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STOCHASTIC SYSTEMS

Analytical Investigation of a Single-Server Queueing System with an Incoming MAP Event Flow

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Abstract—This paper considers a single-server queueing system with an incoming Markovian Arrival Process (MAP) request flow with two states. Explicit expressions are derived for the stationary probability distribution of the states and several numerical characteristics of the system (the probability of idle time of the server, the expected number of requests in the system, and the mean queue length). The resulting numerical characteristics are presented in tables and plotted in graphical form as well. The recurrent MAP flow with two states as a special case of correlated MAP request flows is studied.

Keywords: MAP request flow, single-server queueing system, stationary probability distribution of system states, numerical characteristics

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1. INTRODUCTION

Mathematical models of queueing systems and networks (QSs, QNs) adequately describe the behavior of real physical, technical, economic, and other objects and systems. Therefore, they have become widespread in the scientific community. A basic element of QSs and QNs are random incoming request flows. Almost throughout the 20th century, research on QSs and QNs was based on the assumption of the uncorrelated nature of incoming request flows. In other words, the simplest request flows—stationary Poisson ones—were considered. However, at the end of the century, the stationary Poisson flow model lost its adequacy to real information request flows in telecommunication networks and systems, wireless and mobile communication networks due to their intensive development.

The rapid change of digital technologies ensured the penetration of digital networks into all spheres of human activity. It would be impossible without the use and development of mathematical modeling methods and algorithms for network technologies. Since the end of the 20th century, intensive research in modern queueing theory has been dealing with queueing systems with correlated flows (systems with doubly stochastic flows). The emergence of doubly stochastic event flows, a new mathematical model with the most adequate description of the correlated nature of real information flows, was motivated by the practical studies of modern telecommunication networks with essentially nonstationary and correlated heterogeneous information flows.

Doubly stochastic flows are characterized by two stochastics: requests in the flow arrive at random time instants (the first stochastics), and the flow intensity (the accompanying process) is a random process (the second stochastics). There are two types of doubly stochastic flows depending on their accompanying process (intensity): the ones with a continuous random process [1, 2] and the ones with a piecewise constant random process with a finite (arbitrary) number of states. The studies of second-type flows were first presented almost simultaneously in 1979 in the papers [3–5]. In [3, 4], these flows were called Markov Chain (MC) flows; in [5], Markov Versatile Process (MVP) flows. In [6, 7], the above flows were termed Markovian Arrival Process (MAP) flows. Their main property is correlation. Note that MAP (MC) flows are the most appropriate mathematical model of correlated request flows in real telecommunication systems and networks [8].

The monograph [8], unique in the world literature, systematically presented QSs and QNs with correlated flows. As was emphasized in [8], the analytical investigation of QSs and QNs with correlated flows is a rather difficult process; finding the explicit-form characteristics of QSs and QNs is a nontrivial problem, sometimes unsolvable.

In this paper, we analytically investigate a single-server QS with waiting, the classical incoming MAP request flow with two states [6, 7], and exponential service.

For the stationary operation mode of this QS, explicit analytical formulas are derived for the probability of idle time of the server, the mean queue length, and the expected number of requests in the system.

Note that QSs and QNs with incoming MAP request flows have been analyzed since the 1990s. In particular, the states and parameters of an MAP request flow under perfect and incomplete observability conditions (in the presence of dead time) were estimated by the authors. In this regard, we refer to some publications [9–14].

In addition, the system under consideration differs from the systems operating in a synchronous random environment: in such an environment, synchronous flows are considered in which the state of the control process (accompanying process) changes at random time instants (the instants of events occurrence). Thus, a synchronous random environment always assumes a nonzero probability for changing the states of the control process at the instant of events occurrence in the synchronous flow. In an MAP flow, in contrast, a flow event not necessarily occurs at the instant of changing the state of the control process. (If the probability of event occurrence is always 1, we have a synchronous flow.) Thus, the mathematical model of a random environment considered below generalizes the mathematical model of a synchronous random environment, which is the novelty of this study.

The evolution from the simplest flow to modern mathematical models of information flows in telecommunication systems and networks (to the models of correlated flows, particularly MAP flows) can be traced in the monograph [8]. In addition, it provides an extensive bibliography on QSs and QNs. Among the recent works on this subject, let us mention the paper [15]. Note that numerical analysis is a common feature of research on QSs and QNs with an incoming MAP request flow. This paper continues the investigations initiated in [16].

2. MATHEMATICAL MODEL OF THE SYSTEM. PROBLEM STATEMENT

Consider a single-server QS with waiting. The server receives an incoming MAP flow of events (requests, messages, etc.) whose accompanying process $\lambda(t)$ is a piecewise constant random process with two states S_1 and S_2. If $\lambda(t) = \lambda_i$, then the process $\lambda(t)$ (flow) has the ith state (S_i), $i = 1, 2$; $\lambda_1 > \lambda_2 > 0$. The sojourn time of the process $\lambda(t)$ in the state S_i is a random variable with the exponential distribution function $F_i(t) = 1 - \exp\{-\lambda_i t\}$, $t \geqslant 0$, $i = 1, 2$.

When the ith state of the flow (process $\lambda(t)$) ends, the following instantaneous changes in the system state are possible:

1) A flow event occurs, and the process $\lambda(t)$ passes from the state S_i to the state S_j; the joint probability of this situation is $P_1(\lambda_j|\lambda_i)$, $i, j = 1, 2$.

2) No flow event occurs, and the process $\lambda(t)$ passes from the state S_i to the state S_j; the joint probability of this situation is $P_0(\lambda_j|\lambda_i)$, $i,j=1,2$ $(i\neq j)$.

Note that $P_0(\lambda_j|\lambda_i)+P_1(\lambda_j|\lambda_i)+P_1(\lambda_i|\lambda_i)=1$, $i,j=1,2$ $(i\neq j)$. Here, the occurrence (non-occurrence) of an event in the state S_i is primary, i.e., it precedes the transition from the flow state S_i to the flow state S_j with the probability $P_1(\lambda_j|\lambda_i)$ (the transition from the flow state S_i to the flow state S_j with the probability $P_0(\lambda_j|\lambda_i)$, respectively).

Let the QS operate in a stationary mode. Under the assumptions made, $\lambda(t)$ is the accompanying stationary, piecewise constant, and transitive Markov process with the two states S_1 and S_2. If the process $\lambda(t)$ is in the state S_i, then the request is served in a time $\tau\geqslant 0$ with the exponential distribution law $F^{(i)}(\tau)=1-\exp\{-\mu_i\tau\}$ with the intensity μ_i $(\mu_i>0)$, $i=1,2$.

Remark 1. For an MAP flow, the accompanying random process $\lambda(t)$ does not coincide with the flow intensity: in the states S_1 and S_2, the flow intensity takes the values $\lambda_1[1-P_0(\lambda_2|\lambda_1)]$ and $\lambda_2[1-P_0(\lambda_1|\lambda_2)]$, respectively. Then the mean intensity of this flow is [17]

$$\begin{gathered}\lambda=\lambda_1[1-P_0(\lambda_2|\lambda_1)]\pi_1+\lambda_2[1-P_0(\lambda_1|\lambda_2)]\pi_2,\\ \pi_1=\frac{\lambda_2[1-P_1(\lambda_2|\lambda_2)]}{\lambda_1[1-P_1(\lambda_1|\lambda_1)]+\lambda_2[1-P_1(\lambda_2|\lambda_2)]},\\ \pi_2=\frac{\lambda_1[1-P_1(\lambda_1|\lambda_1)]}{\lambda_1[1-P_1(\lambda_1|\lambda_1)]+\lambda_2[1-P_1(\lambda_2|\lambda_2)]},\end{gathered}\tag{1}$$

where π_1 and π_2 denote the prior probabilities of the states S_1 and S_2 of the process $\lambda(t)$ (flow), respectively, in the stationary mode.

Let $\tau_k=t_{k+1}-t_k$, $k=1,2,\ldots$, be the duration of the kth interval between the arrival time instants t_k and t_{k+1} of flow requests $(\tau_k\geqslant 0)$. Due to the stationary mode, the probability density of the durations is $p(\tau_k)=p(\tau)$, $\tau\geqslant 0$, for any $k\geqslant 1$. Then, without loss of generality, t_k can be supposed 0, i.e., a request arrives at the time instant $\tau=0$. The following explicit formula for the probability density $p(\tau)$ was derived in [11]:

$$\begin{gathered}p(\tau)=\gamma z_1e^{-z_1\tau}+(1-\gamma)z_2e^{-z_2\tau},\quad \tau\geqslant 0,\\ \gamma=\{z_2-\lambda_1\pi_1(0)[1-P_0(\lambda_2|\lambda_1)]-\lambda_2\pi_2(0)[1-P_0(\lambda_1|\lambda_2)]\}\,(z_2-z_1)^{-1},\\ z_{1,2}=\left[(\lambda_1+\lambda_2)\mp\sqrt{(\lambda_1-\lambda_2)^2+4\lambda_1\lambda_2P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)}\right]/2,\\ \pi_1(0)=\frac{P_1(\lambda_1|\lambda_2)+P_1(\lambda_1|\lambda_1)P_0(\lambda_1|\lambda_2)}{P_1(\lambda_1|\lambda_2)+P_1(\lambda_2|\lambda_1)+P_1(\lambda_1|\lambda_1)P_0(\lambda_1|\lambda_2)+P_1(\lambda_2|\lambda_2)P_0(\lambda_2|\lambda_1)},\\ \pi_2(0)=1-\pi_1(0).\end{gathered}\tag{2}$$

In (2), $\pi_i(0)$ is the stationary probability that the process $\lambda(\tau)$ has the state S_i, $i=1,2$, at the time instant $\tau=0$ (the arrival of an MAP flow request); z_1 and z_2 are the roots of the characteristic equation $z^2-(\lambda_1+\lambda_2)z+\lambda_1\lambda_2[1-P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)]=0$, where $0<z_1<z_2$ due to (2); γ is a value that depends on the flow parameters.

Consider two adjacent intervals (t_k,t_{k+1}) and (t_{k+1},t_{k+2}) with the durations $\tau_k=t_{k+1}-t_k$ and $\tau_{k+1}=t_{k+2}-t_{k+1}$, respectively. Since the flow is stationary, they are located arbitrarily on the time axis. Letting $k=1$, we study two intervals (t_1,t_2) and (t_2,t_3) with the durations $\tau_1=t_2-t_1$ and $\tau_2=t_3-t_2$, respectively, where $\tau_1\geqslant 0$ and $\tau_2\geqslant 0$. In this case, $\tau_1=0$ corresponds to the

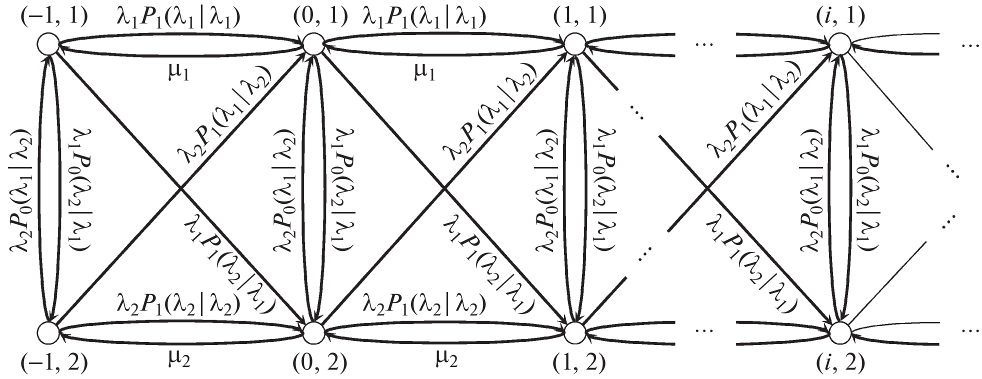

Fig. 1. The stochastic state transition graph for the process $\lambda(t)$.

arrival time instant t_1 of a flow request and $\tau_2 = 0$ to the arrival time instant t_2 of the next flow request. The joint probability density has the form [11, 13]

$$p(\tau_1,\tau_2) = p(\tau_1)p(\tau_2) + \gamma(1-\gamma)\frac{P_1(\lambda_1|\lambda_1)P_1(\lambda_2|\lambda_2) - P_1(\lambda_1|\lambda_2)P_1(\lambda_2|\lambda_1)}{1 - P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)}$$
$$\times \left(z_1e^{-z_1\tau_1} - z_2e^{-z_2\tau_1}\right)\left(z_1e^{-z_1\tau_2} - z_2e^{-z_2\tau_2}\right), \quad \tau_1 \geqslant 0, \quad \tau_2 \geqslant 0, \tag{3}$$

where z_1, z_2, and $p(\tau_k)$ are given by (2) for $\tau = \tau_k$, $k = 1, 2$.

According to (3), an MAP flow is generally a correlated flow; it turns recurrent or degenerates into elementary only in special cases.

Special case 1: $P_1(\lambda_1|\lambda_1)P_1(\lambda_2|\lambda_2) - P_1(\lambda_1|\lambda_2)P_1(\lambda_2|\lambda_1) = 0$, a recurrent MAP request flow with two states. In this case, $p(\tau)$ is given by (2), where $\gamma = [z_2 - \lambda_1P_1(\lambda_1|\lambda_1) - \lambda_2P_1(\lambda_2|\lambda_2)](z_2 - z_1)^{-1}$.

From (3) it follows that $p(\tau_1,\tau_2) = p(\tau_1)p(\tau_2)$. Since the arrival time instants $t_1,\ldots,t_k$ in the flow induce a nested Markov chain $\{\lambda(t_k)\}$, for an arbitrary number k, $k \geqslant 2$, we have $p(\tau_1,\ldots,\tau_k) = p(\tau_1)\ldots p(\tau_k)$.

The product $\gamma(1-\gamma)$ in (3) can be represented as

$$\gamma(1-\gamma) = \frac{z_1z_2}{(z_2-z_1)^2}\{\lambda_1[1 - P_0(\lambda_2|\lambda_1)] - \lambda_2[1 - P_0(\lambda_1|\lambda_2)]\}$$
$$\times \{\pi_1(0)\lambda_1[1 - P_1(\lambda_1|\lambda_1)] - \pi_2(0)\lambda_2[1 - P_1(\lambda_2|\lambda_2)]\} \tag{4}$$
$$\times \{\lambda_1\lambda_2[1 - P_0(\lambda_2|\lambda_1)][1 - P_1(\lambda_2|\lambda_2)] + \lambda_1\lambda_2[1 - P_0(\lambda_1|\lambda_2)][1 - P_1(\lambda_1|\lambda_1)]\}^{-1}.$$

The expression (4) implies special cases 2 and 3; see below.

Special case 2: $\lambda_1[1 - P_0(\lambda_2|\lambda_1)] - \lambda_2[1 - P_0(\lambda_1|\lambda_2)] = 0$, an elementary flow with a parameter z_1. From (2) it follows that $z_1 = \lambda_1[1 - P_0(\lambda_2|\lambda_1)]$, $\gamma = 1$; $p(\tau) = z_1e^{-z_1\tau}$, $\tau \geqslant 0$.

Special case 3: $\pi_1(0)\lambda_1[1 - P_1(\lambda_1|\lambda_1)] - \pi_2(0)\lambda_2[1 - P_1(\lambda_2|\lambda_2)] = 0$, an elementary flow with a parameter z_1. From (2) it follows that $z_1 = \lambda_2[P_1(\lambda_2|\lambda_1) + P_1(\lambda_2|\lambda_2)P_0(\lambda_2|\lambda_1)]$, $\gamma = 1$; $p(\tau) = z_1e^{-z_1\tau}$, $\tau \geqslant 0$.

The problem is to find an explicit analytical form of the numerical characteristics of this QS:

(a) the probability of idle time of the server,

(b) the mean queue length,

(c) the expected number of requests in the system.

Let $i(t)$ be the number of requests in the queue at an arbitrary time instant t ($i(t) = 0, 1, \ldots$). Since the incoming MAP flow is correlated, the random process $i(t)$ is not Markovian. To construct a Markov process, it is necessary to consider the state of the incoming MAP flow. For this purpose, we introduce an additional variable $j(t)$, i.e., the state of the incoming MAP flow (the state of the accompanying process $\lambda(t)$ at an arbitrary time instant t), $j(t) = 1, 2$. If $j(t) = 1$, then $\lambda(t) = \lambda_1$; if $j(t) = 2$, then $\lambda(t) = \lambda_2$, which ensures the Markov property of the two-dimensional process $(i(t), j(t))$.

Remark 2. Because the intensity of the server in the state S_j is μ_j ($\mu_j > 0$), $j = 1, 2$, the component $j(t)$ of the two-dimensional Markov process $(i(t), j(t))$ must be observable in the same way as the component $i(t)$ is. Then the accompanying process $\lambda(t)$, generally unobservable, must be treated as an observable process that controls the change of states in the MAP request flow.

Since the stationary operation mode is considered, the system state will be denoted by (i, j), $i = 0, 1, \ldots$, $j = 1, 2$. There are two more possible states, $(-1, 1)$ and $(-1, 2)$; in these states, the system receives no requests (the queue length is zero and the server is idle).

Under the prerequisites above, the mathematical model of the QS under study can be represented as a connected stochastic graph [18]; see Fig. 1. Here, the vertices reflect the states of the QS; each arc corresponds to infinitesimal characteristics (state transition intensities), without loops in each state; each vertex (each state) is reachable and recurrent.

3. DERIVATION OF NUMERICAL CHARACTERISTICS OF THE SYSTEM

We denote by $P(i, 1)$ and $P(i, 2)$ the stationary (final) probabilities of the system states ($i = -1, 0, \ldots$). The stochastic graph cutsets $G_{i1} = \{(i-1, 1; i, 1), (i, 1; i-1, 1), (i, 1; i+1, 1), (i+1, 1; i, 1), (i, 1; i, 2), (i, 2; i, 1), (i-1, 2; i, 1), (i, 1; i+1, 2)\}$, $G_{i2} = \{(i-1, 2; i, 2), (i, 2; i-1, 2), (i, 2; i+1, 2), (i+1, 2; i, 2), (i, 2; i, 1), (i, 1; i, 2), (i-1, 1; i, 2), (i, 2; i+1, 1)\}$, $i = 0, 1, \ldots$, satisfy the following infinite system of difference equations with constant coefficients:

$$\begin{aligned}
&\mu_1 P(i+1, 1) - (\lambda_1 + \mu_1) P(i, 1) + \lambda_1 P_1(\lambda_1|\lambda_1) P(i-1, 1) \\
&\quad + \lambda_2 P_0(\lambda_1|\lambda_2) P(i, 2) + \lambda_2 P_1(\lambda_1|\lambda_2) P(i-1, 2) = 0, \\
&\mu_2 P(i+1, 2) - (\lambda_2 + \mu_2) P(i, 2) + \lambda_2 P_1(\lambda_2|\lambda_2) P(i-1, 2) \\
&\quad + \lambda_1 P_0(\lambda_2|\lambda_1) P(i, 1) + \lambda_1 P_1(\lambda_2|\lambda_1) P(i-1, 1) = 0, \quad i = 0, 1, \ldots.
\end{aligned} \tag{5}$$

The solution of system (5) is found in the form $P(i, 1) = \xi^i$, $P(i, 2) = C\xi^i$ ($i = 0, 1, \ldots$). The characteristic equation for (5) is

$$\begin{aligned}
&(\xi - 1)\Big\{\mu_1\mu_2\xi^3 - [\lambda_1\mu_2 + \mu_1(\lambda_2 + \mu_2)]\xi^2 \\
&+ [\lambda_1\lambda_2 + \lambda_1\mu_2 P_1(\lambda_1|\lambda_1) + \lambda_2\mu_1 P_1(\lambda_2|\lambda_2) - \lambda_1\lambda_2 P_0(\lambda_1|\lambda_2) P_0(\lambda_2|\lambda_1)]\xi \\
&- \lambda_1\lambda_2[P(\lambda_1|\lambda_1) P_1(\lambda_2|\lambda_2) - P_1(\lambda_1|\lambda_2) P_1(\lambda_2|\lambda_1)]\Big\} = 0.
\end{aligned} \tag{6}$$

Consider conditions for the existence of the stationary operation mode of the QS (the existence of the probabilities $P(i, 1)$ and $P(i, 2)$, $i = -1, 0, \ldots$). The random variable $\boldsymbol{\tau}$, the duration of the time interval between sequential events in the MAP request flow, has the expectation

$$E(\boldsymbol{\tau}) = \int_0^\infty \tau p(\tau) d\tau, \tag{7}$$

where the density $p(\tau)$ is given by (2). Substituting this function into (7) yields $E(\boldsymbol{\tau}) = [\gamma z_2 + (1-\gamma) z_1]/z_1 z_2$. Then the expected number of requests in the incoming correlated MAP flow per unit time can be written as $\lambda = 1/E(\boldsymbol{\tau}) = \lambda_1[1 - P_0(\lambda_2|\lambda_1)]\pi_1 + \lambda_2[1 - P_0(\lambda_1|\lambda_2)]\pi_2$, which coincides with (1). On the other hand, the expected number of requests served per unit time is $\mu = \mu_1\pi_1 + \mu_2\pi_2$.

Consider a situation where $\lambda = \mu$, or $(\mu_1 - \lambda_1[1 - P_0(\lambda_2|\lambda_1)])\pi_1 + (\mu_2 - \lambda_2[1 - P_0(\lambda_1|\lambda_2)])\pi_2 = 0$. Hence, this expression vanishes only if $\mu_1 = \lambda_1[1 - P_0(\lambda_2|\lambda_1)]$, $\mu_2 = \lambda_2[1 - P_0(\lambda_1|\lambda_2)]$. Substituting these formulas for μ_1 and μ_2 into (6), we obtain the characteristic equation

$$\begin{gathered}\lambda_1\lambda_2(\xi-1)^2\Big\{[1-P_0(\lambda_1|\lambda_2)][1-P_0(\lambda_2|\lambda_1)]\xi^2 - [2-P_0(\lambda_1|\lambda_2)-P_0(\lambda_2|\lambda_1)]\xi \\ + [P_1(\lambda_1|\lambda_1)P_1(\lambda_2|\lambda_2) - P_1(\lambda_1|\lambda_2)P_1(\lambda_2|\lambda_1)]\Big\} = 0.\end{gathered} \tag{8}$$

Since Eq. (8) has multiple roots, the general solution of system (5) with $\mu_1 = \lambda_1[1 - P_0(\lambda_2|\lambda_1)]$ and $\mu_2 = \lambda_2[1 - P_0(\lambda_1|\lambda_2)]$ takes the form

$$\begin{gathered}P(i,1) = D_1\xi_1^i + D_2 i\xi_2^i + D_3\xi_3^i + D_4\xi_4^i, \\ P(i,2) = B_1D_1\xi_1^i + B_2D_2 i\xi_2^i + B_3D_3\xi_3^i + B_4D_4\xi_4^i, \quad i = 0,1,\ldots.\end{gathered} \tag{9}$$

In (9), $P_s(i,1) = D_s\xi_s^i$ and $P_s(i,2) = B_sD_s\xi_s^i$, $s = \overline{1,4}$, are partial solutions of system (5); their constants B_s and D_s are determined from the boundary conditions, $\xi_1 = \xi_2 = 1$, and

$$\begin{gathered}\xi_{3,4} = \Big\{[2-P_0(\lambda_1|\lambda_2)-P_0(\lambda_2|\lambda_1)] \\ \mp\Big([2-P_0(\lambda_1|\lambda_2)-P_0(\lambda_2|\lambda_1)]^2 - 4[1-P_0(\lambda_1|\lambda_2)][1-P_0(\lambda_2|\lambda_1)]b\Big)^{\frac{1}{2}}\Big\} \\ \times\Big\{2[1-P_0(\lambda_1|\lambda_2)][1-P_0(\lambda_2|\lambda_1)]\Big\}^{-1}, \\ b = P_1(\lambda_1|\lambda_1)P_1(\lambda_2|\lambda_2) - P_1(\lambda_1|\lambda_2)P_1(\lambda_2|\lambda_1).\end{gathered} \tag{10}$$

Here, three cases are possible: $b > 0$, $b < 0$, and $b = 0$.

The case $b > 0$. From (10) it follows that $0 < \xi_3 < 1 < \xi_4$. Since $P(i,1)$ and $P(i,2)$ are probabilities, they must satisfy the normalization condition

$$\sum_{i=-1}^{\infty} P(i,1) + \sum_{i=-1}^{\infty} P(i,2) = 1.$$

A necessary condition for this equality is the limit relations $\lim P(i,1) = 0$ and $\lim P(i,2) = 0$ as $i \to \infty$. Otherwise, the series $\sum_{i=-1}^{\infty} P(i,1)$ and $\sum_{i=-1}^{\infty} P(i,2)$ will diverge. In view of the aforesaid, the general solution (9) with $D_1 = D_2 = D_4 = 0$ takes the form

$$P(i,1) = D_3\xi_3^i, \quad P(i,2) = B_3D_3\xi_3^i, \quad i = 0,1,\ldots. \tag{11}$$

We find the constant B_3. Substituting (11) into the first equation of system (5) with $\mu_1 = \lambda_1[1 - P_0(\lambda_2|\lambda_1)]$ and $\mu_2 = \lambda_2[1 - P_0(\lambda_1|\lambda_2)]$ gives $B_3 < 0$ after nontrivial transformations. Then (11) implies $D_3 < 0$. The inequality $D_3 < 0$ leads to a contradiction: $P(i,1) < 0$, $i \geqslant 0$; $P(i,2) > 0$, $i \geqslant 0$. Letting $D_3 = 0$ yields $P(i,1) = P(i,2) = 0$, $i \geqslant 0$; in other words, the contradiction is eliminated. Therefore, the final distribution $P(i,1)$, $P(i,2)$, $i \geqslant 0$, does not exist for $\lambda = \mu$ and, a fortiori, for $\lambda > \mu$.

We analyze the situation $\lambda < \mu$. Due to (6), the general solution of system (5) takes the form

$$\begin{gathered}P(i,1) = A_1\xi_1^i + A_2\xi_2^i + A_3\xi_3^i + A_4\xi_4^i, \\ P(i,2) = C_1A_1\xi_1^i + C_2A_2\xi_2^i + C_3A_3\xi_3^i + C_4A_4\xi_4^i, \quad i = 0,1,\ldots,\end{gathered} \tag{12}$$

where $P_s(i,1) = A_s\xi_s^i$ and $P_s(i,2) = C_sA_s\xi_s^i$ are partial solutions of system (5); their constants C_s and A_s, $s = \overline{1,4}$, are determined from the boundary conditions; $\xi_4 = 1$, ξ_1, ξ_2, and ξ_3 are the roots of the cubic equation in (6), positive real numbers: $0 < \xi_1 < \xi_2 < 1 < \xi_3$. In addition, the limit relations $\lim P(i,1) = \lim P(i,2) = 0$ as $i \to \infty$ hold (a necessary condition). Hence, $A_3 = A_4 = 0$, and the general solution of (12) takes the form

$$\begin{aligned} P(i,1) &= A_1\xi_1^i + A_2\xi_2^i, \\ P(i,2) &= C_1A_1\xi_1^i + C_2A_2\xi_2^i, \quad i = 0,1,\ldots. \end{aligned} \tag{13}$$

Substituting the partial solution $P_s(i,1) = A_s\xi_s^i$, $P_s(i,2) = C_sA_s\xi_s^i$, $i = 0,1,\ldots$, into the first equation of system (5), first for $s = 1$ and then for $s = 2$, we obtain the constants

$$C_s = -\frac{\mu_1\xi_s^2 - (\lambda_1+\mu_1)\xi_s + \lambda_1 P_1(\lambda_1|\lambda_1)}{\lambda_2[P_0(\lambda_1|\lambda_2)\xi_s + P_1(\lambda_1|\lambda_2)]}, \quad s = 1,2. \tag{14}$$

The values A_i, $i = 1,2$, and the probabilities $P(-1,1)$ and $P(-1,2)$ are found using the boundary equations and the normalization condition. The stochastic graph cutsets

$$\begin{gathered} G_{-1,1} = \{(-1,1;0,1), (0,1;-1,1), (-1,1;0,2), (-1,1;-1,2), (-1,2;-1,1)\}, \\ G_{-1,2} = \{(-1,2;0,2),\ (0,2;-1,2),\ (-1,2;0,1),\ (-1,2;-1,1), (-1,1;-1,2)\}, \\ G = \{(i,1;i+1,2),\ (i,1;i,2),\ (i,2;i+1,1),\ (i,2;i,1),\quad i = -1,0,1,\ldots\} \end{gathered}$$

determine the corresponding boundary equations:

$$\begin{gathered} \mu_1 P(0,1) - \lambda_1 P(-1,1) + \lambda_2 P_0(\lambda_1|\lambda_2)P(-1,2) = 0, \\ \mu_2 P(0,2) - \lambda_2 P(-1,2) + \lambda_1 P_0(\lambda_2|\lambda_1)P(-1,1) = 0, \\ \lambda_1[1 - P_1(\lambda_1|\lambda_1)]\sum_{i=-1}^{\infty} P(i,1) - \lambda_2[1 - P_1(\lambda_2|\lambda_2)]\sum_{i=-1}^{\infty} P(i,2) = 0. \end{gathered} \tag{15}$$

Supplementing (15) with the normalization condition

$$P(-1,1) + P(-1,2) + \sum_{i=0}^{\infty}[P(i,1) + P(i,2)] = 1,$$

in view of (13), we arrive at the system of equations for the unknowns A_i, $i = 1,2$, $P(-1,1)$, and $P(-1,2)$. Solving (15) yields

$$\begin{gathered} P(-1,1) = a_{11}A_1 + a_{12}A_2, \quad P(-1,2) = a_{21}A_1 + a_{22}A_2, \\ A_1 = (1-\xi_1)\frac{\pi_1[C_2 + a_{22}(1-\xi_2)] - \pi_2[1 + a_{12}(1-\xi_2)]}{[1 + a_{11}(1-\xi_1)][C_2 + a_{22}(1-\xi_2)] - [1 + a_{12}(1-\xi_2)][C_1 + a_{21}(1-\xi_1)]}, \\ A_2 = -(1-\xi_2)\frac{\pi_1[C_1 + a_{21}(1-\xi_1)] - \pi_2[1 + a_{11}(1-\xi_1)]}{[1 + a_{11}(1-\xi_1)][C_2 + a_{22}(1-\xi_2)] - [1 + a_{12}(1-\xi_2)][C_1 + a_{21}(1-\xi_1)]}, \\ a_{11} = \frac{\mu_1 + \mu_2 P_0(\lambda_1|\lambda_2)C_1}{\lambda_1[1 - P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)]}, \quad a_{12} = \frac{\mu_1 + \mu_2 P_0(\lambda_1|\lambda_2)C_2}{\lambda_1[1 - P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)]}, \\ a_{21} = \frac{\mu_2 C_1 + \mu_1 P_0(\lambda_2|\lambda_1)}{\lambda_2[1 - P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)]}, \quad a_{22} = \frac{\mu_2 C_2 + \mu_1 P_0(\lambda_2|\lambda_1)}{\lambda_2[1 - P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)]}. \end{gathered} \tag{16}$$

The values C_1 and C_2 are given by (14); the probabilities π_1 and π_2, by (1). The values ξ_1 and ξ_2 are the roots of the cubic equation in (6) $(0 < \xi_1 < \xi_2 < 1)$.

Table 1. The probability of idle time $P(-1)$ depending on λ_1 for $b > 0$

$P_1(\lambda_1\|\lambda_2)$ \ λ_1	2	4	6	8	10	11
1/4	0.780	0.718	0.680	0.651	0.627	0.616
1/6	0.787	0.728	0.693	0.667	0.645	0.636
1/8	0.790	0.734	0.700	0.675	0.654	0.645
1/10	0.792	0.737	0.704	0.679	0.659	0.651
1/12	0.794	0.739	0.706	0.682	0.663	0.655
1/13	0.794	0.739	0.707	0.684	0.664	0.656

Table 2. The mean queue length $E(I)$ depending on λ_1 for $b > 0$

$P_1(\lambda_1\|\lambda_2)$ \ λ_1	2	4	6	8	10	11
1/4	0.052	0.097	0.145	0.196	0.249	0.276
1/6	0.047	0.085	0.125	0.167	0.209	0.231
1/8	0.045	0.080	0.116	0.153	0.191	0.210
1/10	0.043	0.076	0.110	0.145	0.180	0.198
1/12	0.042	0.074	0.106	0.140	0.173	0.190
1/13	0.042	0.073	0.105	0.138	0.171	0.187

Table 3. The expected number of requests $E(I+1)$ in the system depending on λ_1 for $b > 0$

$P_1(\lambda_1\|\lambda_2)$ \ λ_1	2	4	6	8	10	11
1/4	0.272	0.379	0.465	0.545	0.622	0.659
1/6	0.260	0.357	0.432	0.500	0.564	0.595
1/8	0.254	0.346	0.416	0.478	0.537	0.565
1/10	0.251	0.340	0.406	0.466	0.521	0.547
1/12	0.249	0.335	0.400	0.457	0.510	0.536
1/13	0.248	0.334	0.398	0.454	0.506	0.531

Formulas (13) and (16) allow deriving explicit expressions for the numerical characteristics of the system: $P(-1)$ (the probability of idle time of the server), $E(I)$ (the mean queue length), and $E(I+1)$ (the expected number of requests in the system), where I is the random queue length in the QS. They are:

$$\begin{gathered} P(-1) = (a_{11} + a_{21})A_1 + (a_{12} + a_{22})A_2, \\ E(I) = A_1(1 + C_1)\frac{\xi_1}{(1-\xi_1)^2} + A_2(1 + C_2)\frac{\xi_2}{(1-\xi_2)^2}, \\ E(I+1) = \frac{A_1(1 + C_1)}{(1-\xi_1)^2} + \frac{A_2(1 + C_2)}{(1-\xi_2)^2}, \end{gathered} \tag{17}$$

where C_1 and C_2 are given by (14); A_1, A_2, a_{11}, a_{21}, a_{12}, and a_{22}, by (16). The values ξ_1 and ξ_2 are the roots of the cubic equation in (6) ($0 < \xi_1 < \xi_2 < 1$).

The initial data for calculating the numerical characteristics (17), see the tables below, are chosen to assess the degree of their correspondence to the physical understanding of the service process in the QS.

Tables 1–3 present the characteristics $P(-1)$, $E(I)$, and $E(I+1)$ (17) depending on the parameter λ_1 ($\lambda_1 = 2, 4, \ldots, 10, 11$) under the fixed parameter values $\lambda_2 = 1$, $\mu_1 = 12$, $\mu_2 = 2$; $P_1(\lambda_1|\lambda_1) = P_1(\lambda_2|\lambda_1) = P_0(\lambda_2|\lambda_1) = P_1(\lambda_2|\lambda_2) = \frac{1}{3}$ for $b > 0$ and $P_1(\lambda_1|\lambda_2) = \frac{1}{4}$ ($P_0(\lambda_1|\lambda_2) = \frac{5}{12}$); $P_1(\lambda_1|\lambda_2) = \frac{1}{6}$

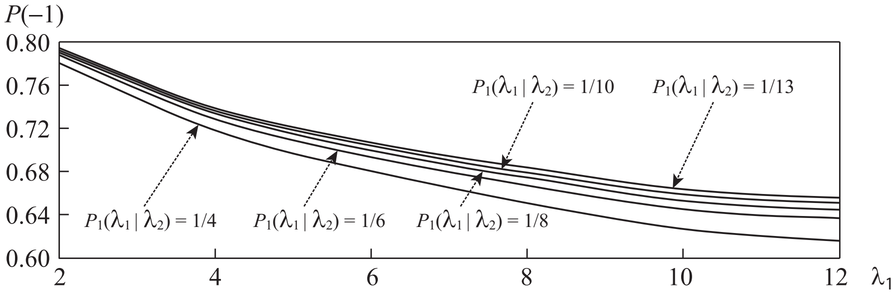

Fig. 2. The probability of idle time $P(-1)$ depending on λ_1 for $b > 0$.

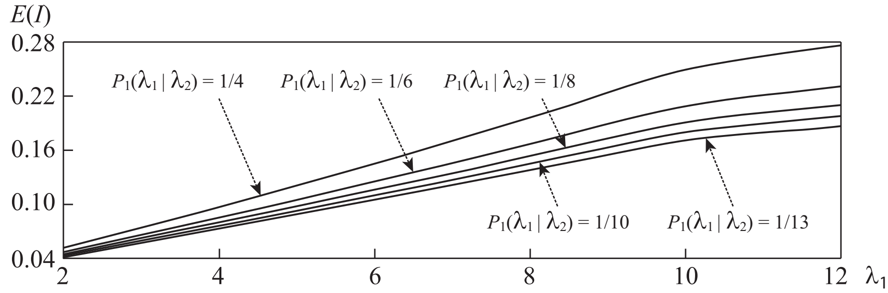

Fig. 3. The mean queue length $E(I)$ depending on λ_1 for $b > 0$.

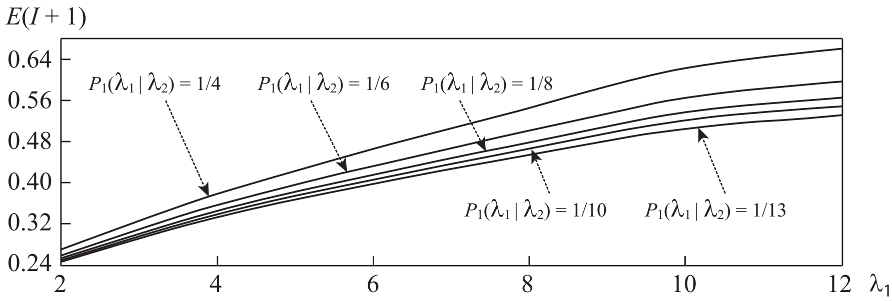

Fig. 4. The expected number of requests $E(I+1)$ in the system depending on λ_1 for $b > 0$

$(P_0(\lambda_1|\lambda_2) = \frac{1}{2})$; $P_1(\lambda_1|\lambda_2) = \frac{1}{8}$ $(P_0(\lambda_1|\lambda_2) = \frac{13}{24})$; $P_1(\lambda_1|\lambda_2) = \frac{1}{10}$ $(P_0(\lambda_1|\lambda_2) = \frac{17}{30})$; $P_1(\lambda_1|\lambda_2) = \frac{1}{12}$ $(P_0(\lambda_1|\lambda_2) = \frac{7}{12})$; $P_1(\lambda_1|\lambda_2) = \frac{1}{13}$ $(P_0(\lambda_1|\lambda_2) = \frac{23}{39})$.

The behavior of these characteristics depending on the parameter λ_1 for $b > 0$ matches the physical understanding of the service process in the single-server QS with an incoming correlated MAP request flow.

Figures 2–4 show the graphs of the numerical characteristics (17) plotted on the numerical values of Tables 1–3, respectively.

The case $b < 0$. First of all, we investigate the existence of the stationary mode, i.e., the situation $\lambda = \mu$. Then it follows from (10) that $\xi_3 < 0$ and $\xi_4 > 1$; similar to the case $b > 0$, the general solution of the system takes the form (11). Since $\xi_3 < 0$, this fact entails the negative probability $P(i,1)$ for $i = 1, 3, \ldots,$ (an obvious contradiction to its definition). This contradiction is eliminated by letting $D_3 = 0$: $P(i,1) = P(i,2) = 0$, $i \geqslant 0$. Therefore, in the case $b < 0$, the final distribution $P(i,1)$, $P(i,2)$, $i \geqslant 0$, does not exist for $\lambda = \mu$ and, a fortiori, for $\lambda > \mu$.

Now we study the situation $\lambda < \mu$. Due to (6), the general solution of system (5) takes the form (12). In the case $b < 0$, we have $\xi_4 = 1$, ξ_1, ξ_2, and ξ_3 are the real roots of the cubic equation in (6): $\xi_1 < 0$, $0 < \xi_2 < 1 < \xi_3$. Hence, it follows that $A_1 = A_3 = A_4 = 0$ in (12), and the general

Table 4. The probability of idle time $P(-1)$ depending on λ_1 for $b < 0$

$P_1(\lambda_2\|\lambda_2)$ \ λ_1	2	4	6	8	10	11
1/4	0.796	0.729	0.687	0.654	0.627	0.615
1/6	0.815	0.748	0.705	0.672	0.645	0.633
1/8	0.824	0.757	0.714	0.681	0.654	0.642
1/10	0.830	0.762	0.719	0.686	0.659	0.647
1/12	0.833	0.765	0.722	0.690	0.662	0.650
1/13	0.834	0.767	0.724	0.691	0.664	0.652

Table 5. The mean queue length $E(I)$ depending on λ_1 for $b < 0$

$P_1(\lambda_2\|\lambda_2)$ \ λ_1	2	4	6	8	10	11
1/4	0.044	0.091	0.143	0.200	0.260	0.291
1/6	0.035	0.077	0.124	0.177	0.232	0.261
1/8	0.031	0.071	0.116	0.166	0.220	0.248
1/10	0.029	0.067	0.112	0.161	0.213	0.241
1/12	0.027	0.065	0.109	0.157	0.209	0.236
1/13	0.027	0.064	0.108	0.156	0.207	0.234

Table 6. The expected number of requests $E(I+1)$ in the system depending on λ_1 for $b < 0$

$P_1(\lambda_2\|\lambda_2)$ \ λ_1	2	4	6	8	10	11
1/4	0.249	0.362	0.457	0.546	0.633	0.675
1/6	0.219	0.329	0.419	0.504	0.587	0.528
1/8	0.206	0.314	0.402	0.485	0.566	0.606
1/10	0.199	0.305	0.392	0.474	0.554	0.594
1/12	0.194	0.300	0.386	0.467	0.547	0.586
1/13	0.193	0.298	0.384	0.465	0.544	0.583

solution of (12) is written as

$$P(i,1) = A_2\xi_2^i, \quad P(i,2) = C_2A_2\xi_2^i, \quad i = 0, 1, \ldots. \tag{18}$$

In (18), the constant C_2 is given by (14) for $s = 2$. The constant A_2 and the probabilities $P(-1,1)$ and $P(-1,2)$ are determined using Eqs. (15) and the normalization condition. As a result,

$$\begin{gathered} P(-1,1) = a_{12}A_2; \quad P(-1,2) = a_{22}A_2; \\ A_2 = \frac{1-\xi_2}{1 + C_2 + (a_{12}+a_{22})(1-\xi_2)}, \end{gathered} \tag{19}$$

where C_2 is given by (14) for $s = 2$; a_{12} and a_{22}, by (16). The value ξ_2 is the root of the cubic equation in (6) ($0 < \xi_2 < 1$).

Formulas (18) and (19) allow deriving the system characteristics:

$$\begin{gathered} P(-1) = (a_{12}+a_{22})A_2; \\ E(I) = A_2\xi_2\frac{1+C_2}{(1-\xi_2)^2}, \quad E(I+1) = \frac{(1+C_2)A_2}{(1-\xi_2)^2}, \end{gathered} \tag{20}$$

where C_2 is given by (14) for $s = 2$; a_{12} and a_{22}, by (16); A_2, by (19). The value ξ_2 is the root of the cubic equation in (6) ($0 < \xi_2 < 1$).

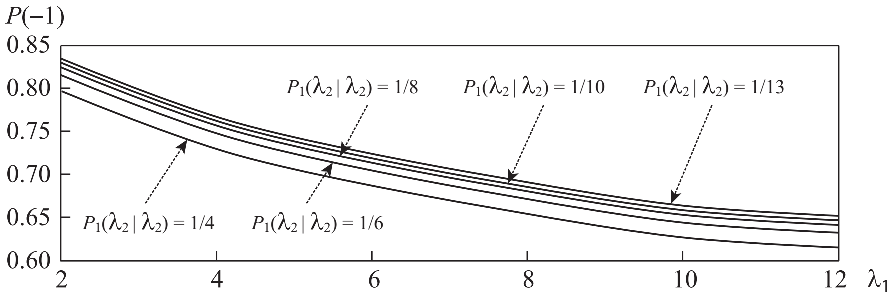

Fig. 5. The probability of idle time $P(-1)$ depending on λ_1 for $b < 0$.

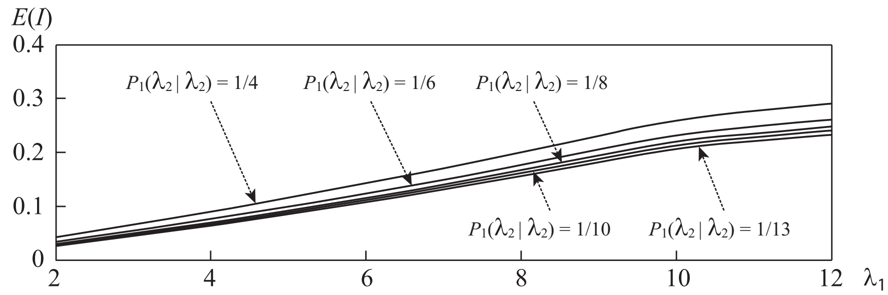

Fig. 6. The mean queue length $E(I)$ depending on λ_1 for $b < 0$.

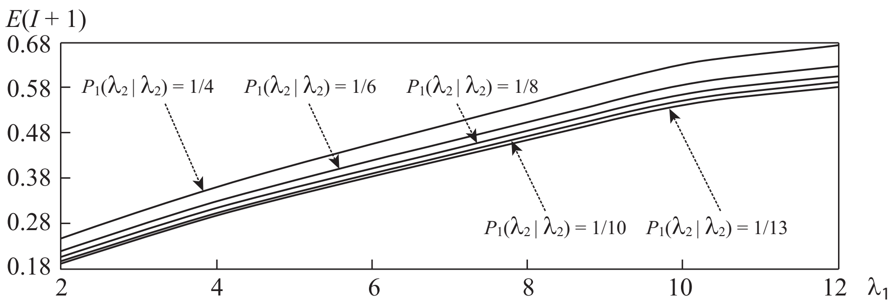

Fig. 7. The expected number of requests $E(I+1)$ in the system depending on λ_1 for $b < 0$.

Tables 4–6 present the characteristics $P(-1)$, $E(I)$, and $E(I+1)$ (20) depending on the parameter λ_1 ($\lambda_1 = 2, 4, \ldots, 10, 11$) for the fixed parameter values $\lambda_2 = 1$, $\mu_1 = 12$, $\mu_2 = 2$; $P_1(\lambda_1|\lambda_1) = P_1(\lambda_2|\lambda_1) = P_0(\lambda_2|\lambda_1) = P_1(\lambda_1|\lambda_2) = \frac{1}{3}$ for $b < 0$ and $P_1(\lambda_2|\lambda_2) = \frac{1}{4}$ ($P_0(\lambda_1|\lambda_2) = \frac{5}{12}$); $P_1(\lambda_2|\lambda_2) = \frac{1}{6}$ ($P_0(\lambda_1|\lambda_2) = \frac{1}{2}$); $P_1(\lambda_2|\lambda_2) = \frac{1}{8}$ ($P_0(\lambda_1|\lambda_2) = \frac{13}{24}$); $P_1(\lambda_2|\lambda_2) = \frac{1}{10}$ ($P_0(\lambda_1|\lambda_2) = \frac{17}{30}$); $P_1(\lambda_2|\lambda_2) = \frac{1}{12}$ ($P_0(\lambda_1|\lambda_2) = \frac{7}{12}$); $P_1(\lambda_2|\lambda_2) = \frac{1}{13}$ ($P_0(\lambda_1|\lambda_2) = \frac{23}{39}$).

Figures 5–7 show the graphs of the numerical characteristics (20) plotted on the numerical values of Tables 4–6, respectively.

The behavior of these characteristics depending on the parameter λ_1 for $b < 0$ also matches the physical understanding of the service process in the single-server QS with an incoming correlated MAP request flow.

4. A SPECIAL CASE: A RECURRENT MAP REQUEST FLOW

In this special case, we have $b = 0$, which implies the recurrence of the MAP request flow; see (3). Consider conditions for the existence of the stationary probabilities $P(i, 1)$ and $P(i, 2)$, $i \geqslant 0$. In the

Table 7. The probability of idle time $P(-1)$ depending on λ_1 for $b = 0$

$P_1(\lambda_1\|\lambda_1)$ \ λ_1	2	4	6	8	10	11
1/4	0.831	0.781	0.750	0.726	0.707	0.698
1/6	0.888	0.854	0.833	0.818	0.805	0.799
1/8	0.916	0.891	0.875	0.863	0.854	0.850
1/10	0.933	0.913	0.900	0.891	0.883	0.880
1/12	0.944	0.927	0.917	0.909	0.902	0.900
1/13	0.949	0.933	0.923	0.916	0.910	0.907

Table 8. The mean queue length $E(I)$ depending on λ_1 for $b = 0$

$P_1(\lambda_1\|\lambda_1)$ \ λ_1	2	4	6	8	10	11
1/4	0.030	0.056	0.083	0.111	0.139	0.152
1/6	0.013	0.023	0.033	0.043	0.053	0.058
1/8	0.007	0.012	0.018	0.023	0.028	0.030
1/10	0.004	0.008	0.011	0.014	0.017	0.019
1/12	0.003	0.005	0.008	0.010	0.0129	0.013
1/13	0.002	0.005	0.006	0.008	0.010	0.011

Table 9. The expected number of requests $E(I + 1)$ in the system depending on λ_1 for $b = 0$

$P_1(\lambda_1\|\lambda_1)$ \ λ_1	2	4	6	8	10	11
1/4	0.199	0.275	0.333	0.385	0.432	0.454
1/6	0.124	0.169	0.200	0.226	0.248	0.259
1/8	0.091	0.122	0.143	0.160	0.174	0.181
1/10	0.071	0.095	0.111	0.124	0.134	0.139
1/12	0.059	0.078	0.091	0.101	0.109	0.113
1/13	0.054	0.072	0.083	0.092	0.100	0.103

situation $\lambda = \mu$, the characteristic Eq. (8) takes the form

$$\lambda_1\lambda_2(\xi - 1)^2\xi\Big\{[1 - P_0(\lambda_2|\lambda_1)][1 - P_0(\lambda_1|\lambda_2)]\xi - [2 - P_0(\lambda_1|\lambda_2) - P_0(\lambda_2|\lambda_1)]\Big\} = 0, \tag{21}$$

and the general solution of system (5) is (9). The characteristic Eq. (21) has the roots

$$\xi_1 = \xi_2 = 1, \quad \xi_3 = 0, \quad \xi_4 = \frac{1}{1 - P_0(\lambda_1|\lambda_2)} + \frac{1}{1 - P_0(\lambda_2|\lambda_1)} > 1. \tag{22}$$

In view of (22), letting $D_1 = D_2 = D_4 = 0$ in the general solution (9) yields $P(i, 1) = P(i, 2) = 0$, $i \geqslant 0$. Therefore, in the case $b = 0$, the final distribution $P(i, 1)$, $P(i, 2)$, $i \geqslant 0$, does not exist for $\lambda = \mu$ and, a fortiori, for $\lambda > \mu$.

We analyze the situation $\lambda < \mu$. In the case $b = 0$, the characteristic Eq. (6) is written as

$$\begin{aligned}\xi(\xi - 1)\Big\{&\mu_1\mu_2\xi^2 - [\lambda_1\mu_2 + \mu_1(\lambda_2 + \mu_2)]\xi \\ &+ [\lambda_1\lambda_2 + \lambda_1\mu_2P_1(\lambda_1|\lambda_1) + \lambda_2\mu_1P_1(\lambda_2|\lambda_2) - \lambda_1\lambda_2P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)]\Big\} = 0.\end{aligned} \tag{23}$$

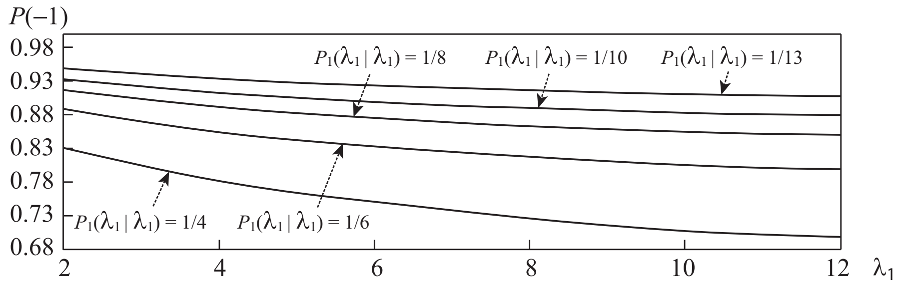

Fig. 8. The probability of idle time $P(-1)$ depending on λ_1 for $b = 0$.

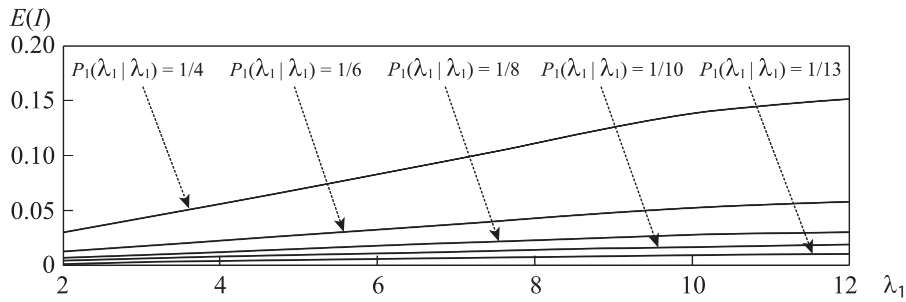

Fig. 9. The mean queue length $E(I)$ depending on λ_1 for $b = 0$.

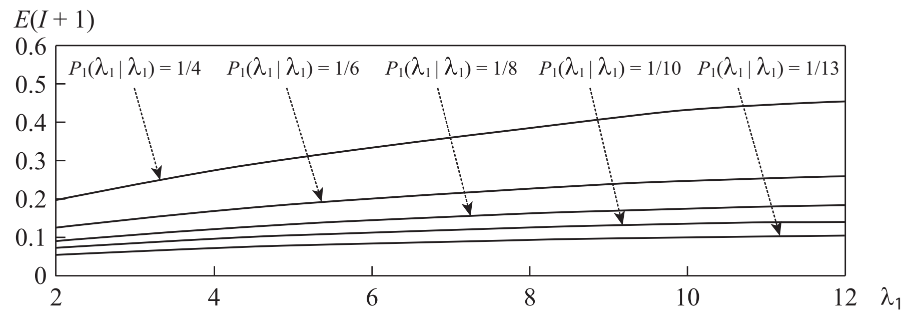

Fig. 10. The expected number of requests $E(I + 1)$ in the system depending on λ_1 for $b = 0$.

The characteristic Eq. (23) has the roots $\xi_3 = 0$, $\xi_4 = 1$, and

$$\begin{aligned}\xi_{1,2} = \Big\{(\lambda_1\mu_2 + \lambda_2\mu_1 + \mu_1\mu_2) \mp \Big[(\lambda_1\mu_2 + \lambda_2\mu_1 + \mu_1\mu_2)^2 \\ - 4\mu_1\mu_2\Big(\lambda_1\lambda_2 + \lambda_1\mu_2 P_1(\lambda_1|\lambda_1) + \lambda_2\mu_1 P_1(\lambda_2|\lambda_2) - \lambda_1\lambda_2 P_0(\lambda_1|\lambda_2)P_0(\lambda_2|\lambda_1)\Big)\Big]^{\frac{1}{2}}\Big\}/2\mu_1\mu_2,\end{aligned} \tag{24}$$

$0 < \xi_1 < 1 < \xi_2$. Due to (23) and (24), the general solution (12) of system (5) takes the form

$$P(i, 1) = A_1\xi_1^i, \quad P(i, 2) = C_1A_1\xi_1^i, \quad i = 0, 1, \ldots. \tag{25}$$

In (25), the constant C_1 is given by (14) for $s = 1$. The constant A_1 and the probabilities $P(-1, 1)$ and $P(-1, 2)$ are determined using Eqs. (15) and the normalization condition. As a result,

$$\begin{aligned}P(-1, 1) = a_{11}A_1; \quad P(-1, 2) = a_{21}A_1; \\ A_1 = \frac{1 - \xi_1}{1 + C_1 + (a_{11} + a_{21})(1 - \xi_1)},\end{aligned} \tag{26}$$

where C_1 is given by (14) for $s = 1$; a_{21} and a_{11}, by (16); ξ_1, by (24).

Formulas (25) and (26) allow deriving the system characteristics:

$$P(-1) = (a_{21} + a_{11})A_1;$$
$$E(I) = A_1\xi_1 \frac{1 + C_1}{(1 - \xi_1)^2}, \quad E(I + 1) = \frac{(1 + C_1)A_1}{(1 - \xi_1)^2}, \tag{27}$$

where C_1 is given by (14) for $s = 1$; a_{21} and a_{11}, by (16); A_1, by (26); ξ_1, by (24).

Tables 7–9 present the characteristics $P(-1)$, $E(I)$, and $E(I+1)$ (27) depending on the parameter λ_1 ($\lambda_1 = 2, 4, \ldots, 10, 11$) for the fixed parameter values $\lambda_2 = 1$, $\mu_1 = 12$, $\mu_2 = 2$ for $b = 0$ and ($P_1(\lambda_i|\lambda_i) = P_1(\lambda_j|\lambda_i) = \frac{1}{4}$; $P_0(\lambda_1|\lambda_2) = P_0(\lambda_2|\lambda_1) = \frac{1}{2}$); ($P_1(\lambda_i|\lambda_i) = P_1(\lambda_j|\lambda_i) = \frac{1}{6}$; $P_0(\lambda_1|\lambda_2) = P_0(\lambda_2|\lambda_1) = \frac{2}{3}$); ($P_1(\lambda_i|\lambda_i) = P_1(\lambda_j|\lambda_i) = \frac{1}{8}$; $P_0(\lambda_1|\lambda_2) = P_0(\lambda_2|\lambda_1) = \frac{3}{4}$); ($P_1(\lambda_i|\lambda_i) = P_1(\lambda_j|\lambda_i) = \frac{1}{10}$; $P_0(\lambda_1|\lambda_2) = P_0(\lambda_2|\lambda_1) = \frac{4}{5}$); ($P_1(\lambda_i|\lambda_i) = P_1(\lambda_j|\lambda_i) = \frac{1}{12}$; $P_0(\lambda_1|\lambda_2) = P_0(\lambda_2|\lambda_1) = \frac{5}{6}$); ($P_1(\lambda_i|\lambda_i) = P_1(\lambda_j|\lambda_i) = \frac{1}{13}$; $P_0(\lambda_1|\lambda_2) = P_0(\lambda_2|\lambda_1) = \frac{11}{13}$); $i, j = 1, 2$ ($i \neq j$).

As in the cases $b > 0$ and $b < 0$, the behavior of these characteristics depending on the parameter λ_1 for $b = 0$ matches the physical understanding of the service process in the single-server QS with an incoming correlated MAP request flow.

Figures 8–10 show the graphs of the numerical characteristics (27) plotted on the numerical values of Tables 7–9, respectively.

5. CONCLUSIONS

The paper has considered a single-server QS with an incoming correlated MAP request flow with two states. The analysis problems formulated in Section 2 have been completely solved for this queueing system.

Let us summarize the results and present the final formulas.

The case $b > 0$. The stationary probabilities $P(i, 1)$ and $P(i, 2)$, $i = 0, 1, \ldots,$ are given by $P(i, 1) = A_1\xi_1^i + A_2\xi_2^i$ and $P(i, 2) = C_1A_1\xi_1^i + C_2A_2\xi_2^i$, respectively, where: the constants C_s, $s = 1, 2$, are calculated using (14); ξ_1 and ξ_2 ($0 < \xi_1 < \xi_2 < 1$) are the roots of the cubic Eq. (6); the probabilities $P(-1, 1)$ and $P(-1, 2)$ as well as the constants A_1 and A_2 are calculated using (16). The numerical characteristics $P(-1)$, $E(I)$, and $E(I + 1)$ are given by (17).

The case $b < 0$. The stationary probabilities $P(i, 1)$ and $P(i, 2)$, $i = 0, 1, \ldots,$ are given by $P(i, 1) = A_2\xi_2^i$ and $P(i, 2) = C_2A_2\xi_2^i$, respectively, where: the constant C_2 is calculated using (14) for $s = 2$; ξ_2 ($0 < \xi_2 < 1$) is the root of the cubic Eq. (6); the probabilities $P(-1, 1)$ and $P(-1, 2)$ as well as the constant A_2 are calculated using (19). The numerical characteristics $P(-1)$, $E(I)$, and $E(I + 1)$ are given by (20).

The case $b = 0$. The stationary probabilities $P(i, 1)$ and $P(i, 2)$, $i = 0, 1, \ldots,$ are given by $P(i, 1) = A_1\xi_1^i$ and $P(i, 2) = C_1A_1\xi_1^i$, respectively, where: the constant C_1 is calculated using (14) for $s = 1$; ξ_1 ($0 < \xi_1 < 1$) is the root (24) of the characteristic Eq. (23); the probabilities $P(-1, 1)$ and $P(-1, 2)$ as well as the constant A_1 are calculated using (26). The numerical characteristics $P(-1)$, $E(I)$, and $E(I + 1)$ are given by (27).

Formulas (17), (20), and (27) have been derived by introducing an additional variable and using the method of transition intensity diagrams (the method of stochastic graph cutsets) [8]. The case $b = 0$ degenerates the incoming correlated MAP request flow into a recurrent one.

The analytical formulas (17), (20), and (27) serve to calculate the numerical characteristics of an MAP request flow with given parameters without involving numerical methods. The graphs of the numerical characteristics presented above match the physical understanding of the service process in this QS.

REFERENCES

1. Cox, D.R., The Analysis of Non-Markovian Stochastic Processes by the Inclusion of Supplementary Variables, *Proceedings of the Cambridge Philosophical Society*, 1955, vol. 51, no. 3, pp. 433–441.

2. Kingman, J.F.C., On Doubly Stochastic Poisson Processes, *Proceedings of the Cambridge Philosophical Society*, 1964, vol. 60, no. 4, pp. 923–930.

3. Basharin, G.P., Kokotushkin, V.A., and Naumov, V.A., On the Equivalent Change Method to Calculate Fragments of Communication Networks. Part 1, *Izv. Akad. Nauk SSSR. Tekh. Kibern.*, 1979, no. 6, pp. 92–99.

4. Basharin, G.P., Kokotushkin, V.A., and Naumov, V.A., On the Equivalent Change Method to Calculate Fragments of Communication Networks. Part 2, *Izv. Akad. Nauk SSSR. Tekh. Kibern.*, 1980, no. 1, pp. 55–61.

5. Neuts, M.F., A Versatile Markov Point Process, *J. Appl. Probab.*, 1979, vol. 16, no. 4, pp. 764–779.

6. Lucantoni, D.M., New Results on the Single Server Queue with a Batch Markovian Arrival Process, *Communications in Statistics. Stochastic Models*, 1991, vol. 7, no. 1, pp. 1–46.

7. Lucantoni, D.M. and Neuts, M.F., Some Steady-State Distributions for the MAP/SM/1 Queue, *Communications in Statistics. Stochastic Models*, 1994, vol. 10, no. 3, pp. 575–598.

8. Vishnevsky, V.M., Dudin, A.N., and Klimenok, V.I., *The Theory of Queuing Systems with Correlated Flows*, Heidelberg: Springer, 2020.

9. Gortsev, A.M. and Solov'ev, A.A., The Probability of Error in Estimating the States of Physical Event Flows, *Izv. Vuzov. Fizika*, 2016, vol. 59, no. 5, pp. 54–60.

10. Gortsev, A.M., Nezhel'skaya, L.A., and Solov'ev, A.A., Optimal State Estimation in MAP Event Flows with Unextendable Dead Time, *Autom. Remote Control*, 2012, vol. 73, no. 8, pp. 1316–1326.

11. Gortzev, A.M. and Soloviev, A.A., The Joint Density of Probability for Intervals of MAP Event Flows and Conditions of Its Recurrence, *Tomsk State Univ. J. Control Comp. Sci.*, 2012, no. 3(20), pp. 32–41.

12. Gortzev, A.M. and Solovev, A.A., Comparison of ML- and M-Estimates for Unextandable Dead Time in MAP Event Flow, *Tomsk State Univ. J. Control Comp. Sci.*, 2015, no. 4(33), pp. 13–22.

13. Gortsev, A.M. and Solov'ev, A.A., The Joint Probability Density of Physical Event Flow Intervals with Unextendable Dead Time, *Izv. Vuzov. Fizika*, 2014, vol. 57, no. 7, pp. 103–111.

14. Gortsev, A.M. and Solov'ev, A.A., A Maximum Likelihood Estimate for Unextendable Dead Time in a Physical Event Flow, *Izv. Vuzov. Fizika*, 2015, vol. 58, no. 11, pp. 141–149.

15. Bin Sun, Dudin, S.A., Dudina, O.S., and Dudin, A.N., A Customer Service Model in an Adaptive-Modulation Mobile Communication Cell with Allowance for Random Environment, *Autom. Remote Control*, 2021, vol. 82, no. 5, pp. 812–826.

16. Gortsev, A.M. and Nezhelskaya, L.A., Analytical Investigation of a Single-Channel QS with Incoming Asynchronous Event Flow, *Autom. Remote Control*, 2022, vol. 83, no. 8, pp. 1200–1212.

17. Nezhel'skaya, L.A., *Otsenka sostoyanii dvazhdy stokhasticheskikh potokov sobytii* (State Estimation for Doubly Stochastic Event Flows), Tomsk: Tomsk State Univ., 2020.

18. Medvedev, G.A., Analysis of Stochastic Graphs Describing the Behavior of Stepped Automatic Search Systems, *Avtomat. Vychisl. Tekh.*, 1968, no. 4, pp. 27–30.

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