ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2023, Vol. 84, No. 6, pp. 671–684.
Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, 2023.

Observation-Based Filtering of State of a Nonlinear Dynamical System with Random Delays

A. V. Bosov

Institute of Informatics Problems of the Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences, Moscow, Russia
e-mail: ABosov@frccsc.ru

Received December 21, 2022
Revised February 14, 2023
Accepted March 30, 2023

Abstract—We present a model of a stochastic observation system that allows for time delays between the received observation and the actual state of the observed object that formed these observations. Such delays can occur when observing the movement of an object in a water medium using acoustic sonars and have a significant impact on the accuracy of position tracking. We present equations to solve the optimal mean square filtering problem. Since the practical use of the optimal solution is barely feasible due to its computational complexity, we pay the main attention to an alternative, suboptimal but computationally efficient approach. Specifically, we adapted a conditional minimax nonlinear filter (CMNF) to the proposed model and formulated sufficient existence conditions for its estimate. We conducted a computational experiment on a model that is close to practical needs. The results of the experiment show the effectiveness of CMNF in the model considered. However, they also show a significant decrease in the quality of estimation compared to the model without random observation delays, which can be considered as a motivation for further research into the model and related problems.

Keywords: nonlinear stochastic observation system, observation with random delays, conditional minimax nonlinear filter, simulation modeling

DOI: 10.25728/arcRAS.2023.50.90.001

1. INTRODUCTION

One of the most versatile tools for describing the behavior of systems that change over time are stochastic dynamic system models. If information about the unknown state of the system is only available through indirect observations, then the key to solving almost any problem is the filtering problem, that is, estimating the current state of the system with the occurrence of the subsequent observation. The solution of the filtering problem, which is important in itself, provides a means of solving other problems such as analysis (e.g., parameter identification) and synthesis (e.g., feedback control) of stochastic systems. These problems have remained relevant for many years for a variety of applications and fundamental research. Among numerous applications, some of the most significant examples of the use of stochastic filtering methods involve tracking and navigation problems [1]. The traditional context for such applications is aircraft (manned or unmanned) and radar observation equipment. In recent years, the same problems have been investigated more frequently in another applied context involving autonomous underwater vehicles (AUVs) or "underwater drones" [2]. This class of robotic systems has many different applications and most of them require solving tracking and navigation problems [3–5]. Well-known methods of stochastic filtering from the Kalman filter [6] and its non-linear suboptimal analogs like the extended Kalman filter [7] to modern trendy concepts of sigma-point filters (the pioneering work on this topic is [8], a well-executed overview is [9]) are applicable to the models of these systems.

However, significant details are revealed for problems related to the water medium. For example, a well-known property of acoustic sensors is the dependence of sound speed on temperature, salinity, and water pressure [10]. This property is well explored in [11] in the problem of building a robust navigation system that combines measurement data from acoustic sensors with information from other sensors in the onboard inertial navigation system. The study [12] applies a similar model and approach, but with a focus on the performance of using Doppler sensors. This work suggests the possibility of investigating an observer model that takes into account observation delays depending on the distance to the object. Indeed, when modeling the movement of underwater vehicles, in particular autonomous vehicles and typical means of observing them, observations of the position of the vehicle are considered immediately available, and the time delay between initiating the next measurement cycle by the sensor and receiving the result of this measurement is not taken into account. Traditionally, they ignore this delay, which is entirely justified for aircraft and various radars and is physically understandable. In addition, models with time delay are not very popular because deterministic delay does not introduce any new quality into the filtering problem. Finally, proposing a meaningful model requires understanding the nature of the delay. The movement of underwater vehicles, unlike aircraft, provides substantive answers to these questions. Specifically, since the sensors use sound waves that are transmitted through the water medium, delays, first of all, can be significant and, secondly, will change along with the distance to the observed object. Based on these premises, this paper proposes a fundamentally new model of a stochastic dynamic observation system, which is described in Section 2. Sections 3 and 4 present solutions to the filtering problem—optimal and conditional minimax filters [13]. Section 5 discusses the results of the numerical experiment that models the movement of an AUV in a plane with constant speed, for which acoustic observations are made from two remote points (range and angle to the target are measured). We highlight the fundamental difference in the effectiveness of estimating the position of the vehicle from observations with and without delay. We also discuss the perspectives for solving other problems related to the model under consideration, in particular, the possibility of identifying motion parameters.

2. MODEL OF DYNAMICAL SYSTEM WITH RANDOM OBSERVATION DELAYS

We describe the observation system in discrete time $t = -T, -T+1, \ldots, 0, 1, \ldots$. We assume the filtering process starts at $t = 0$, when it is required to acquire the first state estimation, but the system started its evolution some time T earlier. The state $x_t \in \mathbb{R}^{p_x}$ of the system is described with standard form difference equations:

$$x_t = \varphi_t\left(x_{t-1},\ w_t\right), \quad x_{-T-1} = \eta, \tag{1}$$

where $w_t \in \mathbb{R}^{p_w}$ is discrete white noise that models perturbations, and $\eta \in \mathbb{R}^{p_x}$ is initial condition.

The observation time delay value τ_t is modeled as a random sequence, the elements of which are discrete random variables with values in the set $\{0, 1, \ldots, T\}$ and is a function of the state x_t :

$$\tau_t = \theta_t\left(x_t\right). \tag{2}$$

Indirect observations $y_t \in \mathbb{R}^{q_y}$ are described by equations of the following form:

$$y_t = \psi_t(x_{t-\tau_t}, v_t), \tag{3}$$

where $v_t \in \mathbb{R}^{q_v}$ is discrete white noise that models measurement errors. We assume that the vectors η, w_t, v_t are mutually independent.

Thus, the only difference between this model and the traditional non-linear observation system with discrete time is the dependence of the current observation y_t on the state calculated before time t by a random variable τ_t. This formulation follows from the above practical interpretation of the problem as tracking the movement of an AUV. We do not aim to propose a model of maximum generality. For example, there is no standalone perturbation in (2), and all observations are delayed by the same amount in (3). If necessary, we can expand the model by considering additional disturbances in (2), simply by expanding its state vector. It is also possible to make τ_t a vector. However, it is more important to form a qualitative idea of the model (1)–(3), even if it is not in the most general form.

Consider the problem of the state x_t estimation based on observations $y_s,\ s = 0, 1, \ldots, t$, the criterion of accuracy $\widehat{x}_t$ is mean-root-square: $E\left\{\|x_t - \widehat{x}_t\|^2\right\}$, $E\{x\}$ is statistical expectation x, $\|x\|$ is common Euclidean norm of the vector x.

Note that there are several reasons to refine both the state (1) and observation (3) models. Therefore, the existence of a solution to the filtering problem is the availability of the second moments of the process x_t, which is ensured by limiting the linear growth of the function φ_t, which is too burdensome for practical purposes. We also require a compromise description for the application of several popular suboptimal filters, such as the extended Kalman filter [7] or Sigma-point filters [8]. Although such filtering methods cannot be applied to this model, the need to replace model (1)–(3) with some diffusion-type alternative is evident. Finally, we need the simplified form (1)–(3) to write the equations of the optimal filter presented in the next section.

3. OPTIMAL FILTRATION

The proposed and possibly only approach to obtaining the optimal filtering equations in the given problem consists of writing the recurrent Bayesian relations for the posterior probability density, which is a typical solution for discrete observation systems [14]. To simplify the representation, we refine the model (1)–(3) as follows:

$$\begin{gathered} x_t = \varphi_t\left(x_{t-1}\right) + w_t, \quad x_{-T-1} = \eta, \\ y_t = \psi_t\left(x_{t-\tau_t}\right) + v_t. \end{gathered} \tag{4}$$

We assume that the probability density functions of the vectors w_t, η, and v_t to be known. To denote these densities, as well as the densities of others, including conditional distributions, we will use the same method. Let the random vector $(x', y')'$, where $x \in \mathbb{R}^p$, $y \in \mathbb{R}^q$, $'$ is the symbol of transposition, has both joint and marginal probability densities. Then, the marginal density of x is denoted by $f_x(X)$, the joint density of x and y is denoted by $f_{x,y}(X, Y)$, and the conditional density of x with respect to y is denoted by $f_{x|y}(X|Y)$ (assuming additionally that $f_y(Y) > 0$). Thus, we use the same lowercase letters to denote the arguments corresponding to the random variables.

Let y^t denote the vector of all observations up to the moment t inclusive, i.e., $y^t = (y_0', \ldots, y_t')'$.

Note that the system with observations (4) is not Markovian, but can be represented as a component of a Markov process by expanding the state vector. Let $\mathbf{x}_t \in \mathbb{R}^{(T+1)p_x}$ be the new state vector defined as follows: $\mathbf{x}_t = \left(x_{t-T}', \ldots, x_{t-1}', x_t'\right)'$, i.e., it includes all states from the moment $t - T$ to the current moment t. Note that we use boldface symbols for the extended state and its corresponding probability characteristics that are used later. The state equations now take the form

$$\begin{aligned} (\mathbf{x}_t)_1^{p_x} &= (\mathbf{x}_{t-1})_{p_x+1}^{2p_x}, \\ &\ldots \\ (\mathbf{x}_t)_{(T-1)p_x+1}^{Tp_x} &= (\mathbf{x}_{t-1})_{Tp_x+1}^{(T+1)p_x}, \\ (\mathbf{x}_t)_{Tp_x+1}^{(T+1)p_x} &= \varphi_t\left((\mathbf{x}_{t-1})_{Tp_x+1}^{(T+1)p_x}\right) + w_t, \end{aligned} \qquad \mathbf{x}_t = \Phi_t\left(\mathbf{x}_t\right) + \mathbf{w}_t, \quad \mathbf{w}_t = \begin{pmatrix} 0 \\ \ldots \\ 0 \\ w_t \end{pmatrix}, \tag{5}$$

where $(\mathbf{x})_i^j$ denotes a sub-vector of vector $\mathbf{x}$ with elements from the ith to the jth. Therefore, we replace the state function φ_t from (4) with Φ_t from (5). The observer function ψ_t will be replaced with Ψ_t:

$$\Psi_t(\mathbf{x}_t) = \left(\psi_t'\left((\mathbf{x}_t)_1^{p_x}\right), \ldots, \psi_t'\left((\mathbf{x}_t)_{Tp_x+1}^{(T+1)p_x}\right)\right)' = \left(\psi_t'(x_{t-T}), \ldots, \psi_t'(x_t)\right)'.$$

Instead of (2), let us define the function $\Theta_t(\mathbf{x}_t)$ that takes the value in the set of unit vectors $\{e_0, \ldots, e_T\}$ of the space $\mathbb{R}^{T+1}$ in such a way that $\Theta_t(\mathbf{x}_t) = e_i$ if $\theta_t(x_t) = i$. This denotation allows us to write the observation equation as follows:

$$y_t = \Theta_t'(\mathbf{x}_t)\, \Psi_t(\mathbf{x}_t) + v_t. \tag{6}$$

Therefore, we have the canonical form of a Markovian observation system with discrete time (5)–(6). We can formally write the recurrent Bayesian relations for the posterior probability density for the conditional distribution of the state $\mathbf{x}_t$ of this system relative to the observations y_s, $s = 0, \ldots, t$, [14]:

$$f_{\mathbf{x}_t|y^t}\left(\mathbf{X}_t\middle|Y^t\right) = \frac{\int f_{\mathbf{x}_t|\mathbf{x}_{t-1}}(\mathbf{X}_t|\mathbf{X}_{t-1})\, f_{\mathbf{x}_{t-1}|y^{t-1}}(\mathbf{X}_{t-1}|Y^{t-1})\, d\mathbf{X}_{t-1} \cdot f_{y_t|\mathbf{x}_t}(Y_t|\mathbf{X}_t)}{\int\int f_{\mathbf{x}_t|\mathbf{x}_{t-1}}(\mathbf{X}_t|\mathbf{X}_{t-1})\, f_{\mathbf{x}_{t-1}|y^{t-1}}(\mathbf{X}_{t-1}|Y^{t-1})\, d\mathbf{X}_{t-1} \cdot f_{y_t|\mathbf{x}_t}(Y_t|\mathbf{X}_t)\, d\mathbf{X}_t}. \tag{7}$$

We write (7) with the assumption that all occurring densities exist. For this reason, certain refinements are required with respect to the transition density $f_{\mathbf{x}_t|\mathbf{x}_{t-1}}(\mathbf{X}_t|\mathbf{X}_{t-1})$, that can be written only using generalized δ-functions. Beyond that, we can refine the form of density $f_{y_t|\mathbf{x}_t}(Y_t|\mathbf{X}_t)$. For this purpose, let us adjust the derivation of (7). While repeating the first steps, let us introduce the posterior density in the following form:

$$\begin{aligned} f_{\mathbf{x}_t|y^t}\left(\mathbf{X}_t\middle|Y^t\right) &= \frac{f_{\mathbf{x}_t,y^t}(\mathbf{X}_t, Y^t)}{f_{y^t}(Y^t)} = \frac{f_{\mathbf{x}_t,y^{t-1},y_t}(\mathbf{X}_t, Y^{t-1}, Y_t)}{f_{y^t}(Y^t)} \\ &= \frac{f_{\mathbf{x}_t,y^{t-1}}(\mathbf{X}_t, Y^{t-1})\, f_{y_t|\mathbf{x}_t}(Y_t|\mathbf{X}_t)}{f_{y^t}(Y^t)}. \end{aligned}$$

Then, for the first multiplier:

$$\begin{aligned} f_{\mathbf{x}_t,y^{t-1}}\left(\mathbf{X}_t, Y^{t-1}\right) &= f_{x_{t-T},\ldots,x_t,y^{t-1}}\left(X_{t-T}, \ldots, X_t, Y^{t-1}\right) \\ &= \int f_{x_{t-T-1},x_{t-T},\ldots,x_t,y^{t-1}}\left(X_{t-T-1}, X_{t-T}, \ldots, X_t, Y^{t-1}\right) dX_{t-T-1} \\ &= \int f_{x_t|\mathbf{x}_{t-1},\ y^{t-1}}\left(X_t\middle|\mathbf{X}_{t-1}, Y^{t-1}\right) f_{\mathbf{x}_{t-1},\ y^{t-1}}\left(\mathbf{X}_{t-1}, Y^{t-1}\right) dX_{t-T-1} \\ &= \int f_{x_t|x_{t-1}}(X_t|X_{t-1})\, f_{\mathbf{x}_{t-1}|y^{t-1}}\left(\mathbf{X}_{t-1}\middle|Y^{t-1}\right) dX_{t-T-1} \cdot f_{y^{t-1}}\left(Y^{t-1}\right) \\ &= \int f_{\mathbf{x}_{t-1}|y^{t-1}}\left(\mathbf{X}_{t-1}\middle|Y^{t-1}\right) dX_{t-T-1} \cdot f_{x_t|x_{t-1}}(X_t|X_{t-1})\, f_{y^{t-1}}\left(Y^{t-1}\right) \\ &= \int f_{\mathbf{x}_{t-1}|y^{t-1}}\left(\mathbf{X}_{t-1}\middle|Y^{t-1}\right) dX_{t-T-1} \cdot f_{w_t}\left(X_t - \varphi_t(X_{t-1})\right) f_{y^{t-1}}\left(Y^{t-1}\right). \end{aligned}$$

We made the last manipulations assuming that $T > 1$. This condition must be met in order for the considered delay model to be meaningful.

The step we made with the first multiplier allows us to use the transition density $f_{x_t|x_{t-1}}(X_t|X_{t-1})$ of the assumed state instead of the transition density $f_{\mathbf{x}_t|\mathbf{x}_{t-1}}(\mathbf{X}_t|\mathbf{X}_{t-1})$ of the expanded state, as well as calculating the $\mathbb{R}^{p_x}$ space integral $\int \cdot \, dX_{t-T-1}$ instead of the $\mathbb{R}^{(T+1)p_x}$ space integral $\int \cdot \, d\mathbf{X}_{t-1}$.

Now, for the second multiplier:

$$
\begin{aligned}
f_{y_t|\mathbf{x}_t}\left(Y_t|\mathbf{X}_t\right) &= \sum_{i=0}^{T} I\left(\Theta_t\left(\mathbf{X}_t\right) = e_i\right) f_{y_t|\mathbf{x}_t}\left(Y_t|\mathbf{X}_t, \Theta_t\left(\mathbf{X}_t\right) = e_i\right) \\
&= \sum_{i=0}^{T} I\left(\theta_t\left(X_t\right) = i\right) f_{y_t|\mathbf{x}_t,\tau_t=i}\left(Y_t|\mathbf{X}_t, \tau_t = i\right) \\
&= \sum_{i=0}^{T} I\left(\theta_t\left(X_t\right) = i\right) f_{v_t}\left(Y_t - \psi_t\left(X_{t-i}\right)\right).
\end{aligned}
$$

Finally we obtain

$$
f_{\mathbf{x}_t|y^t} = \frac{\int f_{\mathbf{x}_{t-1}|y^{t-1}} dX_{t-T-1} \cdot f_{w_t}(X_t - \varphi_t(X_{t-1})) \sum\limits_i I(\theta_t(X_t) = i) f_{v_t}(Y_t - \psi_t(X_{t-i}))}{f_{y^t}\left(Y^t\right) / f_{y^{t-1}}\left(Y^{t-1}\right)}
$$

$$
= \frac{f_{w_t}(X_t - \varphi_t(X_{t-1})) \sum\limits_i I(\theta_t(X_t) = i) f_{v_t}(Y_t - \psi_t(X_{t-i})) \int f_{\mathbf{x}_{t-1}|y^{t-1}} dX_{t-T-1}}{\int f_{w_t}(X_t - \varphi_t(X_{t-1})) \sum\limits_i I(\theta_t(X_t) = i) f_{v_t}(Y_t - \psi_t(X_{t-i})) \int f_{\mathbf{x}_{t-1}|y^{t-1}} dX_{t-T-1} d\mathbf{X}_t}.
$$

We omit the arguments of the densities in the last equality: $f_{\mathbf{x}_t|y^t}(\mathbf{X}_t|Y^t)$ and $f_{\mathbf{x}_{t-1}|y^{t-1}}(\mathbf{X}_{t-1}|Y^{t-1})$, furthermore, we denote the sum $\sum_{i=0}^{T}$ as $\sum_i$ and account for the coefficient $f_{y^t}\left(Y^t\right)/f_{y^{t-1}}\left(Y^{t-1}\right)$ being a normalizing factor.

Therefore, we obtain the following statement.

Theorem 1. *Let the following probability densities exist for the system* (4)*: perturbation* $f_{w_t}(W_t)$, $t = -T, -T+1, \ldots$, *observation errors* $f_{v_t}(V_t)$, $t = 0, 1, \ldots$, *and initial condition* $f_\eta(X_{-T-1})$. *Then the posterior probability density* $\rho_t = \rho_t\left(\mathbf{X}_t|Y^t\right)$ *of the expanded state* $\mathbf{x}_t = \left(x'_{t-T}, \ldots, x'_{t-1}, x'_t\right)'$, $t = 0, 1, \ldots$, *of this system with respect to the observations* $y^t = \left(y'_0, \ldots, y'_t\right)'$, $t = 0, 1, \ldots$, *and the optimal filter estimation* $x^*_t = E\left\{x_t|Y^t\right\}$ *of the state* x_t *lead to the following recurrent equalities being satisfied:*

$$
\begin{gathered}
\rho_t = \frac{f_{w_t}\left(X_t - \varphi_t\left(X_{t-1}\right)\right) \sum\limits_i I\left(\theta_t\left(X_t\right) = i\right) f_{v_t}\left(Y_t - \psi_t\left(X_{t-i}\right)\right) \int \rho_{t-1} dX_{t-T-1}}{\int f_{w_t}(X_t - \varphi_t(X_{t-1})) \sum\limits_i I(\theta_t(X_t) = i) f_{v_t}(Y_t - \psi_t(X_{t-i})) \int \rho_{t-1} dX_{t-T-1} d\mathbf{X}_t}, \\
\mathbf{x}^*_t = \int \mathbf{X}_t \rho_t\left(\mathbf{X}_t\middle|Y^t\right) d\mathbf{X}_t, \; x^*_t = \left(\mathbf{x}^*_t\right)^{(T+1)p_x}_{Tp_x+1},
\end{gathered} \tag{8}
$$

with initial condition

$$
\begin{aligned}
\rho_{-1}\left(\mathbf{X}_{-1}\middle|Y^{-1}\right) &= \rho_{-1}\left(\mathbf{X}_{-1}\right) = \rho_{-1}\left(X_{-T-1}, \ldots, X_{-1}\right) \\
&= f_\eta\left(X_{-T-1}\right) f_{w_{-T}}\left(X_{-T} - \varphi_{-T}\left(X_{-T-1}\right)\right) \cdot \ldots \cdot f_{w_{-1}}\left(X_{-1} - \varphi_{-T}\left(X_{-2}\right)\right).
\end{aligned}
$$

Here, as above, we omit the arguments of the densities $\rho_t\left(\mathbf{X}_t|Y^t\right)$, and simplified the sum. Note that there are no formal reasons that prevent computer implementation of the obtained relation (8). Unlike (7), all multipliers in (8) are described by ordinary functions, so integrals can be computed using any approximate method. However, the feasibility of such a computer calculation is another question. It depends on the dimensions, which are mainly determined by the value of (T) rather than the dimensions of the observation system p_x and q_y. In the example discussed later in the article, $T = 75$, and this choice is confirmed by examples of real-life modeled trajectories. Under more realistic conditions, T can reach several hundred. Consequently, even with the low state ($p_x = 2$) and observation ($q_y = 4$) dimensions used in the model example, the integrals in (7) would have to be computed over $\mathbb{R}^{150}$ and $\mathbb{R}^{300}$, which is practically impossible. The purpose of relations (8) and the canonical representation of the studied observation system (5)–(6) is to justify the impossibility of practical implementation of even an approximate optimal filter, as well as the application of common suboptimal filtering methods. Thus, the formal representation (5)–(6) does not prevent the application of well-known filters, such as extended Kalman filters, polynomial, sigma-point, and other similar filters, as well as more precise constructions such as particle filters [15]. However, the mentioned features show the futility of such attempts. The only concept of practical filtering that remains relevant for the considered case is conditional minimax filtering.

4. CONDITIONAL MINIMAX FILTERING

The ideas and detailed description of the conditional minimax nonlinear filter (CMNF) are presented in several accessible works [13, 16], so we briefly present the following main provisions with the notation used in the article.

The CMNF estimate $\widehat{x}_t$ of the state x_t based on the observations y^t is sought in the form of a prediction correction $\widehat{x}_t = \widetilde{x}_t + \Delta\widehat{x}_t$. To calculate the prediction, we use a basic prediction function $\xi_t = \xi_t(x)$, a typical example of which is a prediction based on the system (1), i.e., $\xi_t\left(\widehat{x}_{t-1}\right) = \varphi_t\left(\widehat{x}_{t-1},\ E\left\{w_t\right\}\right)$ and $\xi_t \in \mathbb{R}^{p_x}$. To calculate the correction, we use a basic correction function $\zeta_t = \zeta_t(x, y)$, a typical example of which is the disparity of the observations (3), i.e., $\zeta_t\left(\widetilde{x}_t, y_t\right) = y_t - \psi_t(\widetilde{x}_t, E\left\{v_t\right\})$ and $\zeta_t \in \mathbb{R}^{q_y}$. The conditional minimax prediction $\widetilde{x}_t$ and the correction $\Delta\widehat{x}_t$ are solutions to the following optimization problems:

$$\begin{gathered}
\widetilde{x}_t = \widetilde{\Xi}_t\left(\xi_t\right), \quad \widetilde{\Xi}_t = \operatorname{argmin}_{\Xi_t} \max_{\mathcal{F}_z} E\left\{\left|\left|x_t - \Xi_t\left(\xi_t\right)\right|\right|^2\right\}, \\
\Delta\widehat{x}_t = \widehat{x}_t - \widetilde{x}_t = \widehat{Z}_t\left(\zeta_t\right), \ \widehat{Z}_t = \operatorname{argmin}_{Z_t} \max_{\mathcal{F}_Z} E\left\{\left|\left|x_t - \widetilde{x}_t - Z_t\left(\zeta_t\right)\right|\right|^2\right\},
\end{gathered} \tag{9}$$

where $\mathcal{F}_z$ denotes the distribution of the vector z, with respect to which we suggest that $\mathcal{F}_z \in \mathcal{F}(m_z, D_z)$ — a class of all probability distributions with mean m_z and covariance D_z. Accordingly, in the first problem, $z = (x_t', \xi_t')'$, and in the second problem $z = (x_t' - \widetilde{x}_t', \zeta_t')'$. Thus, we refine the basic prediction and correction in the best way (in terms of mean square closeness to the estimated state) with the assumption that only the second-order moment characteristics of the estimated and observable variables are known.

It should be noted that in general, the choice of basic prediction and correction may be an independent problem with a solution that ensures the consideration of the specific features of the particular dynamic system. For instance, ξ_t and ζ_t are not limited to the dimensions p_x and q_y. The typical choice of the basic correction proposed in the considered problem $\zeta_t\left(\widetilde{x}_t, y_t\right) = y_t - \psi_t(\widetilde{x}_t, E\left\{v_t\right\})$ essentially ignores observation delays. Perhaps a more correct choice would be the basic correction in the form of $\zeta_t\left(\widetilde{x}_t, y_t\right) = y_t - \psi_t\left(\widetilde{x}_{t-\widetilde{\tau}_t}, E\left\{v_t\right\}\right)$, $\widetilde{\tau}_t = \theta_t\left(\widetilde{x}_t\right)$, i.e., utilizing a delay model (2) in the basic filter structure. We consider further investigation of this prediction structure, but currently it seems excessive. The optimization of the parameters in (9) should use the difference $y_t - \psi_t(\widetilde{x}_t, E\left\{v_t\right\})$ no less effectively, while the dimension of the filter will be the same as in the case without considering time delay, whereas the "more complex" correction $y_t - \psi_t\left(\widetilde{x}_{t-\widetilde{\tau}_t}, E\{v_t\}\right)$, $\widetilde{\tau}_t = \theta_t(\widetilde{x}_t)$, will lead to a significant increase in the dimension of the filter since it requires storing all predictions $\widetilde{x}_{t-\tau}$, $\tau = 0, 1, \dots, T$.

Since the worst distribution in problem (9) is normal and the corresponding best mean square estimate is determined by the normal correlation theorem [17], the optimal minimax functions sought $\widetilde{\Xi}_t(\xi_t)$ and $\widehat{Z}_t(\zeta_t)$ are linear, i.e.,

$$\begin{gathered} \widetilde{x}_t = F_t\xi_t + f_t,\ \xi_t = \xi_t(\widehat{x}_{t-1}), \\ \widehat{x}_t = \widetilde{x}_t + H_t\zeta_t + h_t,\ \zeta_t = \zeta_t(\widetilde{x}_t,\ y_t), \end{gathered} \tag{10}$$

where

$$\begin{gathered} F_t = \operatorname{cov}(x_t, \xi_t)\operatorname{cov}^+(\xi_t, \xi_t),\ f_t = E\{x_t\} - F_tE\{\xi_t\}, \\ H_t = \operatorname{cov}(x_t - \widetilde{x}_t, \zeta_t)\operatorname{cov}^+(\zeta_t, \zeta_t),\ h_t = -H_tE\{\zeta_t\}. \end{gathered} \tag{11}$$

In (11) we use denotations $\operatorname{cov}(x, y)$ for covariance of x and y, $^+$ for the Moore–Penrose pseudoinverse operation.

In this case, the prediction $\widetilde{x}_t$ and the state estimate $\widehat{x}_t$ are unbiased and provide the following quality of estimation:

$$\begin{gathered} \widetilde{K}_t = \operatorname{cov}(x_t - \widetilde{x}_t, x_t - \widetilde{x}_t) = \operatorname{cov}(x_t, x_t) - F_t\operatorname{cov}(\xi_t, x_t), \\ \widehat{K}_t = \operatorname{cov}(x_t - \widehat{x}_t, x_t - \widehat{x}_t) = \widetilde{K}_t - H_t\operatorname{cov}(\zeta_t, x_t - \widetilde{x}_t). \end{gathered} \tag{12}$$

It is important to understand that the relations (10)–(11) define the conditionally optimal Pugachev filter [18, 19], the linear structure of which is initially postulated. Accordingly, the concept of CMNF complements this filter with a minimax justification of the structure. However, the second element of the CMNF concept is no less important—it is the way of practically determining the coefficients F_t, f_t, H_t, and h_t using the Monte Carlo method, that is, computer simulation modeling, which is used to obtain the filter by substituting the mathematical expectations and covariances in (11) with their statistical estimates obtained through simulation modeling.

Applying it to the considered model (1)–(3) or its particular case (4), it remains to formulate the conditions for the existence of a solution (9). The following statement gives a convenient set of sufficient conditions for the existence of the CMNF estimation.

Theorem 2. *Let one of the following conditions be met*:

1) *for the observation system* (1)–(3)

a) *the function exists* $C_\varphi(w) > 0$, $w \in \mathbb{R}^{p_w}$, *such that* $E\{C_\varphi(w_t)\} < \infty$, $t = -T, -T+1, \dots, 0, 1, \dots,$ *and* $\|\varphi_t(x, w)\|^2 < C_\varphi(w)\left(1 + \|x\|^2\right)$, $x \in \mathbb{R}^{p_x}$;

b) *the function exists* $C_\psi(v) > 0$, $v \in \mathbb{R}^{q_v}$, *such that* $E\{C_\psi(v_t)\} < \infty$, $t = 0, 1, \dots,$ *and* $\|\psi_t(y, v)\|^2 < C_\psi(v)\left(1 + \|y\|^2\right)$, $y \in \mathbb{R}^{q_y}$;

c) *constants* $C_\xi > 0$ *and* $C_\zeta > 0$ *exist such that structural functions of CMNF satisfy* $\|\xi_t(x)\|^2 < C_\xi\left(1 + \|x\|^2\right)$ *and* $\|\zeta_t(x, y)\|^2 < C_\zeta\left(1 + \|x\|^2 + \|y\|^2\right)$;

2) *for the observation system* (4)

a) *perturbations* w_t, *observation errors* v_t *and initial conditions* η *have normal distributions* (*or any distributions with all moments finite*);

b) *constants* $C > 0$ *and* $D > 0$ *exist such that* $\|\varphi_t(x)\|^2 + \|\psi_t(y)\|^2 + \|\xi_t(x)\|^2 + \|\zeta_t(x, y)\|^2 < C\left(1 + \|x\|^D + \|y\|^D\right)$.

Then the prediction $\widetilde{x}_t$ *and the estimation* $\widehat{x}_t$ *of the conditional minimax filter* (9) *exist, are described by the relations* (10), (11), *and provide the quality of filtration* (12).

This statement is an adaptation of the existence theorems [13]. The proof consists of two essential points. Firstly, it is the solution of minimax problems (9), which provides the already mentioned property of a linear estimate determined by the theorem of normal correlation, together with the worst normal distribution in the class $\mathcal{F}(m_z, D_z)$. The second element of the proof is the existence of second moments of vectors $(x_t', \xi_t')'$ and $(x_t' - \widetilde{x}_t', \zeta_t')'$ assuming the moment $E\left\{\|\widehat{x}_{t-1}\|^2\right\} < \infty$. The sufficiency of conditions 1) for the model (1)–(3) or 2) for (4) can be conveniently confirmed by using the expanded system (5). Specifically, by expressing the observation system using the variable $\mathbf{x}_t$ and taking into account that $\|\Theta_t(\mathbf{x})\| = 1$, we obtain the same canonical notation for the observation system used in [13], and therefore the sufficiency of conditions 1) and 2) for the existence of the required second moments.

Qualitatively, the content of the conditions 1) and 2) implies the limitation of the growth rate of the model functions and filter structure at infinity: in conditions 1), it is linear growth and bounded random factors, and in conditions 2), it is polynomial growth and the existence of all moments of random factors.

Thus, to confirm the applicability of the CMNF estimate in the problem considered with random observation delay, it remains to verify, through numerical experiments, the possibility of approximate calculation of the filter parameters (11) and compare the actual filtering quality with the computed values (12), obtained by Monte Carlo estimation of the parameters (11).

5. TRACKING AN UNDERWATER TARGET MOVING IN A PLANE WITH CONSTANT SPEED

To conduct a practical experiment on the investigated problem and filtering algorithm, we used a simple planar motion model with constant velocity. We assume that an autonomous underwater vehicle moves at depth in the horizontal plane Oxy. Figure 1 schematically illustrates the motion of the AUV and observations of it. We use the common notations $x(t)$ and $y(t)$ to denote the coordinates of the motion trajectory, and take kilometers (km) as the unit of its measurement.

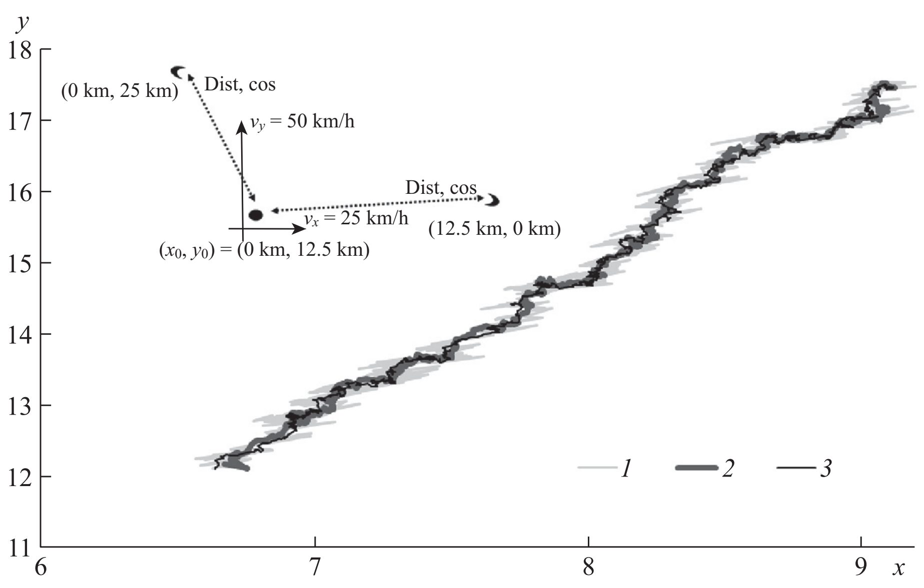

Fig. 1. Motion and observation schematic, and an example of typical trajectories: (*1*)—direct position estimation $\breve{x}_t$, (*2*)—AUV trajectory x_t, (*3*)— CMNF estimation $\widehat{x}_t$.

The use of x and y here should not cause confusion in connection with the general notations x_t and y_t used above for states and observations. The initial position of the AUV is given by the vector $(x(-T-1), y(-T-1))'$, which has a normal distribution with mean of $(0,\ 12.5)'$ and covariance of diag $\{25,\ 100\}$. The coordinate components of the velocities are constant and equal to $v_x = 25$ km/h and $v_y = 50$ km/h, respectively. The time unit is hours (h), and the observation sampling rate is $h = 0.0001$ h, which corresponds to a frequency of about three measurements per second. The calculation is performed for 1000 sampling rates, which corresponds to 6 minutes of motion. During this time, the AUV moves an average distance of about 5.5 km.

The delay parameter T will take two values. First, the results of the calculation for $T = 0$ will be shown, i.e., under normal observation conditions without delay. Then, we assume $T = 75$, i.e., the maximum delay of 0.0075 h or 27 s. The figures below show that for the selected case parameters, such a delay value corresponds to the physical interpretation used, i.e., there are trajectories for which the observation delay is close to the specified maximum.

We assume that the motion of the AUV is influenced by uncontrolled disturbances, which are modeled by additive perturbations $w_x(t)$ and $w_y(t)$. The vector $(w_x(t), w_y(t))'$ has a normal distribution with mean of $(0,\ 0)'$ and covariance of diag $\{1,\ 4\}$. Thus, we obtain the following dynamics:

$$\begin{aligned} x(t) &= x(t-1) + hv_x + \sqrt{h}\, w_x(t), \\ y(t) &= y(t-1) + hv_y + \sqrt{h}\, w_y(t). \end{aligned} \tag{13}$$

The observers are located at two points on the same plane Oxy: the first one has coordinates $(0,\ l_y)$, $l_y = 25$ km, i.e., located at a distance of 25 km from the origin along the Oy axis, and the second one has coordinates $(l_x,\ 0)$, $l_x = 12.5$ km, i.e., located at a distance of 12.5 km from the origin along the Ox axis. The observers are identical, each measuring the range, $d_1(t)$ and $d_2(t)$, and the directional cosine to the object, $c_1(t)$ and $c_2(t)$. We assume the observation error vector $(v_{d_1}(t),\ v_{d_2}(t),\ v_{c_1}(t),\ v_{c_2}(t))'$ to be Gaussian with mean of $(0,\ 0,\ 0,\ 0)'$ and covariance of diag $\{0.001^2,\ 0.005^2,\ 0.001^2,\ 0.005^2\ \}$. Since in practice such measurements can only be implemented with sonars, it should be noted that the assigned accuracy parameters do not fully reflect the accuracy of real devices [20]. In the described experiment, such accuracy characteristics are used to maintain at least the apparent competitiveness of the alternative filtering algorithm, the algorithm for direct position calculation, described below. Increasing observation errors leads to the loss of meaning of this algorithm.

Now then, we have the following observations:

$$\begin{aligned} d_1(t) &= \sqrt{(x\,(t-\tau_t))^2 + (y\,(t-\tau_t) - l_y)^2} + v_{d_1}(t), \\ c_1(t) &= \frac{(y\,(t-\tau_t) - l_y)}{\sqrt{(x\,(t-\tau_t))^2 + (y\,(t-\tau_t) - l_y)^2}} + v_{c_1}(t), \\ d_2(t) &= \sqrt{(x\,(t-\tau_t) - l_x)^2 + (y\,(t-\tau_t))^2} + v_{d_1}(t), \\ c_2(t) &= \frac{(x\,(t-\tau_t) - l_x)}{\sqrt{(x\,(t-\tau_t) - l_x)^2 + (y\,(t-\tau_t))^2}} + v_{c_2}(t). \end{aligned} \tag{14}$$

It remains to define the model τ_t. For the first calculation, the assumption $T = 0$ automatically implies $\tau_t = 0$. For the second calculation, it is convenient to define the model τ_t with two random variables $\tau_1(t)$ and $\tau_2(t)$, reflecting the observation delays for the first and second observers, respectively. Taking into account the sampling rate of $h = 0.0001$ h and the approximate speed of sound in water of $v_s = 6000$ km/h, we express $\tau_1(t)$ and $\tau_2(t)$ in terms of the distances from the observers to the AUV, i.e.,

$$\begin{aligned} \tau_1(t) &= \min\left\{T, \left[\frac{\sqrt{(x(t))^2 + (y(t) - l_y)^2}}{(hv_s)}\right]\right\}, \\ \tau_2(t) &= \min\left\{T, \left[\frac{\sqrt{(x(t) - l_x)^2 + (y(t))^2}}{(hv_s)}\right]\right\}. \end{aligned} \tag{15}$$

Here, we use the notation $[x]$ for the integer part of x, and take into account the potential possibility of the object being removed to a distance for which the delay exceeds the maximum value T. Note that in (14), instead of the general delay notation τ_t, we use the values $\tau_1(t)$ for $d_1(t)$ and $c_1(t)$, and the values $\tau_2(t)$ for $d_2(t)$ and $c_2(t)$. Thus, the delay model (15) has an even more general form than (2). In fact, there is no fundamental generalization here, and it is easy to adjust the model (2) to account for the different delays obtained in this case study for different observations. Since these circumstances do not affect the filtering, we will not dwell on them further.

To analyze the quality of filtering provided by the CMNF estimate, an alternative method for estimating the position of the AUV is required. As noted above, known suboptimal filtering algorithms are not applicable to the model under consideration. For a different reason of computational nature but with the same result, it is not possible to calculate the estimate of the optimal filter, defined by Theorem 1. The only implementable alternative at this stage of the study is the direct position calculation algorithm or the least-squares method. The estimate $\breve{x}_t = (\breve{x}(t), \breve{y}(t))'$ of this filter is obtained as follows. First, assume that there is no delay and no observation errors in (14). Then, each pair of measured ranges and cosines can be obviously recalculated to coordinates $(\breve{x}_1(t), \breve{y}_1(t))$ and $(\breve{x}_2(t), \breve{y}_2(t))$. Now, the estimate of the position $\breve{x}_t$ is the midpoint of the segment that connects these two points. This rather primitive estimate is not that bad for this problem. The point is that using other algorithms, such as the extended Kalman filter, various versions of sigma-point filters, and others, which are used without taking into account the delay, is likely to lead to filter divergence, whereas the estimate of direct position calculation filter cannot diverge, as it does not consider any dynamics and only uses a single point observation.

To synthesize the CMNF and analyze its quality, we modeled two independent sets of 100 000 trajectories. For the first set, we calculated the filter parameters using formulas (11) and (12), where the mathematical expectations were replaced by the statistical means, i.e., using the Monte Carlo method. For the second set we evaluated the real quality of the CMNF estimate $\widehat{x}_t = (\widehat{x}(t), \widehat{y}(t))'$ and the estimate $\breve{x}_t$. We compared the accuracies of the coordinates estimation of the AUV position, determined by the root mean square deviations of the estimation errors denoted as $\sigma_{\widehat{x}}(t)$, $\sigma_{\widehat{y}}(t)$ for the CMNF estimate $\widehat{x}_t$ and as $\sigma_{\breve{x}}(t)$, $\sigma_{\breve{y}}(t)$ for the direct position estimate $\breve{x}_t$.

First, let us consider the results obtained in the first calculation, assuming that $T = 0$. Figure 1 shows a typical trajectory of the AUV movement $(x(t), y(t))$ and the corresponding estimates $\breve{x}_t$ and $\widehat{x}_t$, in addition to the already mentioned motion schematic. Besides the very high accuracy of the CMNF estimate, it should be noted that, compared to it, the direct position estimate is not very informative. Figure 2 presents the formal characteristics. Additionally, it should be noted that the experiment confirms the nonbias of the CMNF estimate, and the estimate $\breve{x}_t$ is slightly biased—about 1 m along the x variable and 0.5 m along the y variable, which is unlikely to be significant given such quality of estimation. With quite accurate range measurements, the low quality of point observations is provided by the error in measuring the directional cosine, which, at the existing distance to the AUV, leads to a very large error in the scale of kilometers of the

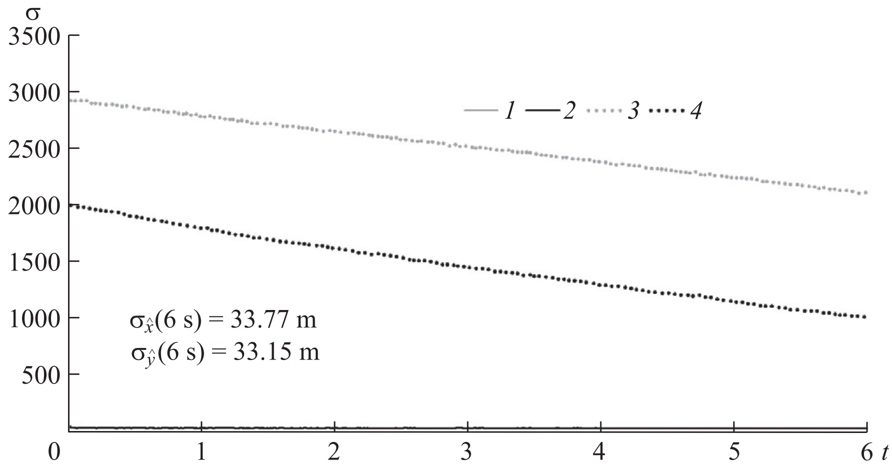

Fig. 2. Root mean square deviations: (*1*) — $\sigma_{\widehat{x}}(t)$, (*2*) — $\sigma_{\widehat{y}}(t)$, (*3*) — $\sigma_{\breve{x}}(t)$, (*4*) — $\sigma_{\breve{y}}(t)$.

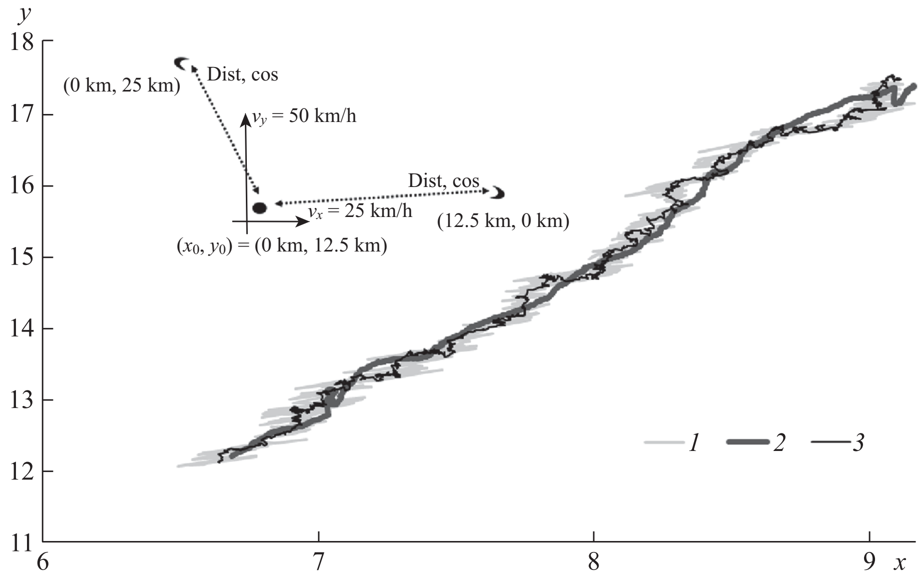

Fig. 3. Cases of typical trajectories of the model with delays: (*1*) — direct position estimation $\breve{x}_t$, (*2*) — AUV trajectory x_t, (*3*) — CMNF estimation $\widehat{x}_t$.

position estimate. Of course, this estimate should not be used. However, in the next calculation, its quality will not seem that unacceptable.

In the next calculation, we keep all the parameters, and show the same trajectory $(x(t), y(t))$ in Fig. 3 as in Fig. 1, but with $T = 75$. Visually, this example shows that the accuracy of the CMNF estimate has significantly decreased, while the accuracy of the direct observation estimate has not changed much. This was to be expected for an estimation that uses only a single point observation. Its accuracy could not be greatly affected by the delay, since compared to the speed of sound waves, even in water, the speed of the AUV is small compared to the existing error in the direct position calculation filter estimate. Figure 4 formally illustrates the quality and shows the root mean square deviations for the estimation errors of the coordinates of the AUV position. And the main thing here is the decrease in the quality of the CMNF estimate by an order, although, of course, it continues to significantly exceed the direct observation estimate. The reason for this decrease is the process τ_t, or more specifically, $\tau_1(t)$ and $\tau_2(t)$. Figure 5 illustrates the real delay

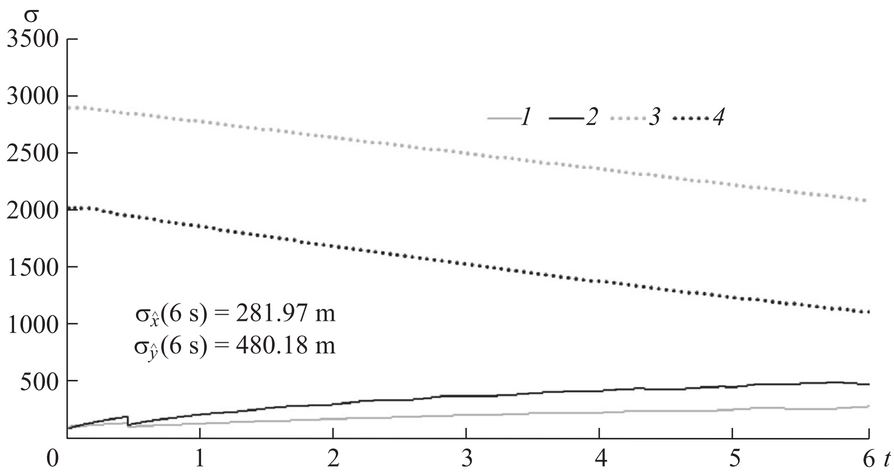

Fig. 4. Root mean square deviations in the model with delays: (*1*) — $\sigma_{\widehat{x}}(t)$, (*2*) — $\sigma_{\widehat{y}}(t)$, (*3*) — $\sigma_{\breve{x}}(t)$, (*4*) — $\sigma_{\breve{y}}(t)$.

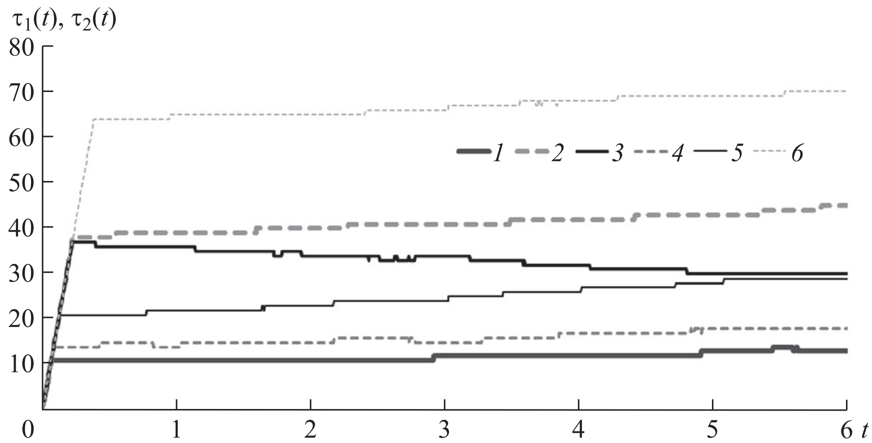

Fig. 5. Cases of observation delay trajectories: (*1*), (*3*), (*5*) — $\tau_1(t)$, (*2*), (*4*), (*6*) — $\tau_2(t)$, pairs (*1*)–(*2*), (*3*)–(*4*), (*5*)–(*6*) correspond to similar trajectories x_t.

values. As can be seen, it may take tens of seconds—the time in which the AUV can swim hundreds of meters. The insights from the experiment are formulated in the conclusion. Note, however, that the sections of linear growth of $\tau_1(t)$ and $\tau_2(t)$ at the initial stage are related to the absence of observations at the initial moments $t = 0, 1, \dots,$ until the values of t greater than the delays for a given trajectory are reached. Such a difference in the implementation of the experiment compared to the formal description of the model (1)–(3) is necessary in order to model both cases with and without observation delays on the same set of trajectories.

6. CONCLUSION

The attempt presented in this paper to approach the physical reality of sonar observation of underwater targets is expressed by a simple model of random observation delay, which is unknown and dependent on the state of the system. Such a simple delay model resulted in an observation system that is rather complex for a filtration problem. In fact, the delay is a multidimensional multiplicative noise in the observations. Dealing with such noise is extremely difficult. Many suboptimal filtering algorithms even exclude the possibility of such noise right at the level of observation modeling. The goals of the conducted experiment were, first, to confirm the performance of the conditional minimax filter concept and, secondly, to understand the magnitude of the loss in estimation quality compared to the delay-free model. We achieved both goals, but there are still a number of questions for further study. The first thing that needs improvement is the quality of the state estimation provided by the CMNF. We used the simplest filter structure in the paper, while the physics of the case study allows reasonable proposals for more flexible solutions, which is the direction of the conditional optimal filtration concept and CNMF in particular. The second direction is the generalization of the motion model, involving the absence of information on the movement parameters. Indeed, at least the velocity of the AUV movement, or rather its systematic component, can hardly be considered known. Thus, we must solve the identification problem in parallel with filtration. This is a typical situation in practice, but here it will be significantly burdened by random delay. Finally, the third question is the use of Doppler observations. The study [12] shows how essential the estimation using simple sonar observations improves if there are even not very accurate velocity data. In the model considered, this effect can be even more significant.

FUNDING

The infrastructure for this work was provided by Core Facility Center "High-performance computing and big data" (CFC "Informatics" FRC IC RAS, Moscow).

REFERENCES

1. Bar-Shalom, Y., X.-R., Li, and Kirubarajan, T., *Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software*, John Wiley & Sons, Inc. 2002.

2. *Autonomous Underwater Vehicles: Design and Practice (Radar, Sonar & Navigation)*, Ehlers, F., Ed., London, UK: SciTech Publishing, 2020.

3. Luo, J., Han, Y., and Fan, L., Underwater Acoustic Target Tracking: A Review, *Sensors*, 2018, vol. 18, no. 1(112).

4. Ghafoor, H. and Noh, Y., An Overview of Next-Generation Underwater Target Detection and Tracking: An Integrated Underwater Architecture, *IEEE Access*, 2019, vol. 7, pp. 98841–98853.

5. Su, X., Ullah, I., Liu, X., and Choi, D., A Review of Underwater Localization Techniques, Algorithms, and Challenges, *J. Sensors*, 2020, vol. 2020 (6403161).

6. Kalman, R.E., A New Approach to Linear Filtering and Prediction Problems, *Trans. ASME-J. Basic Engineer.*, 1960, vol. 82, Ser. D, pp. 35–45.

7. Bernstein, I. and Friedland, B., Estimation of the State of a Nonlinear Process in the Presence of Nongaussian Noise and Disturbances, *J. Franklin Inst.*, 1966, vol. 281, no. 6, pp. 455–480.

8. Julier, S.J., Uhlmann, J.K., and Durrant-Whyte, H.F., A New Approach for Filtering Nonlinear Systems, *Proc. IEEE Am. Control Conf. (ACC'95)*, 1995, pp. 1628–1632.

9. Menegaz, H.M.T., Ishihara, J.Y., Borges, G.A., and Vargas, A.N., A Systematization of the Unscented Kalman Filter Theory, *IEEE Trans. Autom. Control*, 2015, vol. 60, no. 10, pp. 2583–2598.

10. Christ, R.D. and Wernli, R.L., *The ROV Manual: A User Guide for Remotely Operated Vehicles*, 2nd ed., Oxford, UK: Butterworth-Heinemann, 2013.

11. Miller, A., Miller, B., and Miller, G., Navigation of Underwater Drones and Integration of Acoustic Sensing with Onboard Inertial Navigation System, *Drones*, 2021, vol. 5, no. 3 (83).

12. Borisov, A., Bosov, A., Miller, B., and Miller, G., Passive Underwater Target Tracking: Conditionally minimax Nonlinear Filtering with Bearing-Doppler Observations, *Sensors*, 2020, vol. 20, no. 8 (2257).

13. Pankov, A.R. and Bosov, A.V., Conditionally minimax Algorithm for Nonlinear System State Estimation, *IEEE Trans. Autom. Control*, 1994, vol. 39, no. 8, pp. 1617–1620.

14. Bertsekas, D.P. and Shreve, S.E., *Stochastic Optimal Control: The Discrete-Time Case*, New York: Academic Press, 1978.

15. Chen, Z., Bayesian Filtering: From Kalman Filters to Particle Filters, and Beyond, *Statist.*, 2003, vol. 182, no. 1, pp. 1–69.

16. Borisov, A.V., Bosov, A.V., Kibzun, A.I., Miller, G.B., and Semenikhin, K.V., The Conditionally Minimax Nonlinear Filtering Method and Modern Approaches to State Estimation in Nonlinear Stochastic Systems, *Autom. Remote Control*, 2018, vol. 79, no. 1, pp. 1–11.

17. Shiryaev, A.N., *Veroyatnost'* (Probability), Moscow: Nauka, 1989.

18. Pugachev, V.S., Recurrent Estimation of Variables and Parameters in Stochastic Systems Described by Difference Equations, *Dokl. Akad. Nauk SSSR*, 1978, vol. 243, no. 5, pp. 1131–1133.

19. Pugachev, V.S., Estimation of Variables and Parameters in Discrete-time Nonlinear Systems, *Autom. Remote Control*, 1979, vol. 40, no. 4, pp. 39–50.

20. Weirathmueller, M., Weber, T.C., Schmidt, V., McGillicuddy, G., Mayer, L., and Huff, L., Acoustic Positioning and Tracking in Portsmouth Harbor, New Hampshire, *Proc. OCEANS 2007*, Vancouver, BC, Canada, 29 September–4 October, 2007, pp. 1–4.

This paper was recommended for publication by B.M. Miller, a member of the Editorial Board