2) times less than the expected error on the program test signal, with $C = 1$—more than 2.7 (2.2) times, with $C = 0.5$—more than 1.6 (1.2) times. Out of 20 000 realizations of the values b and x_0, out of a priori possible ones, the share of realizations for which the ratio of expected identification errors on the test signal and test control was more than two was equal to: when $C = 2$—93%, when $C = 1$—78%, when $C = 0.5$—28%.

We note that, within the framework of the comparison, the formulation of the problem of optimizing the program test signal fit to conditions favorable for identification for conducting a test maneuver with an open control loop.

The optimal values of μ and $L_{i,j}$ in the problem under consideration were such that: $\max\limits_{i,j} L_{i,j} = C$; $\mu = 0.64$ for $C = 2$, $\mu = 0.58$ for $C = 1$, $\mu = 0.45$ for $C = 0.5$. We can assume that the optimal (maximum achievable) values of the parameter μ in control (7) are limited by the value of the parameter C in (8). To confirm this assumption, criterion (4) in this problem was replaced by the criterion $J = \mu$, which was maximized over μ and L under the same restrictions (16), (8) and in that class controls (17). The values of μ and $L_{i,j}$, optimal for the criterion $J = \mu$, obtained at $C = 0.5, 1, 2$ practically did not differ from the corresponding previously obtained values. We note that the problem of maximizing μ is significantly simpler than the problem of minimizing the nonlinear criterion (4).

The a priori uncertainty of the initial conditions of the test maneuver significantly affects the effectiveness of the test control. The influence of this uncertainty can be weakened if feedback is introduced gradually at the beginning of the test maneuver (see [20]). In the considered example, this technique leads to a decrease in the average expected identification error on the test control by 4.2 times (at $C = 2$) compared to the error on the program test signal $u^P(t)$.

5. CONCLUSION

The problem of planning an experiment for parametric identification of an object's motion model is considered under restrictions on permissible disturbances of the object's state vector in the experiment and a priori uncertainty regarding the initial conditions of the experiment. Methods are proposed for solving this problem in the class of feedback controls. This ensures tracking of an object's trajectory that satisfies the specified restrictions and has good informativeness about the identified parameters.

The scope of application of the methods proposed in the article is limited to the tasks of planning experiments to clarify the characteristics of automatically controlled objects, in particular the aerodynamic characteristics of automatically controlled aircraft. It should be expected that the effectiveness of the proposed methods in such problems increases with the increase in the uncertainty of the prior estimates of the identified characteristics and the tightening of restrictions on the permissible disturbances of the object's state vector in the experiment.

The control synthesized for active parametric identification in the class of controls with feedback is proposed to be called test control by analogy with test signals selected in the class of program controls.

The results of statistical modeling, carried out with a fifty percent a priori uncertainty regarding the true values of the identified parameters, confirmed that by choosing a test control, the identification error can be significantly reduced compared to the identification error on the optimal program test signal, both on average and "by probability," i.e., for most priori possible trajectories of object movement.

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