ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2023, Vol. 84, No. 7, pp. 810–822.

Period-Time Parametric Identification Method for Solving Location and Navigation Tasks

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Received June 16, 2022
Revised May 4, 2023
Accepted May 21, 2023

Abstract—With regard to location and navigation tasks for single-position passive observer, a bearing-free method for identifying parameters of a polynomial model of object motion has been developed taking into account evolution of the discrepancy between the periodic radiated and received quasi-periodic signal. The passage of a signal in an arbitrary physical environment is considered, at the same time, knowledge of the period of the emitted signal and assessing the current Doppler frequency are not required. The method is based on counting the number of periods of the received signal in a given surveillance intertissue. The issues related to the analysis of the resulting discrepancy by the observability of the method and its accuracy characteristics are considered. Useful practical recommendations and an illustrative context are given.

Keywords: radiating target, periodic signal, quasi-periodic signal, single-position passive observer, bearing-free method, time mistie, period-time method, polynomial motion, parametric identification, observability of the method, complete correlation matrix of estimation errors, adaptation

DOI: 10.25728/arcRAS.2023.75.79.001

1. INTRODUCTION

Methods of passive location and navigation of a radiating target based on a single-position passive observer are widely reflected in the well-known literature [1–20]. Among them, doppler-time bearing-free methods are quite popular, operating with periodic signals and geared towards measurement capability the continuous displacement of the doppler frequency the received signal at the observation point caused by target movement (for location tasks) motion of the observer (for navigational tasks); [6] on pp. 169–173 an exhaustive list of literature on this issue is given, and it is available in the open press. In this case, measurements can be implemented at any characteristic frequency from the spectrum of the emitted signal (for example, on the central) or modulating function; as well as by comparing the moments of the arrival of the fronts of consecutive pulses taking into account the known period. These methods are based on the idea of "base synthesis," which ultimately leads to the formation of several observation points on the guidepath and possibilities of using well-known methods of multi-position location and navigation (for example, triangulation, difference-rangefinder, trilateration and their combinations [21, 22]). In this case, as a rule, such path functions are considered, which are either known at the observation site (for example, orbital ones with known motion parameters), or are approximated with sufficient accuracy for practice by a model of straight-line uniform motion (both with known and unknown motion parameters). At the same time, the fundamental point is accounting of information given a priori about the speed of the target or observer, which is often unacceptable for practice.

In [20] the period-time method is developing (PTM), which removes the restriction, related to obtaining of information given a priori about the speed value, and also the question of parametric identification is considered in relation to the model of nonlinear motion, taking into account the possible maneuver of the target or observer. At the same time, a preliminary current estimate of the Doppler frequency is not required, which is equivalent to finding the derivative of time mistie between the periods of the radiated (periodic) and accepted (quasi-periodic) signal. However, the results obtained in [20], apply only to radio signals (spreading as an electromagnetic wave at the speed of light) with a known period, and the dependence of the resulting time mistie on the parameters of the target movement has not been investigate. This article is a further development of the well-known PTM in terms of eliminating these shortcomings in relation to signals, spreading in arbitrary physical environments.

2. PROBLEM STATEMENT

Let the moving RT form in the current t periodic signal $S_0(t)$ (periodic signal T_S = const may be unknown), spreading in a given physical medium in the form of a wave at a speed of v_S (we can talk about different waves, for example, electromagnetic or acoustic). At the observation point associated with SOPO, at the surveillance intertissue $[0, T]$ a quasi-deterministic signal is received $S(t)$ with a variable period.

According to the PTM, the observation segment is represented as

$$[0, T] = \bigcup_{n=1}^{N} [t_{n-1}, t_n], \quad t_n > t_{n-1}, \quad t_0 = 0, \quad t_N \leq T, \tag{2.1}$$

where $t_0 = 0$ is a fixed moment of time corresponding to the beginning of the received signal (for example, the arrival of the first pulse), t_n is a fixed time of receipt $M_n = \sum_{p=1}^{n} \Delta M_p$ periods of the received quasi-periodic signal (ΔM_p — the number of periods counted on the segment $[t_{p-1}, t_p]$), at the same time, at the moment of time t_n number M_n the whole period fits into the segment $[0, t_n]$.

Theoretical and practical issues related to the calculation of these periods, are solved using electronic digital frequency meters and are described in detail in the well-known technical literature [23, pp. 148–161].

At the observation point (where the SOPO is located) taking into account the movement of the RT, the signal becomes quasi-periodic, because there is a time mistie $\delta(t)$ between the periods of the emitted and received signals

$$\delta(t) = v_S^{-1} \Delta_R(t) = v_S^{-1} [R(t) - R_0], \quad t \in [0, T], \tag{2.2}$$

where $R(t)$ — current range to RT, $R_0 = R(0)$ — initial range.

In a rectangular cartesian reference system XYZ (in the center of which there is SOPO) the motion of the RT is described by a polynomial model (to simplify the calculations and clarity of the method instead of a generalized finite polynomial with arbitrary basis functions, we restrict ourselves to a power polynomial of the second degree with an initial condition $\mathbf{r}_0 = \mathbf{r}(0)$, $\|\mathbf{r}_0\| = R_0$)

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + 2^{-1} \mathbf{a}_0 t^2, \quad t \in [0, T], \tag{2.3}$$

where $\mathbf{r}(t) = \mathbf{r} = [x, y, z]^{\mathrm{T}}$ — position vector ($\|\mathbf{r}(t)\| = R(t)$),

$\mathbf{v}_0 = [v_{x0}, v_{y0}, v_{z0}]^{\mathrm{T}}$ — initial velocity vector ($v_0 = \|\mathbf{v}_0\|$ — speed value),

$\mathbf{a}_0 = [a_{x0}, a_{y0}, a_{z0}]^{\mathrm{T}}$ — acceleration vector ($a_0 = \|\mathbf{a}_0\|$ — acceleration value), while the vectors $\mathbf{r}_0$, $\mathbf{v}_0$ and $\mathbf{a}_0$ are a priori unknown.

If we take the value t_n as the measured parameter, then we can use the following vector equation of observation:

$$\mathbf{h} = \mathbf{t} + \boldsymbol{\xi} = \bar{\mathbf{t}} + \boldsymbol{\delta} + \boldsymbol{\xi}, \tag{2.4}$$

where $\mathbf{h} = \left[h_n, n = \overline{1, N}\right]^{\mathrm{T}}$, $\mathbf{t} = \left[t_n, n = \overline{1, N}\right]^{\mathrm{T}}$, $\bar{\mathbf{t}} = \left[\bar{t}_n, n = \overline{1, N}\right]^{\mathrm{T}}$, $\boldsymbol{\delta} = \left[\delta_n, n = \overline{1, N}\right]^{\mathrm{T}}$, $\boldsymbol{\xi} = \left[\xi_n, n = \overline{1, N}\right]^{\mathrm{T}}$, $h_n = h(t_n)$, $\xi_n = \xi(t_n)$.

In (2.4), $\boldsymbol{\xi} = \left[\xi_n, n = \overline{1, N}\right]^{\mathrm{T}}$ is understood as the Gaussian measurement with zero mathematical expectation and the correlation matrix $\mathbf{K}_{\xi}$, measured parameter t_n connected with number of counted periods by the ratio

$$t_n = M_n T_S + \delta_n = \bar{t}_n + \delta_n = \bar{t}_n + v_S^{-1}\left[R_n - R_0\right], \tag{2.5}$$

where $\delta_n = \delta(t_n)$ is unknown time discrepancy, $\bar{t}_n = M_n T_S$, $R_n = R(t_n)$, $t_0 = 0$.

Formula (2.5) can be commented as follows [6, p. 154]: during the time $\bar{t}_n = M_n T_S$ the distance between the RT and SOPO the range will change by $\Delta R_n = R_n - R_0$, which corresponds to the time mistie $\delta_n = v_S^{-1}\Delta R_n$ between the periods of the emitted and received signals. If the target was stationary or moving in a circle (in the center of which there is SOPO) that range increment would be missing and $\delta_n = 0$ for all n. It is the passage of an additional section of the path length by the wave ΔR_n with speed v_S is the cause of the time mistie δ_n.

Recall that for a known period T_S as a measured parameter, it was possible to take the value $\delta_n = t_n - M_n T_S$ (this is how the observation equation was formed in [6, 20]), for an unknown period T_S only the values t_n and M_n are available for measurement.

If the distance between the RT and the SOPO decreases, then $\delta_n < 0$, otherwise $\delta_n > 0$. The appearance of the time mistie $\delta_n = \delta(t_n)$ is due to the effect of compression or stretching of the initial periodic signal at the observation point due to the movement of the RT.

It is required, taking into account (2.1)–(2.5) to develop a method of parametric identification of RT with a curved (polynomial) movement based on a period-a temporary SOPO that does not require knowledge of the period T_S the emitted signal and the calculation of the current Doppler frequency. The method should include solving the following issues:

— obtaining dependencies that allow us to assess the nature of the evolution of the received signal period (caused by the movement of RT), is fundamental for this method;

— formation of an algorithm for identification of the inclined range and a number of characteristic parameters of the RT movement based on accurate data (taking $\boldsymbol{\xi}_n = 0$, $n = \overline{1, N}$);

— determination of the conditions for the correct application of the method on accurate data (i.e., determination of the observability conditions of the method);

— accounting for random measurement errors;

— solving the identification problem on redundant data ($\mathbf{h}$) taking into account measurement noise (smoothing problem based on the least squares method (PTM)) and obtaining a ratio for calculating the correlation matrix of identification errors;

— conducting a computational experiment to demonstrate the capabilities of the method.

3. INVESTIGATION OF THE EVOLUTION OF THE SIGNAL PERIOD

The foray $\delta(t)$ is described by the expression (for the case of rectilinear uniform motion)

$$\delta(t) = v_S^{-1}\left\{\left[R_0^2 + 2tR_0 v_0 \cos\gamma_0 + t^2 v_0^2\right]^{1/2} - R_0\right\}, \quad t \geq 0, \quad \delta(0) = 0, \tag{3.1}$$

where γ_0 — angle between vectors $\mathbf{r}_0$ and $\mathbf{v}_0$.

For $0 < \gamma_0 \le \pi/2$ the function $\delta(t)$ is non-negative, smooth and strictly convex, $\delta^{(1)}(t) = d\delta(t)/dt = 0$ at the point $t = 0$. Ror $\pi/2 < \gamma_0 < \pi$ the function $\delta(t)$ is smooth and strictly convex, has two roots ($t = 0$ and $t = -2R_0 \cos\gamma_0/v_0$), at the point $t = -R_0 \cos\gamma_0/v_0$ reaches the minimum value $(v_S^{-1} R_0 (\sin\gamma_0 - 1))$. For $\gamma_0 = 0$ we have $\delta(t) = (v_0/v_S)\, t$, the raid is a linear non-negative function independent of R_0. For $\gamma_0 = \pi$ we have $\delta(t) = -(v_0/v_S)\, t$ for $0 \le t \le R_0/v_0$, so $\delta(t)$ is a linear function and reaches its minimum $(-R_0/v_S)$ at the point $t = R_0/v_0$. Since for $\gamma_0 = 0$ and $\gamma_0 = \pi$ the foray $\delta(t)$ does not depend on R_0, then, for these incorrect cases associated with the movement of the RT along the line of sight, it is impossible to determine the range taking into account the evolution of the signal period at the observation point.

For a more detailed study of $\delta(t)$ we will find the first few occurrences in time (at the point $t = 0$):

$$\begin{cases} \delta_0^{(1)} = v_S^{-1} v_R, \\ \delta_0^{(2)} = (v_S R_0)^{-1}\, v_\tau^2, \\ \delta_0^{(3)} = -3\,(v_S R_0^2)^{-1}\, v_\tau^2 v_R, \end{cases} \tag{3.2}$$

where $v_R = R_0 \cos\gamma_0$ and $v_\tau = v_0 \sin\gamma_0$ — respectively, the values of the radial and tangential velocity.

As a result, you can use the decomposition based on the Taylor series

$$\delta(t) = v_S^{-1} t \left(v_R + \frac{v_\tau^2 t}{2R_0} - \frac{v_\tau^2 v_R t^2}{2R_0^2} + \ldots \right) = v_S^{-1} t \left[v_R + \frac{v_\tau^2 t}{2R_0} \left(1 - \frac{v_R t}{R_0} \right) + \ldots \right], \tag{3.3}$$

from which it follows that the spectral composition of the function $\delta(t)$ significantly depends on the observation conditions, and in many practically important cases it is not possible to neglect derivatives of the second and higher orders, especially for long observation intervals and short ranges.

Formulas (3.1)–(3.3) are very useful in substantiating the possibility of practical implementation of the developed PTM in each specific case, taking into account the accepted initial data.

4. BUILDING A PARAMETRIC IDENTIFICATION ALGORITHM BASED ON ACCURATE DATA

Taking into account (2.3) we can use the following dependency

$$R^2(t) - R_0^2 = 2t \langle \mathbf{r}_0, \mathbf{v}_0 \rangle + t^2 \left(v_0^2 + \langle \mathbf{r}_0, \mathbf{a}_0 \rangle \right) + t^3 \langle \mathbf{v}_0, \mathbf{a}_0 \rangle + 4^{-1} t^4 a_0^2, \tag{4.1}$$

where $\langle \cdot, \cdot \rangle$ is the symbol of the scalar product of two vectors, $\|\cdot\|$ is the symbol of the vector norm.

Formula (4.1) represents the first basic ratio of the developed PTM.

The second basic relation follows directly from the formula (2.2):

$$R^2(t) - R_0^2 = 2 v_S R_0 \delta(t) + v_S^2 \delta^2(t). \tag{4.2}$$

Equating expressions (4.1) and (4.2), after simple transformations we obtain the equation

$$-2 v_S \delta(t) \chi_1 + 2t\chi_2 + t^2 \chi_3 + t^3 \chi_4 + 4^{-1} t^4 \chi_5 = v_S^2 \delta^2(t), \tag{4.3}$$

where

$$\begin{cases} \chi_1 = R_0, \\ \chi_2 = \langle \mathbf{r}_0, \mathbf{v}_0 \rangle, \\ \chi_3 = \left(v_0^2 + \langle \mathbf{r}_0, \mathbf{a}_0 \rangle\right), \\ \chi_4 = \langle \mathbf{v}_0, \mathbf{a}_0 \rangle, \\ \chi_5 = a_0^2 \end{cases} \tag{4.4}$$

— unknown fraction that have a clear physical meaning and are subject to identification.

Since the values of δ_n are unknown, then, taking into account (2.5) for discrete time, we write down the equation with respect to unknown quantities T_S and χ_i, $i = \overline{1,5}$:

$$-2v_S\left(t_n - M_nT_S\right)\chi_1 + 2t_n\chi_2 + t_n^2\chi_3 + t_n^3\chi_4 + 4^{-1}t_n^4\chi_5 = v_S^2\left[\left(t_n - M_nT_S\right)\right]^2. \tag{4.5}$$

After simple but cumbersome transformations, formula (4.5) can be represented as a new equation (relative to the coefficients A_i)

$$\sum_{i=1}^{6} B_{in}A_i = D_n, \tag{4.6}$$

where

$$\begin{cases} A_1 = (v_S\chi_1 - \chi_2)v_S^{-2}T_S^{-1}, \quad A_2 = -\chi_1 v_S^{-1}, \\ A_3 = \left(v_S^2 - \chi_3\right)\left(2v_S^2T_S\right)^{-1}, \quad A_4 = 2^{-1}T_S, \\ A_5 = -\chi_4\left(2T_Sv_S^2\right)^{-1}, \quad A_6 = \chi_5\left(8T_Sv_S^2\right)^{-1}, \\ B_{1n} = t_n, \quad B_{2n} = M_n, \quad B_{3n} = t_n^2, \\ B_{4n} = M_n^2, \quad B_{5n} = t_n^3, \quad B_{6n} = t_n^4, \\ D_n = M_nt_n. \end{cases} \tag{4.7}$$

The relations (4.6) and (4.7) are the basis for identifying the parameters of the curvilinear motion of the RT at an unknown period of the emitted signal. In (4.6) the unknown coefficients are A_i, $i = \overline{1,6}$, those that are uniquely related to the desired parameters of the motion of the RT and the period of the emitted signal. If Eq. (4.6) $n = \overline{1,N}$, where $N \geq 6$, then we get a system of linear algebraic equations (SLAE) (with a rectangular matrix $\mathbf{B}$)

$$\mathbf{BA} = \mathbf{D}, \tag{4.8}$$

where $\mathbf{B} = \left[B_{in}, n = \overline{1,N}, i = \overline{1,6}\right]$, $\mathbf{A} = \left[a_i, i = \overline{1,6}\right]^{\mathrm{T}}$, $\mathbf{D} = \left[D_n, n = \overline{1,N}\right]^{\mathrm{T}}$.

This SLAE allows us to solve the problem of estimating these coefficients and parameters, as well as the signal period for redundant measurements. For $N > 6$ we are talking about the problem of smoothing based on OLS using orthogonal-singular decomposition [24].

Consider a special case when the RT moves rectilinearly and uniformly, and the signal period is unknown. Now instead of (4.3) we have the equation

$$-2v_S\delta(t)\chi_1 + 2t\chi_2 + t^2\chi_3 = v_S^2\delta^2(t), \tag{4.9}$$

where

$$\begin{cases} \chi_1 = R_0, \\ \chi_2 = \langle \mathbf{r}_0, \mathbf{v}_0 \rangle, \\ \chi_3 = v_0^2. \end{cases} \tag{4.10}$$

In this case, instead of (4.6) we have

$$t_n A_1 + M_n A_2 + t_n^2 A_3 + M_n^2 A_4 = M_n t_n. \tag{4.11}$$

If we assume that the signal period is known, i.e., the values are known δ_n, then, taking into account (4.9) to find the parameters of a rectilinear uniform motion of the RT, it is sufficient to solve the SLAE (regarding $\chi_i,\ i = \overline{1,3}$)

$$-2v_S\delta_n\chi_1 + 2t_n\chi_2 + t^2\chi_3 = v_S^2\delta_n^2, \quad n = \overline{1,N}. \tag{4.12}$$

At the same time, we find the range R_0, the speed value $v_0 = \|\mathbf{v}_0\|$ and the angle γ_0 between the vectors $\mathbf{r}_0$ and $\mathbf{v}_0$ taking into account the obvious relations:

$$\begin{cases} R_0 = \chi_1, \\ v_0 = \sqrt{\chi_3}, \\ \gamma_0 = \arccos\left[\chi_2 \left(R_0 v_0\right)^{-1}\right]. \end{cases} \tag{4.13}$$

In the case of rectilinear equidistant motion of the RT (when the vectors $\mathbf{v}_0$ and $\mathbf{a}_0$ are collinear) it is necessary to solve the SLAE (regarding χ_i, $i = \overline{1,5}$)

$$-2v_S\delta_n\chi_1 + 2t_n\chi_2 + t_n^2\chi_3 + t_n^3\chi_4 + 4^{-1}t_n^4\chi_5 = v_S^2\delta_n^2. \tag{4.14}$$

Now we have

$$\begin{cases} \chi_1 = R_0, \\ \chi_2 = R_0 v_0 \cos\gamma_0, \\ \chi_3 = \left(v_0^2 + R_0 a_0 \cos\gamma_0\right), \\ \chi_4 = v_0 a_0, \\ \chi_5 = a_0^2. \end{cases} \tag{4.15}$$

Based on the found values $\chi_1, \ldots, \chi_5$ we calculate the following parameters of the movement of the RT:

$$\begin{cases} R_0 = \chi_1, \\ a_0 = \sqrt{\chi_5}, \\ v_0 = \chi_4 a_0^{-1}, \\ \gamma_0 = \arccos\left[\chi_2 \left(R_0 v_0\right)^{-1}\right]. \end{cases} \tag{4.16}$$

Expressions (4.1)–(4.16) form the mathematical basis of the developed PTM.

In the next section we will analyze the observability conditions of the developed method, i.e., we will identify situations in which it becomes incorrect from a computational point of view.

5. ANALYSIS OF THE OBSERVABILITY OF THE METHOD

The developed PTM can be implemented on any set of nodes from the set $\{t_1, \ldots, t_N\}$, which allows not only to reduce the amount of calculations, but also in some cases to increase the reliability of the generated estimates (especially in the absence of reliable a priori information about the weighting factors necessary for the implementation of LSM). To do this, we introduce vectors of temporary nodes $\mathbf{t}_{[l]} = \left[t_{[l]p}, p = \overline{1, P_{[l]}}\right]^{\mathrm{T}}$, where $l = \overline{1, L}$, $t_{[l]p} \in \{t_1, \ldots, t_N\}$, $t_{[l]p+1} > t_{[l]p}$. Here L is the number of sets, $P_{[l]}$ — the number of nodes in the l set, $t_{[l]p}$ is the node with number $[l]\,p$ (this is a natural number belonging to the set $\{1,\ldots,N\}$). Based on (4.12) we will form the following SLAE:

$$\mathbf{C}_{[l]}\boldsymbol{\chi}_{[l]} = \mathbf{Y}_{[l]}, \tag{5.1}$$

where $\mathbf{Y}_{[l]} = \left[\delta^2_{[l]p}, p=\overline{1,P_{[l]}}\right]^{\mathrm{T}}$, $\boldsymbol{\chi}_{[l]} = \left[\chi_{i[l]}, i=\overline{1,5}\right]^{T}$, and the matrix $\mathbf{C}_{[l]}$ (size $P_{[l]}\times 5$) is formed by strings $v_S^{-2}\left(-2v_S\delta_{[l]p},\, 2t_{[l]p},\, t^2_{[l]p},\, t^3_{[l]p},\, 4^{-1}t^4_{[l]p}\right)$, $p=\overline{1,P_{[l]}}$.

The introduction of $\mathbf{t}_{[l]}$ makes it possible to find such sets of nodes taking into account the observation geometry, the characteristics of the RT and the SOPN, to find such sets of nodes on which the identification issue is solved most qualitatively (this refers to the well-known problem of experiment planning [25]).

Without reducing the generality of reasoning, we will limit ourselves to the flat case (assuming $z=0$) and a signal with a known period, and also, we will ask $P_{[l]}=5$, what corresponds to a square matrix $\mathbf{C}_{[l]}$. It is obvious that for the correct application of the developed method, related to the SLAE solution (5.1), it is necessary and sufficient to fulfill the condition $\det \mathbf{C}_{[l]} \neq 0$, what leads to the desired result $\boldsymbol{\chi}_{[l]} = \mathbf{C}^{-1}_{[l]}\mathbf{Y}_{[l]}$. To identify cases in which this condition is violated, we write down the columns of the matrix $\mathbf{C}_{[l]}$ in the form of vectors:

$$\mathbf{C}_{[l]1} = \left[-2v_S\delta_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}}, \quad \mathbf{C}_{[l]2} = \left[2t_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}},$$
$$\mathbf{C}_{[l]3} = \left[t^2_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}}, \quad \mathbf{C}_{[l]4} = \left[t^3_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}}, \quad \mathbf{C}_{[l]5} = \left[4^{-1}t^4_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}}.$$

It is light to notice that the columns $\mathbf{C}_{[l]2}$, $\mathbf{C}_{[l]3}$ and $\mathbf{C}_{[l]4}$ are linearly independent, therefore, to check the condition $\det \mathbf{C}_{[l]} \neq 0$ it is enough to show that the column $\mathbf{C}_{[l]1}$ cannot be represented as a linear combination of these columns.

Since $R_{[l]p} = \left[x^2_{[l]p} + y^2_{[l]p}\right]^{-2}$ $\left(\text{where } R_{[l]p} = R\left(t_{[l]p}\right),\ x^2_{[l]p} = \left(x_0 + v_{x0}t_{[l]p} + 2^{-1}a_{x0}t^2_{[l]p}\right)^2 \text{ and } y^2_{[l]p} = \left(y_0 + v_{y\,0}t_{[l]p} + 2^{-1}a_{y0}t^2_{[l]p}\right)^2\right)$, that violation of the condition $\det \mathbf{C}_{[l]} \neq 0$ it is equivalent to the fact that the vectors $\boldsymbol{\mu}_{[l]} = \left[x^2_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}}$ and $\boldsymbol{\eta}_{[l]} = \left[y^2_{[l]p}, p=\overline{1,5}\right]^{\mathrm{T}}$ are not bound by the collinearity condition: $\boldsymbol{\mu}_{[l]} = k\boldsymbol{\eta}_{[l]}$, where k — where is the proportionality coefficient. Otherwise we have

$$R_{[l]p} = \left[x^2_{[l]p} + y^2_{[l]p}\right]^{-2} = \left[k^2y^2_{[l]p} + y^2_{[l]p}\right]^{-2} = q\left|y_{[l]p}\right|, \tag{5.2}$$

$$-2v_S\delta_{[l]p} = -2\left[R_{[l]p} - R_0\right] = -2\left[q\left|y_{[l]p}\right| - R_0\right], \tag{5.3}$$

where $q = \left(k^2+1\right)^{-2}$.

It follows from (5.2) and (5.3) that the coordinates of vector $\mathbf{C}_{[l]1}$ can be represented by a linear combination of the coordinates of vectors $\mathbf{C}_{[l]2}$, $\mathbf{C}_{[l]3}$ and $\mathbf{C}_{[l]4}$. The physical meaning of the condition $\boldsymbol{\mu}_{[l]} = k\boldsymbol{\eta}_{[l]}$ (the condition of computational incorrectness of the method) is that the RT moves rectilinearly along the line of sight SOPN.

Thus, for the correctness of the method, it is necessary to exclude cases when the RT moves along the specified line or in its vicinity. This imposes certain restrictions on the conditions for monitoring RT, which must be provided for in practice.

If we limit ourselves to the model of rectilinear uniform motion and a signal with a known period in this case, $\left(\text{in (5.1) we must put } p=\overline{1,3} \text{ and } \mathbf{t}_{[l]} = \left[t_{[l]1}, t_{[l]2}, t_{[l]3}\right]^{\mathrm{T}}\right)$, the solution of SLAE (5.1) with the correct application of the method, allows us to determine the desired parameters of the motion of the RT

$$\begin{cases} R_{0[l]} = 2^{-1} v_S \left(\dfrac{\delta^2_{[l]1} \Delta^t_{[l]23} - \delta^2_{[l]2} \Delta^t_{[l]13} + \delta^2_{[l]3} \Delta^t_{[l]12}}{-\delta_{[l]1} \Delta^t_{[l]23} + \delta_{[l]2} \Delta^t_{[l]13} - \delta_{[l]3} \Delta^t_{[l]12}} \right), \\ \langle \mathbf{r}_0, \mathbf{v}_0 \rangle_{[l]} = 2^{-1} v_S^2 \left(\dfrac{t^2_{[l]1} \Delta^\delta_{[l]23} - t^2_{[l]2} \Delta^\delta_{[l]13} + t^2_{[l]3} \Delta^\delta_{[l]12}}{-\delta_{[l]1} \Delta^t_{[l]23} + \delta_{[l]2} \Delta^t_{[l]13} - \delta_{[l]3} \Delta^t_{[l]12}} \right), \\ v_{0[l]} = \left[\dfrac{t_{[l]3} \Delta^\delta_{[l]12} - t_{[l]2} \Delta^\delta_{[l]13} + t_{[l]1} \Delta^\delta_{[l]23}}{\delta_{[l]1} \Delta^t_{[l]23} - \delta_{[l]2} \Delta^t_{[l]13} + \delta_{[l]3} \Delta^t_{[l]12}} \right]^{1/2}, \\ \gamma_{0[l]} = \arccos \left[\dfrac{\langle \mathbf{r}_0, \mathbf{v}_0 \rangle_{[l]}}{R_{0[l]} v_{0[l]}} \right], \end{cases} \tag{5.4}$$

where $\Delta^t_{[l]12} = t_{[l]1} t_{[l]2} \left(t_{[l]1} - t_{[l]2} \right)$, $\Delta^\delta_{[l]12} = \delta_{[l]1} \delta_{[l]2} \left(\delta_{[l]1} - \delta_{[l]2} \right)$ and, if you do not take into account measurement and calculation errors, $R_{0[l]} = R_0$, $v_{0[l]} = v_0$, $\langle \mathbf{r}_0, \mathbf{v}_0 \rangle_{[l]} = \langle \mathbf{r}_0, \mathbf{v}_0 \rangle$, $\gamma_{0[l]} = \gamma_0$.

Therefore, it becomes possible to determine the motion parameters R_0, v_0 and γ_0 $\Big($where $R_0 = \chi_1$, $v_0 = \sqrt{\chi_3}$, $\gamma_0 = \arccos \left[\chi_2 \left(R_0 v_0 \right)^{-1} \right] \Big)$, without resorting to the numerical solution of SLAE, which is an undoubted advantage of the developed PTM.

6. ACCOUNTING FOR RANDOM MEASUREMENT ERRORS

Assuming the signal period is known, we use the traditional procedure for calculating the elements of the correlation matrix to assess the effect of random measurement errors on the accuracy characteristics of the method $\mathbf{K}_{\chi[l]}$ errors in estimating the coordinates of the vector $\boldsymbol{\chi}$ in linear approximation [26]. To do this, taking into account SLAE (5.1) (assuming for simplicity the matrix $\mathbf{C}_{[l]}$ square size 5×5) let's use the representation $\boldsymbol{\chi}_{[l]} = \mathbf{C}^{-1}_{[l]} \mathbf{Y}_{[l]} = \left[\chi_k \left(\boldsymbol{\delta}_{[l]} \right), k = \overline{1,5} \right]^{\mathrm{T}}$ (where $\boldsymbol{\delta}_{[l]} = \left[\delta_{[l]p}, p = \overline{1,5} \right]^{\mathrm{T}}$) and partial derivatives of the following form: $\partial \chi_{k[l]} \left(\boldsymbol{\delta}_{[l]} \right) / \partial \delta_{[l]p}$. The correlation matrix is found by the rule

$$\mathbf{K}_{\chi[l]} = \mathbf{F}_{\chi[l]} \mathbf{K}_\xi \mathbf{F}^{\mathrm{T}}_{\chi[l]}, \tag{6.1}$$

where $\mathbf{F}_{\chi[l]} = \left[\partial \chi_{k[l]} \left(\boldsymbol{\delta}_{[l]} \right) / \partial \delta_{[l]p},\ k = \overline{1,5},\ p = \overline{1,5} \right]$.

Expression (6.1) allows a priori, based on the mathematical expectations of the measured parameters, to assess the potential capabilities of the developed PTM and develop practical recommendations for its best use under specific conditions of observation of RT, and also reasonably approach the choice of the main parameters of the method (the length of the observation interval (T), the number of nodes (N) and time sets ($\mathbf{t}_{[l]}$)). So, the number $l^* \in \{1, \ldots, L\}$ of optimal set $\boldsymbol{\delta}_{[l^*]}$, ensuring the minimization of the estimation error, is found according to the following adaptive rule:

$$l^* = \arg \min_l \left\| \mathbf{K}_{\chi[l]} \right\|, \tag{6.2}$$

where $\left\| \mathbf{K}_{\chi[l]} \right\|$ — this is any of the norms of the matrix $\mathbf{K}_{\chi[l]}$, used in evaluation tasks.

In the practical implementation of the developed PTM, the factor should be taken into account that for large values of v_S (for example, when $v_S = c$, where c is the speed of light), the solution of the square SLA (4.8) in the presence of random measurement errors can lead to incorrect results. Let us explain this fact for the case $N = 6$ by the example of calculating the velocity v_0. Because $v_0 = c\sqrt{1 - 2T_S A_3}$, that's a mistake $\Delta_3 = \hat{A}_3 - A_3$ (where $\hat{A}_3$ — calculated coefficient value A_3 by solving SLAE (4.8) taking into account measurement errors) leads to the following speed estimate: $\hat{v}_0 = \sqrt{v_0^2 + 2c^2 T_S \Delta_3}$. That is, a correct assessment of the speed is possible only if the condition is met $\Delta_3 > -v_0^2 \left(2c^2 T_S\right)^{-1}$, which imposes a very strict restriction on the magnitude of the error Δ_3. This effect also applies to all SLOUGH coefficients (4.8), except A_2 and A_4.

To overcome this incorrectness (at high v_S speeds), a two-step approach to identification is recommended. At the first stage, SLAE is solved (4.8), of which only an assessment will be required $\hat{A}_4$ for A_4. This allows you to form the desired estimate $\hat{T}_S = 2\hat{A}_4$ for period T_S, and based on it, estimates for residuals $\hat{\delta}_n = t_n - M_n \hat{T}_S$. All estimates of the parameters of the RT movement are based on the SLAE (5.1), in which the value of δ_n is substituted instead of $\hat{\delta}_n$.

7. ACCOUNTING FOR REDUNDANT MEASUREMENTS

Now consider the case of redundant measurements when the matrix $\mathbf{C}_{[l]}$ and the vector $\mathbf{Y}_{[l]}$ in (5.1) have an arbitrary number of lines $P_{[l]} \leq N$, which, as a rule, significantly exceeds the number of estimated parameters. To simplify the calculations, we will consider in SLAU (5.1) the component. $\mathbf{Y}_{[l]} = \left[v_S^2 \delta_{[l]p}^2, p = \overline{1, P_{[l]}}\right]^{\mathrm{T}}$ as a vector of secondary measured parameters $h_{[l]1}, \ldots, h_{[l]P_{[l]}}$ and primary measurements (5.1) the correlation matrix of measurement errors of the coordinates of the vector $\mathbf{Y}_{[l]}$ we can imagine it like this

$$\mathbf{K}_{\mathbf{Y}[l]} = \mathbf{F}_{\delta[l]} \mathbf{K}_{\xi} \mathbf{F}_{\delta[l]}^{\mathrm{T}}. \tag{7.1}$$

Assuming that the matrix $\mathbf{K}_{\xi}$ is diagonal, we have $\mathbf{K}_{Y[l]} = \operatorname{diag}\left[4\delta_{[l]1}^2, 4\delta_{[l]2}^2, \ldots, 4\delta_{[l]P_{[l]}}^2\right]$. Under the condition of sufficiently small measurement errors, the least squares method can be used to construct a smoothed estimate of the vector χ [25]

$$\chi_{[l]}^* = \left(\mathbf{C}_{[l]}^{\mathrm{T}} \mathbf{K}_{Y[l]}^{-1} \mathbf{C}_{[l]}\right)^{-1} \mathbf{C}_{[l]}^{\mathrm{T}} \mathbf{K}_{\mathbf{Y}[l]}^{-1} \mathbf{h}_{\mathbf{Y}[l]}, \tag{7.2}$$

where $\mathbf{h}_{\mathbf{Y}[l]} = \left[h_{\mathbf{Y}[l]p}, p = \overline{1, P_{[l]}}\right]^{\mathrm{T}}$ — the vector of secondary measurements.

The correlation matrix of estimation errors is found as follows:

$$\mathbf{K}_{\chi_{[l]}^*} = \left(\mathbf{C}_{[l]}^{\mathrm{T}} \mathbf{K}_{\mathbf{Y}[l]}^{-1} \mathbf{C}_{[l]}\right)^{-1}. \tag{7.3}$$

To select the optimal set with a number $l^* \in \{1, \ldots, L\}$ we use an adaptive algorithm of type (6.2).

It should be noted that the approach (7.1)–(7.3) is not strictly optimal, since the elements of the matrix $\mathbf{C}_{[l]}$ depend on the results of observations. But with certain limitations on measurement errors, it gives a completely acceptable result.

For more accurate smoothing, well-known nonlinear optimal estimation procedures can be used, which in practice lead to time-consuming recurrent computational algorithms involving the setting of a sufficiently high-quality initial condition.

Another simplest and fairly reliable way to construct a smooth estimate of $\chi_{[l]}^*$ is to pre-smooth the primary measurements $h_{[l]1}, \ldots, h_{[l]P_{[l]}}$ by the corresponding polynomial $\delta_{[l]}^*(t)$ and the application of the results obtained to the solution of SLAE (5.1). In addition, you can find a smoothed range estimate for any $t \in [0, T]$, exactly,

$$R_{[l]}(t) = R_{0[l]}^* + c\delta_{[l]}^*(t). \tag{7.4}$$

Here we take a set with a number as the optimal one $l = l^* \in \{1, \ldots, L\}$.

8. SOME GENERALIZATIONS AND PRACTICAL RECOMMENDATIONS

The case of estimating the initial range was considered above $R_0 = R(0)$ for time $t = 0$. However, if the Taylor series used to describe the curvilinear motion of the RT is written with respect not to the initial, but to any arbitrary $t = t_* \in [0, T]$, then, by analogy with the above, it is possible to solve the identification problem precisely for the moment of time t^*, in particular, to find the range $R_* = R(t_*)$.

The developed method is easy to implement in the form of the following algorithms: by sampling an increasing volume, on a "sliding grid" or in the form of a filter [25]. At the same time, the movement of RT in the observation interval can be considered as piecewise polynomial (in [20] it was considered as piecewise linear).

During the practical implementation of the method, questions arise (for example, the choice of the degree of the polynomial describing the motion of the RT or the number of counted pulses) related to the organization of the measuring experiment. [25] provides practical recommendations for solving these issues in full. It is obvious that the developed method is most effective when it comes to large distances traveled (i.e., a base of sufficient size is "synthesized"), and this sets certain restrictions on the type of RT (in particular, on his speed, maneuverability, etc.), on the adequacy of the polynomial used at a given observation interval and on the technical characteristics of the SOPN.

For cases related to the movement of RT along the line of sight, a hybrid variant of using the developed and well-known energy method can be proposed [27]. It is proved that this method, operating with the relative level of the received signal, implements its potential capabilities when moving RT along the line of sight. In a sense, the developed and energetic methods are "orthogonal" to each other in terms of accuracy. Therefore, by combining these methods, it is possible to align the working area of the hybrid method and achieve acceptable accuracy characteristics for various conditions of observation of the RT.

For a more effective application of the energy method, clustering and majority processing procedures should be used to reduce and eliminate unreliable measurements.

9. ILLUSTRATIVE EXAMPLE

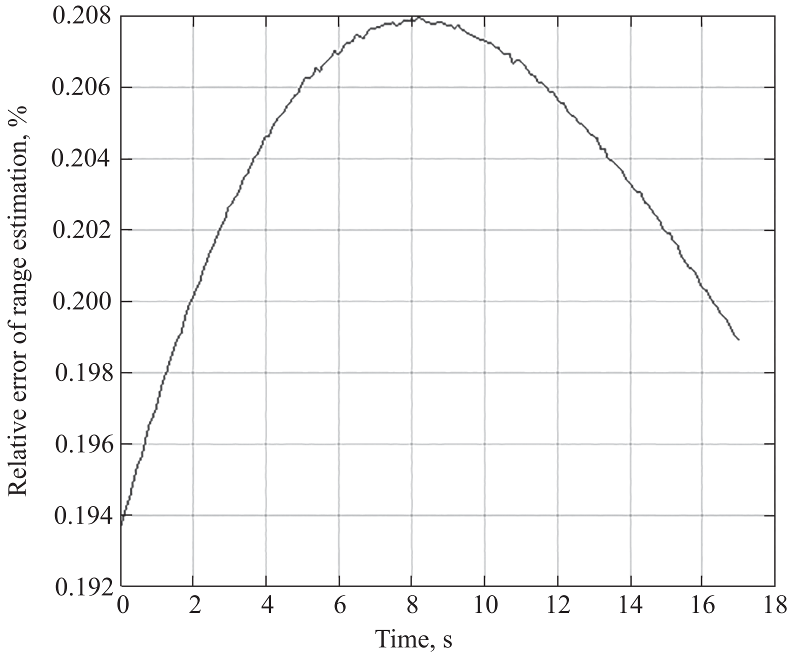
Suppose that the RT carries out a planar movement $x(t) = x_0 + v_{x0}t,\ y(t) = y_0 + v_{y0}t$, where $x_0 = y_0 = 11 \times 10^3$, $v_{x0} = -5 \times 10^2$, $v_{y0} = 6 \times 10^2$, $\gamma_0 = 85$. Here and further, the time and measurement errors of time intervals are set in seconds (s), coordinates and range — in meters (m), speed — in m/s, acceleration — in m/s^2, frequency — in hertz (Hz), angle — in degrees, relative error — as a percentage.

The RT generates a pulsed radio signal

$$S_0(t) = \sum_{k=1}^{K} \operatorname{rect}\left[(t - kT_S)\,\tau^{-1}\right] \cos(2\pi f_0 t),$$

where $T_S = 10^{-2},\ \tau = 10^{-5},\ f_0 = 10^{10}$. Parameters of the SOPN operation: $T = 18$, $v_S = c = 3 \times 10^8$, $L = 1$ (that is, one single set of nodes is used), $P_{[1]} = 4$ (set size), $\Delta M_p = \Delta M = 10$, $\mathbf{K}_\xi = \operatorname{diag}\left[\sigma^2, \ldots, \sigma^2\right]$, at the same time, the measurement errors of the time position of the pulse fronts were assumed to be uncorrelated and were set according to the normal distribution law with zero mathematical expectation and the value of the standard deviation $\sigma = 10^{-9}$.

The method was implemented in two stages using a random number sensor and averaging over a thousand experiments. At the first stage, SLAE (4.8) was solved with a square matrix $\mathbf{B}$ of size 4×4



Relative error of range estimation.

(since the RT with zero acceleration is considered), at the same time, a vector is used to calculate the elements of matrix $\mathbf{B}$ and column $\mathbf{D}$ $\mathbf{t}_{[1]} = \bar{\mathbf{t}}_{[1]} + \boldsymbol{\delta}_{[1]} = \left[t_{[1]p}, p = \overline{1,4}\right]^{\mathrm{T}}$ with node numbers: $[1]1 = 12$, $[1]2 = 65$, $[1]3 = 118$, $[1]4 = 171$, $\bar{\mathbf{t}}_{[1]} = \left[\bar{\mathbf{t}}_{[1]p}, p = \overline{1,4}\right]^{\mathrm{T}} = \left[[1]p \times 10^{-1}, p = \overline{1,4}\right]^{\mathrm{T}}$. From all four estimates of unknown coefficients, only the estimate of the signal period is selected $\hat{T}_{S[1]} = 9.999999731646 \times 10^{-3}$ (obtained based on the set $\mathbf{t}_{[1]} = \bar{\mathbf{t}}_{[1]} + \boldsymbol{\delta}_{[1]}$), which corresponds to the relative error $\delta T_{S[1]} = 2.683540941544882 \times 10^{-6}$.

At the second stage, taking into account $\hat{\delta}_{[1]p} = t_{[1]p} - M_{[1]p}\hat{T}_{S[1]} = t_{[1]p} - [1]\,p\Delta M\hat{T}_{S[1]}$ The SLAE (5.1) was solved with a square matrix $\mathbf{C}_{[l]}$ (3×3 in size), with $\chi_4 = \chi_5 = 0$ and a set was used $\bar{\mathbf{t}}_{[1]} = \left[\bar{t}_{[1]p}, p = \overline{1,3}\right]^{\mathrm{T}} = [1.1;\, 9.1;\, 17.1]^{\mathrm{T}}$. The matrix itself is formed by strings $c^{-2}\left(-2c\hat{\delta}_{[1]p},\, 2t_{[1]p},\, t^2_{[1]p}\right)$, $p = \overline{1,3}$. As a result of the true range $R_0 = 1.555634918 \times 10^4$ match rating $\hat{R}_{0[1]} = 1.559672203 \times 10^4$ measure of inaccuracy $\Delta R_{0[1]} = 0.259526489$, true speed $v_0 = 7.810249675 \times 10^2$ — assessed value $v_{0[1]} = 7.821417156 \times 10^2$ measure of inaccuracy $\Delta v_{0[1]} = 0.142984942$, true angle $\gamma_0 = 84.805571092$ — assessed value $\gamma_{0[1]} = 84.761511501$ measure of inaccuracy $\Delta\gamma_{0[1]} = 0.051953650$.

The figure shows a graph of the dependence of the relative error of the range estimation, obtained taking into account (7.4).

For more effective use of the method developed in the article, the question of choosing the size of the observation interval and the nodes of the time grid is, as well as their coordination with the dynamics of the RT movement and the magnitude of measurement errors should be solved in the optimization formulation. When solving SLAE, well-known regularization methods should be used. The results of the numerical experiment show that the greater the distance between the nodes of the time grid used, the less influence random measurement errors have on the resulting estimation accuracy. This distance must be consistent with the dynamics of the RT, namely: the lower the speed of movement of the RT, the greater the step of this grid and the duration of the observation interval should be.

10. CONCLUSION

The developed PTM makes it possible to identify a model of curvilinear polynomial motion of the RT based on the results of recording the time discrepancy between the periods of the emitted signal and the same periods, calculated at the observation point. The method does not require knowledge of the signal period and a preliminary estimate of the current Doppler frequency, as well as knowledge of any a priori data on the parameters of the accepted motion model of the RT. The observability and the main limitations of the method, the conditions for its most effective application are investigated. Analytical relations are obtained that allow us to estimate the evolution of the time discrepancy taking into account the characteristics of the RT and the SOPO, as well as the accuracy characteristics of the method for various observation conditions.

The method can be implemented in various ways: by a fixed sample of measurements, by a sample of measurements of increasing volume, in the form of a dynamic filtration algorithm (linear, quasi-linear or nonlinear), etc.

The method can be implemented either independently or as part of a hybrid method, combining other well-known approaches of passive single-position and multi-position location and navigation of RT. Since the developed method allows you to determine the range, it can be used in rangefinder-rangefinder systems of multi-position location when solving the well-known trilateration problem [21, 22].

If there are not only fluctuation errors in the period-time measurements, but also singular errors, it is advisable to initially subject these measurements to the procedure of generalized invariant-unbiased estimation [28], compensation for these errors, achieving the smoothing effect and optimal estimation of various numerical characteristics (linear functionals, e.g., derivatives, integrals, spectral coefficients, etc.), useful not only for improving the computational stability of the method, but also for evaluating its effectiveness. To solve the SLAE using the regularization procedure, a well-known approach can be applied [29].

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This paper was recommended for publication by O.N. Granichin, a member of the Editorial Board