

# On Preset Homing and Synchronizing Sequences for Observable Input/Output Automata

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**Abstract**—The paper is devoted to the problem of deriving synchronizing and homing experiments for nondeterministic Input/Output automata; corresponding input sequences are widely used in testing (non-initialized) discrete event systems. In active testing, there is an opportunity to set a system under test into a known initial state; in passive testing, a known current state allows to reduce the number of properties to be checked. In the paper, we note that such experiments for Input/Output automata are different from so-called “gedanken” experiments with classical Finite State Machines; the existence check conditions of such experiments are established for a predefined discipline of applying inputs and a method for its derivation is proposed when such an experiment exists. The obtained results allow to reduce the problem of deriving synchronizing and homing experiments for Input/Output automata to the well developed problem of deriving such experiments for appropriate classes of Finite State Machines.

*Keywords:* finite input/output automaton, homing sequence, synchronizing sequence

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## 1. INTRODUCTION

The state identification problem for Finite State Machines (FSMs) and Automata is actively studied nowadays. It is widely used in various applications, and in particular, for minimizing the verification efforts in active and passive testing [1, 4, 5]. In the active testing mode, state identification sequences allow to reduce length of a test suite. In [5], the authors demonstrate how homing and synchronizing sequences can accelerate/optimize passive testing of telecommunication components. The state identification usually is based on so-called *gedanken* experiments with FSMs [1] and has the following steps: the input sequence application to an FSM under test, the output response observation and the conclusion drawing about an initial or a current state of the FSM. Homing and synchronizing experiments allow determining the final/current FSM state, i.e., the state after the experiment. When performing a synchronizing experiment there is no need to observe output responses of an FSM under test, while in the homing experiment the current state is uniquely determined based on the FSM output response. If an applied input sequence is fixed in advance and is not modified during the experiment then the experiment is called *preset*. In this case, it is possible to consider corresponding homing and synchronizing sequences and there are many papers in which such sequences are studied for deterministic and nondeterministic FSMs, complete and partial FSMs (see, for example, [2, 7]).

At the same time, it should be noticed that the FSMs have limited expressivity when it comes to describing properties of telecommunication components. Instead, Input/Output automata [3, 8] are more generic, and are also widely used for test derivation. In an Input/Output Automaton, transitions between states are labeled not by a pair  $\langle \text{input}, \text{output} \rangle$  but by an action, which can be an input or an output. In [6], the Input/Output automaton state identification has been studied for a special class of such automata, namely for the case when at every state only inputs or outputs are allowed. In this paper, the approach is extended to automata with states where both, inputs and outputs are allowed to be accepted and/or produced. The main differences are the following: the class of considered automata is much wider and correspondingly a preset experiment is defined over automata traces rather than over input sequences. Moreover, a special class of so called *stable* homing sequences is considered. These sequences are of use in active testing and correspond to complete traces, i.e., traces which take an automaton to a stable state where there are no transitions under outputs. The main contribution of the paper is therefore the definition of synchronizing and homing *experiments* for finite Input/Output automata and the adaptation of the known techniques for the existence check and derivation of such experiments.

The paper is organized as follows. Section 2 contains the definitions and notations used in the paper. In Section 3, the notions of synchronizing and homing experiments are introduced while Section 4 is devoted to the existence check and derivation of such experiments. Section 5 concludes the paper and presents the future work.

## 2. DEFINITIONS AND NOTATIONS

A finite Input/Output *Automaton* an Automaton for short, is a 4-tuple  $\mathbf{S} = (S, I, O, h_S)$  where  $S$  is a finite nonempty set of states,  $I$  is a finite set of inputs,  $O$  is a finite set of outputs,  $I \cap O = \emptyset$ , and  $h_S \subseteq (S \times (I \cup O) \times S)$  is the transition relation. For practical reasons, the set  $(I \cup O)$  is assumed to be non-empty. An automaton  $\mathbf{S}$  is *observable* if at each state at most one transition under any action is defined. An automaton  $\mathbf{S}$  is *non-initialized* if any state can be an initial state. In this paper, observable non-initialized automata are considered.

There is a transition from state  $s$  to state  $s'$  under action  $a$  if and only if the triple  $(s, a, s') \in h_S$  is in the transition relation  $h_S$ . An automaton is *nondeterministic* if there exists a state at which several transitions under outputs are defined [6] and further only observable possibly nondeterministic automata are considered if the contrary is not explicitly stated. An automaton is a trace model where a *trace* at state  $s$  is a sequence of inputs and outputs of the set  $(I \cup O)$  defined at this state. The set of all states where there are no transitions under outputs, is denoted  $S_{st}$ ; such states are called *stable* since the automaton can stay at this state infinitely long until an input is applied. In particular, an automaton can have states where no transitions are defined; the set of such states is denoted as  $S_{und}$ . A state of the automaton is a *mixed* state if at this state there are transitions under inputs and outputs.

A trace at state  $s$  is *complete* if it takes the automaton to a state from the set  $S_{st}$ . For the observation of such traces a special output  $\delta \notin I \cup O$  (*quiescence*) is introduced [8]; in other words, at each state where there are no transitions under outputs, a loop labeled by  $\delta$  is added and this symbol (action) is considered as an output, i.e., the automaton  $\mathbf{S}^\delta$  is obtained. The automaton  $\mathbf{S}^\delta$  has the output alphabet  $O \cup \{\delta\}$ . Thus, a trace  $\sigma$  of the automaton  $\mathbf{S}$  is complete at state  $s$  if and only if the automaton  $\mathbf{S}^\delta$  has a trace  $\sigma\delta$  at state  $s$ ; the latter means that this trace cannot be appended with any output of the set  $O$  and such traces are called  $\delta$ -traces. By definition, given a trace of the automaton  $\mathbf{S}^\delta$ , a trace of the automaton  $\mathbf{S}$  is obtained after erasing all symbols  $\delta$ , and vice versa, given a trace  $\sigma$  of the automaton  $\mathbf{S}$ , a trace of the automaton  $\mathbf{S}^\delta$  is obtained after adding to  $\sigma$  any number of  $\delta$  actions after every prefix that is a complete trace.

3. HOMING EXPERIMENTS WITH INPUT/OUTPUT AUTOMATA

Similar to FSMs [1], “gedanken” experiments with automata have three steps: an input sequence is applied to an automaton under test, that in our case, can be the empty sequence, a produced output sequence is observed, that also can be the empty sequence, and the conclusion is drawn about some properties of the automaton. The experiment is *synchronizing* or *homing* if after the experiment the current automaton state is known. Differently from FSMs, in general case, “gedanken” experiments with automata cannot be described as sets of finite input sequences with possible output responses. The reason is that for the same input sequence there can exist different traces with the same output projection. For example, for an input sequence  $i_1 i_2$  there can be traces  $i_1 o_1 i_2 o_2$  and  $i_1 i_2 o_1 o_2$ . In the former case, input  $i_2$  is applied only after getting a response  $o_1$  before the proper timeout  $T_{in}$  expires, while in the latter case, input  $i_2$  is applied during the  $T_{in}$  and  $o_1$  cannot be produced before input  $i_2$  is applied. Correspondingly, an input sequence should be applied under proper conditions. If the initial state when an input sequence is applied is unknown then the conditions of the application of an input sequence have to be held for every initial state.

For formal representation of the conditions for the application of an input sequence, a special input/symbol  $\omega$  is introduced and a copy  $s'$  is created for every state  $s$  of the automaton  $\mathbf{S}^\delta$  where all the transitions from state  $s$  under outputs are added. At state  $s$  there are only inputs and an artificial input  $\omega$  that takes the automaton from state  $s$  to its copy  $s'$ . As the result, the automaton  $\mathbf{S}^{\delta\omega}$  is obtained. Correspondingly, given a trace of the automaton  $\mathbf{S}^{\delta\omega}$ , a trace of  $\mathbf{S}$  is obtained after deleting  $\delta$  and  $\omega$  actions, and vice versa, given a trace  $\sigma$  of the automaton  $\mathbf{S}$ , if any number of  $\delta$  actions are added to  $\sigma$ -prefix that is a complete trace while adding  $\omega$  in front of every output including  $\delta$ , then a trace of  $\mathbf{S}^{\delta\omega}$  is obtained.

Given an automaton  $\mathbf{S}$ , “gedanken” experiments with it can be described using traces which have inputs and outputs and also artificial symbols  $\delta$  and  $\omega$ . There are proper timeouts for these symbols: a timeout  $T_{in}$  for  $\omega$  and a timeout  $T_{out}$  for  $\delta$ . When there is  $\omega$  action in an input sequence no input is applied while an output is expected during the timeout  $T_{out}$ ; if there is no output then it is assumed that the automaton produced the output  $\delta$ .

As usual, an automaton is assumed to have at least two states since there is no homing problem for an automaton with a single state. The state identification experiment for setting an automaton under test into a known state is performed as follows. An input sequence for the experiment has the following shape  $\alpha = \omega^{t_1} i_1 \dots \omega^{t_k} i_k \omega^{t_{k+1}}$ ,  $i_j \in I$ ,  $j = 1, \dots, k$ . If at a current moment, an input  $i_j \in I$ ,  $j = 1, \dots, k$ , should be applied then the tester applies this input during the time interval  $T_{in}$ , and after this, the timer is reset. If there is  $\omega$  in the input sequence then the tester is waiting for an output. If an output is produced then the timer is reset and the next input of  $\alpha$  is analyzed. If no output is produced during  $T_{out}$ , then the system is supposed to produce  $\delta$  and the timer is reset.

Let  $\alpha = \omega^{t_1} i_1 \dots \omega^{t_k} i_k \omega^{t_{k+1}}$ ,  $i_j \in I$ ,  $j = 1, \dots, k$ , be a sequence with inputs and  $\omega$ . A trace  $\sigma$  at state  $s$  of the automaton  $\mathbf{S}^\delta$  is *compatible* with  $\alpha$  if  $\sigma$  has the shape  $\beta_1 i_1 \dots \beta_k i_k \beta_{k+1}$  where  $\beta_j$  is a sequence of length  $t_j$  containing outputs and  $\delta$ ,  $j = 1, \dots, k + 1$ . For Input/Output automata, a homing sequence can be defined in two ways; these definitions correspond to active and passive testing modes. In the active testing mode, a homing sequence has to set a system under test into a known state where further test cases can be applied at any following moment. Therefore, a homing sequence has to take the automaton to a stable state. In the passive testing mode, a homing sequence can take the automaton under test to any state. Correspondingly, two definitions of a homing sequence are proposed.

A sequence  $\alpha \in (I \cup \{\omega\})^*$  is *homing* for the automaton  $\mathbf{S}$  if 1) at each state of  $\mathbf{S}^\delta$ , there exists a trace compatible with  $\alpha$  and 2) for any two different states  $s_1, s_2$  and a common trace  $\sigma$  at these states that is compatible with  $\alpha$ ,  $\sigma$  takes the automaton from states  $s_1$  and  $s_2$  to the same state.

If  $\alpha$  is a homing sequence for  $\mathbf{S}$  and there exists a state  $s$  such that every trace  $\sigma$  that is compatible with  $\alpha$  takes the automaton from any state to state  $s$ , then  $\alpha$  is a *synchronizing* sequence for  $\mathbf{S}$ .

A sequence  $\alpha$  is a *stable homing* sequence for the automaton  $\mathbf{S}$  if 1) at every state of  $\mathbf{S}^\delta$  there exists a trace compatible with  $\alpha$  and only complete traces are compatible, 2) for any two different states  $s_1, s_2$  and a common trace  $\sigma$  at these states that is compatible with  $\alpha$ ,  $\sigma$  takes the automaton from states  $s_1$  and  $s_2$  to the same state. In this case, after the application of a homing sequence, the automaton is at a stable state where it can stay infinitely long, for example, until a further test case is applied. If  $\alpha$  is a stable homing sequence for  $\mathbf{S}$  and there exists a state  $s$  such that any complete trace  $\sigma$  compatible with  $\alpha$  takes the automaton from any state to state  $s$ , then  $\alpha$  is a *stable synchronizing* sequence for  $\mathbf{S}$ . By definition of a homing/synchronizing sequence, the following proposition holds.

**Proposition 1.** 1. *An automaton has no homing (synchronizing) sequence if for each input sequence  $\gamma \in I^*$  there exists a state where there is no trace with such input projection.* 2. *An automaton has no stable homing (synchronizing) sequence if for each input sequence  $\gamma \in I^*$  there exists a state where there is no complete trace with such input projection.* 3) *An automaton with at least two states has no (stable) homing (synchronizing) sequence if the set  $S_{und}$  has more than one state.* 4) *An automaton has no stable homing (stable synchronizing) sequence if at each state at least one output is defined that is different from  $\delta$ , i.e., the set  $S_{st}$  is empty.*

If there are no transitions in  $\mathbf{S}$  labeled by an output of the set  $O$  then  $\mathbf{S}$  becomes a classical automaton without outputs for which only synchronizing sequences are considered [7]. If there are no transitions in  $\mathbf{S}$  labeled by an input of the set  $I$  then for an observable automaton a stable homing/synchronizing sequence is a sequence that has only  $\omega$ , i.e., the automaton only produces outputs with the interval  $T_{out}$ . Such a sequence becomes stable for  $\mathbf{S}$  if and only if a trace at any state of  $\mathbf{S}^\delta$  compatible with such a sequence is complete and for any two states  $s_1$  and  $s_2$  and any common output sequence  $\beta$  at these states,  $\beta$  takes the automaton from states  $s_1$  and  $s_2$  to the same state.

Let  $\alpha$  be a homing sequence for an observable automaton  $\mathbf{S}$  and  $b \in I \cup \{\omega\}$ . If at each state for  $ab$  ( $b\alpha$ ) there exists a trace of  $\mathbf{S}^\delta$  compatible with  $ab$  ( $b\alpha$ ) then  $ab$  ( $b\alpha$ ) is also a homing sequence. The same proposition holds for stable homing sequences.

**Proposition 2.** *If  $\alpha$  is a homing sequence of the automaton  $\mathbf{S}$ , then sequences  $\alpha b$  and  $b\alpha$ ,  $b \in I \cup \{\omega\}$ , for which at each state of  $\mathbf{S}^\delta$  there exists a trace compatible with  $\alpha b$  ( $b\alpha$  respectively), are also homing sequences of the automaton  $\mathbf{S}$ .* 2. *If  $\alpha$  is a stable homing sequence of the automaton  $\mathbf{S}$ , then sequences  $\alpha b$  and  $b\alpha$  for which at each state of  $\mathbf{S}^\delta$  there exists a trace compatible with  $\alpha b$  ( $b\alpha$  respectively) and all such compatible traces are complete traces, are also stable homing sequences for the automaton  $\mathbf{S}$ .*

#### 4. CHECKING THE EXISTENCE AND DERIVING HOMING AND SYNCHRONIZING SEQUENCES FOR INPUT/OUTPUT AUTOMATA

Similar to [6], when deriving homing/synchronizing sequences, an automaton  $\mathbf{S}$  is transformed into an FSM in order to use the well known state identification methods for FSMs. The FSM  $M_S^{\delta\omega}$  has all the states of the automaton  $\mathbf{S}$  and is derived as follows:

- given an input  $i \in I$ , there is a transition from state  $s$  to state  $q$  with output  $\delta$  if and only if the initial automaton has a transition at state  $s$  to state  $q$  labeled by input  $i$ ;
- given an input  $\omega \notin I$ , there is a transition from state  $s$  to state  $q$  with output  $o$  if and only if the initial automaton has a transition at state  $s$  to state  $q$  labeled by output  $o$ ;
- there is a loop at state  $s$  of the FSM labeled by input/output pair  $\omega/\delta$  if and only if  $s \in S_{st}$ .

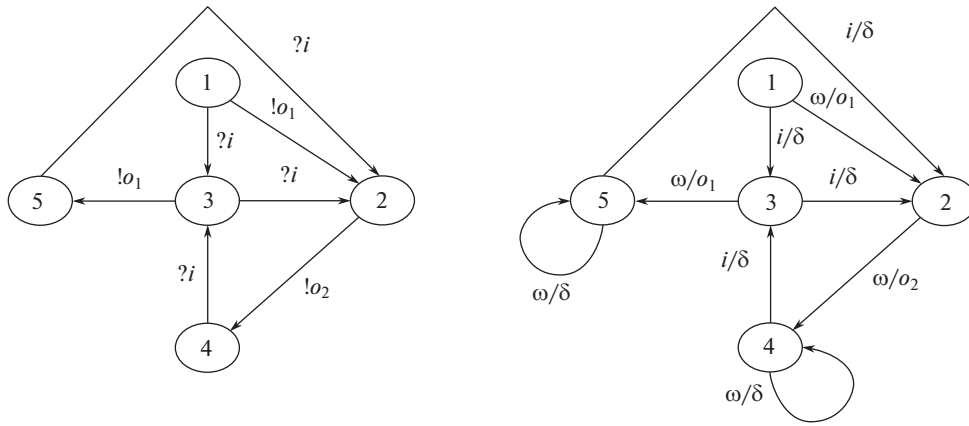


Fig. 1. Automaton **S** and corresponding FSM  $M_S^{\delta\omega}$ .

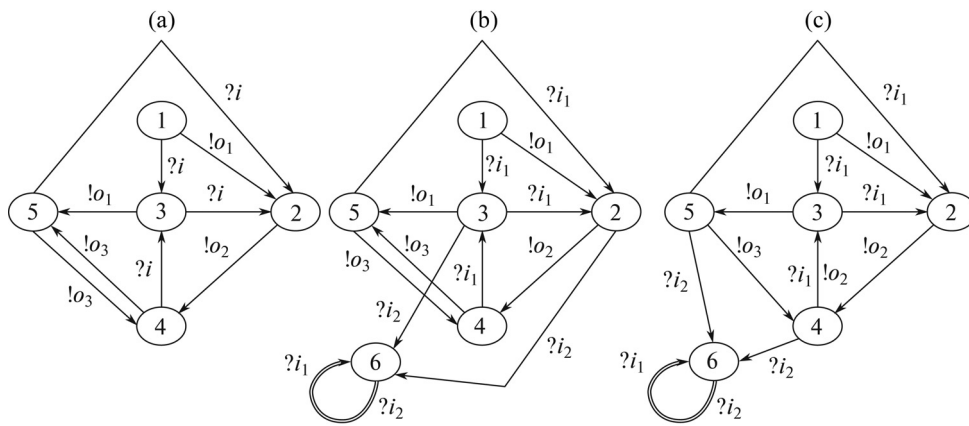


Fig. 2. Automata with (stable) homing sequences.

It is shown that the automaton **S** has a homing (synchronizing) sequence if and only if the FSM  $M_S^{\delta\omega}$  has such a sequence with appropriate features.

Consider the automaton **S** in Fig. 1 for which FSM  $M_S^{\delta\omega}$  is constructed. We derive a homing sequence for this automaton. This sequence cannot be headed by input  $i$  since the transition under this input is not defined at state 2. Correspondingly, at the beginning input  $\omega$  is applied, i.e., in fact, no input is applied. An output  $o_2$  is expected when the initial automaton reaches state 4 or  $o_1$  when the initial automaton reaches state 5 or 2, or, after the output timeout, the current state is 4 or 5. Since a transition under  $i$  is not defined at state 2, another output is expected after the output timeout. In this situation, it is known that the automaton is at state 4 (output  $o_2$ ) or it stayed at state 5, or is at state 4 or 5. An input  $i$  is applied and the automaton reaches state 2 or 3, and after that another output is expected (input  $\omega$ ). If output  $o_2$  is produced, then the automaton reached state 4, if output  $o_1$  is produced then the automaton reached state 5. Thus, the automaton has a homing sequence  $\omega wiw$  that in fact, is a stable homing sequence, since at each state every trace compatible with  $\omega wiw$  is complete.

However, if the initial automaton has some states which are not stable then not every homing sequence of the FSM  $M_S^{\delta\omega}$  is stable homing, since such a sequence has to take the automaton to a stable state of the FSM; nevertheless, such an automaton can have a stable homing sequence. In Fig. 2a there is an example of an automaton for which a sequence  $\omega wiw$  is homing but the automaton has no stable homing sequence. The automaton in Fig. 2b has homing sequences  $\omega wi_1w$  and  $\omega wi_1i_2$  but only the latter is a stable homing sequence. The automaton in Fig. 2c has a stable

homing sequence  $\omega\omega i_1\omega i_2$  that is a prolongation of a homing sequence  $\omega\omega i_1\omega$  that is not stable homing. When deriving a stable homing sequence the notion of an  $S_{st}$ -homing sequence is utilized ( $S'$ -homing sequence [7]).

A homing sequence  $\gamma$  of the FSM  $M_S^{\delta\omega}$  is an  $S_{st}$ -homing sequence if all traces of  $M_S^{\delta\omega}$  with the input projection  $\gamma$  and the same output projection, take the FSM to the same state of the set  $S_{st}$  where  $S_{st}$  is the set of all stable states of the automaton  $\mathbf{S}$ , i.e., all the states where there are no transitions under output actions. If all the traces of the FSM  $M_S^{\delta\omega}$  with the input projection  $\gamma$  take the FSM to the same state of the set  $S_{st}$  independently of the initial state and output projection then an  $S_{st}$ -homing sequence is an  $S_{st}$ -synchronizing sequence. If an FSM has no homing (synchronizing) sequence then the FSM has no  $S_{st}$ -homing ( $S_{st}$ -synchronizing) sequence; the converse is not always true.

**Proposition 3.** 1. An automaton  $\mathbf{S}$  has a homing sequence if and only if the FSM  $M_S^{\delta\omega}$  has a homing sequence. 2. An automaton  $\mathbf{S}$  has a stable homing sequence if and only if the FSM  $M_S^{\delta\omega}$  has a  $S_{st}$ -homing sequence.

**Proof.** Consider a trace  $\beta_1\alpha_1\dots\beta_k\alpha_k\beta_{k+1}$  of the automaton  $\mathbf{S}^\delta$  over alphabets  $I$  and  $O$  where  $\alpha_1, \dots, \alpha_k$  are input sequences,  $\beta_1, \dots, \beta_k, \beta_{k+1}$  are output sequences which can have action  $\delta$  as an output and every sequence can be the empty sequence. By construction of the FSM  $M_S^{\delta\omega}$ , the automaton  $\mathbf{S}$  has a transition from state  $s$  to state  $s'$  under output  $o$  if and only if such a transition labeled by  $\omega/o$  exists in the FSM  $M_S^{\delta\omega}$ . The automaton  $\mathbf{S}$  has a transition from state  $s$  to state  $s'$  under input  $i$  if and only if such a transition labeled by  $i/\delta$  exists in the FSM  $M_S^{\delta\omega}$ . Therefore, at state  $s$ , the automaton  $\mathbf{S}$  has a trace  $\beta_1\alpha_1\dots\beta_k\alpha_k\beta_{k+1}$  that takes the automaton to state  $s'$  if and only if FSM  $M_S^{\delta\omega}$  has a trace  $\omega^{|\beta_1|}/\beta_1.\alpha_1/\delta^{|\alpha_1|}\dots\omega^{|\beta_k|}/\beta_k.\alpha_k/\delta^{|\alpha_k|}.\omega^{|\beta_{k+1}|}/\beta_{k+1}$  that takes the FSM from state  $s$  to state  $s'$  where  $i_1i_2\dots i_k/o_1o_2\dots o_k$  denotes the sequence  $i_1/o_1, i_2/o_2, \dots, i_k/o_k$ .

$\Leftarrow$  Let the FSM  $M_S^{\delta\omega}$  have a homing sequence  $\omega^{t_1}i_1\dots\omega^{t_k}i_k\omega^{t_{k+1}}$ ,  $i_j \in I, j = 1, \dots, k$ . By definition of a homing sequence, the sequence  $\omega^{t_1}i_1\dots\omega^{t_k}i_k\omega^{t_{k+1}}$  is defined at each state of the FSM  $M_S^{\delta\omega}$  and for very output sequence  $\beta_1\delta\dots\beta_k\delta\beta_{k+1}$  for which a trace  $\omega^{|\beta_1|}/\beta_1.i_1/\delta\dots\omega^{|\beta_k|}/\beta_k.i_k/\delta\omega^{|\beta_{k+1}|}/\beta_{k+1}$  exists at least at one state  $s$  of the FSM, there exists a state  $s'$  such that  $\omega^{|\beta_1|}/\beta_1.i_1/\delta\dots\omega^{|\beta_k|}/\beta_k.i_k/\delta.\omega^{|\beta_{k+1}|}/\beta_{k+1}$ -successor of any state  $s$  is either the empty set or a singleton  $\{s'\}$ . Thus,  $\omega^{|\beta_1|}/\beta_1.i_1/\delta\dots\omega^{|\beta_k|}/\beta_k.i_k/\delta.\omega^{|\beta_{k+1}|}/\beta_{k+1}$  is a homing sequence for the automaton  $\mathbf{S}$ .

$\Rightarrow$  Let now the automaton  $\mathbf{S}$  have a homing sequence  $\alpha = \omega^{t_1}i_1\dots\omega^{t_k}i_k\omega^{t_{k+1}}$ . Then by definition, at each state of the automaton  $\mathbf{S}^\delta$  there exists a trace compatible with  $\alpha$ ; and for any two states  $s_1, s_2$  and a common trace  $\sigma$  at these states that is compatible with  $\alpha$ ,  $\sigma$  takes the automaton from states  $s_1$  and  $s_2$  to the same state. In other words, for each trace  $\sigma$  compatible with  $\alpha$ , the final state is uniquely determined independently from the initial state but these states can be different for traces  $\sigma$  with different output projections.

By definition of the FSM  $M_S^{\delta\omega}$ , if at each state of the automaton there exists a trace compatible with  $\omega^{t_1}i_1\dots\omega^{t_k}i_k\omega^{t_{k+1}}$ , then the FSM behavior is defined at each state under this input sequence and moreover, for each output response  $\beta_1\delta\dots\beta_k\delta\beta_{k+1}$  to this sequence there exists a state  $s'$  such that once  $\omega^{t_1}/\beta_1.i_1/\delta\dots\omega^{t_k}/\beta_k.i_k/\delta.\omega^{t_{k+1}}/\beta_{k+1}$  is a trace at least at one state  $s$  of the automaton, it holds that the  $\omega^{t_1}/\beta_1.i_1\dots\omega^{t_k}/\beta_k.i_k.\omega^{t_{k+1}}/\beta_{k+1}$ -successor of such state  $s$  is either the empty set or the singleton  $\{s'\}$ . Therefore,  $\omega^{t_1}i_1\dots\omega^{t_k}i_k\omega^{t_{k+1}}$  is a homing sequence for the FSM  $M_S^{\delta\omega}$ .

Property 2 of the proposition can be proven in the same way but only complete traces are considered in the automaton while in the FSM only traces which take the FSM to states of the set  $S_{st}$  are considered.

We are not aware of any technique for deriving an  $S_{st}$ -homing sequence for an FSM; this technique can be easily developed based on the prolongation of an FSM successor tree [5], the root of

which is labeled by the set of all pairs of different states up to the derivation of singletons  $\{s\}$ ,  $s \in S_{st}$ .

Similar to Proposition 3, the following proposition can be proven.

**Proposition 4.** 1. *The automaton  $\mathbf{S}$  has a synchronizing sequence if and only if  $M_S^{\delta\omega}$  has a synchronizing sequence.* 2. *The automaton  $\mathbf{S}$  has a stable synchronizing sequence if and only if  $M_S^{\delta\omega}$  has a  $S_{st}$ -synchronizing sequence.*

Finally, it is necessary to notice that the length and complexity estimates of deriving homing/synchronizing sequences for Input/Output automata are the same as for FSMs. It is interesting to describe automata classes for which there exists a homing/synchronizing sequence of polynomial length.

## 5. CONCLUSION

In this paper, the problem of deriving homing and synchronizing experiments for Input/Output automata is discussed. The notion of “gedanken” experiments is introduced which in fact is different from that for FSMs and a method for deriving homing and synchronizing experiments is proposed, based on the construction of an FSM with the same set of traces. The complexity estimates of such sequences coincide with those for appropriate classical FSMs.

Since for FSMs, the adaptivity can sometimes reduce the complexity of the existence check and derivation of homing and synchronizing sequences [2], as a future work the authors plan to consider adaptive homing and synchronizing experiments for Input/Output automata as well as extracting appropriate classes with “good” complexity estimates for such experiments.

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## REFERENCES

1. Gill, A., *Vvedenie v teoriyu konechnykh avtomatov* (Introduction to the Theory of Finite-State Machines), Moscow: Nauka, 1966.
2. Evtushenko, N.V. and Kushik, N.G., *Nekotorye zadachi identifikatsii sostoyanii dlya nedeterminirovannykh avtomatov* (Some Problems of Identifying the States of Non-Deterministic Finite State Machines), Tomsk: STT Publishing, 2018.
3. Burdonov, I., Yevtushenko, N., Kossatchev, A., Separating Input/Output Automata with Nondeterministic Behavior, *Rus. Digital Librar. J.*, 2020, vol. 23, no. 4, pp. 634–655.
4. Hennie, F., Fault Detecting Experiments for Sequential Circuits, *5th Annual Symposium on Switching Circuit Theory and Logical Design*, Princeton, New Jersey, USA, November 11–13, 1964, pp. 95–110.
5. Kushik, N., López, J., Cavalli, A., and Yevtushenko, N., Improving Protocol Passive Testing through “Gedanken” Experiments with Finite State Machines, *2016 IEEE International Conference on Software Quality, Reliability and Security, QRS 2016*, Vienna, Austria, August 1–3, 2016, pp. 315–322.
6. Kushik, N., Yevtushenko, N., Burdonov, I., and Kossachev, A., Synchronizing and Homing Experiments for Input/Output Automata, *Syst. Inform.*, 2017, no. 10, pp. 1–10.
7. Sandberg, S., Homing and Synchronizing Sequences, *Model-Based Testing Of Reactive Systems, Advanced Lectures* [The Volume Is the Outcome of a Research Seminar That Was Held in Schloss Dagstuhl in January 2004], 2004, pp. 5–33.
8. Tretmans, J., A Formal Approach to Conformance Testing, *Protocol Test Systems, VI, Proceedings of the IFIP TC6/WG6.1 Sixth International Workshop on Protocol Test Systems*, Pau, France, 28–30 September, 1993, pp. 257–276.

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