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On the General Problem Statement of Cargo Carriages Scheduling and Ways to Solve It

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Abstract—A new mathematical model of transportation along the transport network represented by an undirected multigraph is formulated. A new criterion for the optimality of cargo carriages schedule is proposed. The criterion in addition to the time characteristics of transportation includes their cost, the number of undelivered cargoes. The problem to find the optimal schedule is formulated as a problem of mixed integer linear programming. Various variants of the algorithm for searching for an approximate solution to the problem are proposed. Informative examples are considered.

Keywords: transport network, multigraph, cargo carriage, schedule, mixed integer linear programming

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1. INTRODUCTION

Problems to find the optimal route and time of movement along the transport network, taking into account various restrictions on bandwidth, carriage time and cost have long attracted the attention of researchers. In such problems, an (un)directed (multi)graph is often used to model a transport network. Studies devoted to finding the optimal schedule for movement of cargoes between the vertices of the graph can be divided into two categories based on the fixed time of carriage.

In the classical formulations of the vehicle routing problem [1, 2] and the problem to find a route through the points of sale of goods (traveling purchaser problem) [3] movement between the vertices of the graph can be carried out at any time. In the English-language literature on railway topics, the travel time between the nodes of the transport network is often not given. This time is determined when constructing the schedule [4–9]. [4, 5] considers a single-track railway. [6, 7] considers the problem of constructing a schedule for railway networks of a general form with a fixed set of routes for trains. As well as in [6, 7], in [8] a general railway network was considered. However, a somewhat more complex method for generating a schedule was proposed, involving an iterative procedure in which the set of routes for trains and the time of their movement along railway network were modified in series. In [9], the problem to find route of trains and time of their movement along the railway network was solved simultaneously. Time in [9] was assumed to be discrete, which can lead to a very large dimension of the problem.

In fact, it is proposed that a certain vehicle is instantly ready to transport a cargo/customer from one vertex to another in problems with a non-fixed time of movement from vertex to vertex. However, in real life this is not always possible. So, for example, regular rail and air carriages are carried out according to the schedule, while irregular ones may simply be unavailable. And in view of traffic jams on roads that occur, for example, in the morning and in the evening, the duration of the trip between vertices is not the same. In other words, the capacity of the edges/arcs of the transport network graph depends on time. In this regard, this paper considers the problem statement in which movement between the vertices of the multigraph is possible only at predetermined time intervals.

We single out [10–16] among publications devoted to the formation of a carriages schedule, when movement between vertices is carried out according to some predetermined schedule. In [10], the problem of the simultaneous formation of the schedule and routes for movement of cars, trains along a general railway network was considered. We note that in [10] problem statements with non-fixed time of movement between vertices were also proposed. Time in [10], as well as in [9], was assumed to be discrete. In [11], among other things, the problem of moving cars, which are transported by trains with a fixed departure time, between two railway stations was studied. In [12, 13] the problem of minimizing the fleet of locomotives required for the carriage of trains was considered. In [14], the problem of track possession – a period of time in which some sections of the railway track are closed for repair work, was studied. In [14], the schedule of trains, its routes along the railway network, and the time interval for repair work were built at the same time. Due to the large dimension of the problem formulated in [14], the search for its solution took a long time. Therefore, in [15] the model of movement along the multigraph of the railway network from [14] has been improved. In [16], the track possession problem was formulated in a new formulation on the base of the carriage model from [15]. Algorithms for the finding of an approximate solution for this problem were proposed.

It should be highlighted that in statements from [11–16] there is a problem associated with the finiteness of the time interval for which the schedule is built (hereinafter referred to as the planning horizon). Within this period of time, it is necessary to have time to deliver all cargoes. However, this is not always possible due to a lack of carriage capacity within the planning horizon. (in other words, there is no one to transport), and because the need for carriage arises shortly before the end of the planning horizon (in other words, there is not enough time). Thus, the solution of the problem to find the optimal schedule may not exist, since there will be undelivered cargoes. At the same time, the need for the delivery of cargoes does not disappear. This problem was raised in [10], where it was proposed to expand the planning horizon, or to abandon the restriction on the arrival of all cargoes within the planning horizon. The first approach leads to an increase in the dimension of the optimization problem, and the second approach can lead to the fact that the cargo will be delivered to the point, from which it is almost impossible to get to the destination station. Therefore, it is relevant to develop a mathematical model of movement along the graph of the transport network, which would take into account the possibility of the cargo to be in motion even after the end of the planning horizon, if the expected travel time is acceptable. Such a model is formulated in this paper. As in [9, 10], the mathematical model of carriages along the transport network in present paper allows any kind of graph, the time of movement and the route of movement are searched simultaneously, while fixing the route of movement of the cargo is not necessary.

In addition to the new mathematical model of carriages along the transport network multigraph, a criterion of optimality is proposed, which takes into account various aspects of carriage: cost, carriage time, the amount of undelivered cargoes. The problem of finding optimal schedule is formulated as a problem of mixed integer linear programming. An algorithm to find an approximate solution to the problem is proposed and discussed. With illustrative examples comparison of various variants of the proposed algorithm is carried out.

2. BASIC DESIGNATIONS AND ASSUMPTIONS

Let us consider a transport network represented by an undirected multigraph $G =< V, E >$, where V is a set of vertices (cities, railroad stations, plants, airports, seaports) and E is a set of edges (highways, railroad tracks, seaways, airways), connecting these vertices. Let $|V| = M \geqslant 2$. By renumbering vertices of multigraph G from 1 to M, we compose a set of indices $V' = \{1, 2, \ldots, M\}$. Each element of this set uniquely determines the vertex of multigraph G.

We will count the time in minutes relative to a certain moment of reference. By the planning horizon we mean the time interval $[0, T_{\text{max.}})$, for which the timetable is scheduling. If the timetable is scheduled on a day (1440 minutes), then $T_{\text{max.}} = 1440$.

Let us have I cargoes (parcels, containers, trains), for each of that there are given:

- index of departure vertex $v_i^{\text{dep.}} \in V'$;
- index of arrival (destination) vertex $v_i^{\text{arr.}} \in V'$;
- time of readiness for departure $t_i^{\text{dep.}} \in [0, T_{\text{max.}})$;
- maximal amount of time d_i, during which the cargo is allowed to be at the departure vertex from the moment of readiness;
- cargo travel time T_i, i.e. maximal amount of time during which the cargo is allowed to be on the transport network (excluding time at the departure vertex) computed in minutes;
- mass of the cargo $w_i \in \mathbb{R}_+$.

The cargo is assumed to be indivisible in sense that it can not be sent in parts.

Cargoes carriages between vertices can only be carried out at certain intervals. Let K be a quantity of available movements/transportation (by aircrafts, sea ships, trains, trucks) between vertices. Each transportation is represented by 7-element row $z_k \stackrel{\text{def}}{=} (v_k^{\text{beg.}}, v_k^{\text{end}}, n_k, t_k^{\text{beg.}}, t_k^{\text{end}}, W_k, C_k)$, where $v_k^{\text{beg.}} \in V'$ is the index of starting vertex of the movement, $v_k^{\text{end}} \in V'$ is the index of ending vertex of the movement, moreover $v_k^{\text{beg.}}$ and $v_k^{\text{end}}$are indices of adjacent vertices in multigraph G, n_k is the number of the track, connecting vertices with indices $v_k^{\text{beg.}}$ and v_k^{end}, $t_k^{\text{beg.}} \in [0, T_{\text{max.}})$ is starting time of movement, t_k^{end} is ending time of movement, W_k is maximum transportable mass during transportation, C_k is the transportation cost of unit mass, $k = \overline{1, K}$. Let us designate using $\mathcal{Z}$ the set of all vectors z_k, $k = \overline{1, K}$. We renumber elements of set $\mathcal{Z}$ from 1 to K. Thus, number from 1 to K determines parameters of transportation uniquely.

When transportation is carried out, the warehouses in which cargoes are stored can be filled. In addition some operations may be performed with cargoes, for example, repacking. Therefore we introduce minimal and maximal possible duration of stay at the vertex with index v_k^{end} after using transportation with number k by cargo with number i: $t_{i,k}^{\text{st. min.}}$ and $t_{i,k}^{\text{st. max.}}$, $i = \overline{1, I}$, $k = \overline{1, K}$. Obviously, $\forall i = \overline{1, I}$, $k = \overline{1, K}$ $\;0 \leqslant t_{i,k}^{\text{st. min.}} \leqslant t_{i,k}^{\text{st. max.}}$.

For some cargoes, it may be possible to arrive at the destination vertex even after the end of the planning horizon. However, while movements, it is necessary to take into account the restriction on the time spent in the transport system. For this purpose, it is necessary to set the value τ_{m_1,m_2} that is expected duration (starting from the moment of readiness for departure) of a cargo carriage from vertex with index m_1 to vertex with index m_2, $m_1, m_2 = \overline{1, M}$. Obviously, $\tau_{m_1,m_1} = 0$, $m_1 = \overline{1, M}$. If historical observations on carriages from vertex with index m_1 to vertex with index m_2, are available then as τ_{m_1,m_2} one can select sample mean by existing observations, $m_1, m_2 = \overline{1, M}$. If this data is unavailable then the indicated value can be estimated by an expert. Also we introduce value η_{m_1,m_2} that designates expected duration from the moment of readiness for departure till the departure from vertex with index m_1 to vertex with index m_2. This value is set by analogy with τ_{m_1,m_2}, $m_1, m_2 = \overline{1, M}$.

3. MATHEMATICAL MODEL OF CARRIAGES ALONG TRANSPORT NETWORK

Let us formulate a mathematical model of movements of the above introduced I cargoes over the transport network defined by the multigraph G, based on transportation set $\mathcal{Z}$. As the route of cargo with number i we will understand the chain from transportation numbers used in series by this cargo, $i = \overline{1, I}$. As consequence one can determine the chain of vertices traversed in series by this cargo using the route. We limit the maximal quantity of transportation in the route during the planning horizon by some predetermined value J. As jth phase of the route of ith train we will mean movement of this train when there is used jth transportation in the route, $i = \overline{1, I}$, $j = \overline{1, J+1}$. We will name the vertex intermediate for ith cargo if it's neither the vertex of departure nor the vertex of arrival for that, $i = \overline{1, I}$.

We introduce auxiliary variables $\delta_{i,j,k}$, characterizing the usage of kth transportation by cargo with number i at jth phase, $i = \overline{1, I}$, $j = \overline{1, J+1}$, $k = \overline{1, K}$. Variable $\delta_{i,j,k}$ is equal to zero, if transportation with number k is not used by ith cargo at jth phase, and to zero, otherwise

Now we formulate the control set.

By the definition of variable $\delta_{i,j,k}$ we have

$$\delta_{i,j,k} \in \{0, 1\}, \quad i = \overline{1, I}, \quad j = \overline{1, J+1}, \quad k = \overline{1, K}. \tag{1}$$

We use constraints from [15, 16], that set movement only on adjacent vertices of the graph G

$$\sum_{k=1}^{K} \delta_{i,j,k} v_k^{\text{end}} \leqslant \sum_{k=1}^{K} \delta_{i,j+1,k} v_k^{\text{beg.}} + \left(1 - \sum_{k=1}^{K} \delta_{i,j+1,k}\right) M^3, \quad i = \overline{1, I}, \quad j = \overline{1, J-1}, \tag{2}$$

$$\sum_{k=1}^{K} \delta_{i,j,k} v_k^{\text{end}} \geqslant \sum_{k=1}^{K} \delta_{i,j+1,k} v_k^{\text{beg.}} - \left(1 - \sum_{k=1}^{K} \delta_{i,j+1,k}\right) M, \quad i = \overline{1, I}, \quad j = \overline{1, J-1}. \tag{3}$$

According to [15] if for some $\tilde{i} \in \{1, \ldots, I\}$ and some $\tilde{j} \in \{1, \ldots, J\}$ it is true that

$$\sum_{k=1}^{K} \delta_{\tilde{i},\tilde{j},k} = 0, \quad \text{then} \quad \sum_{k=1}^{K} \delta_{\tilde{i},j+1,k} = 0, \quad j = \overline{\tilde{j}, J}.$$

If

$$\sum_{k=1}^{K} \delta_{\tilde{i},\tilde{j},k} = 1, \quad \text{then} \quad \sum_{k=1}^{K} \delta_{\tilde{i},\tilde{j}+1,k} = 0 \quad \text{or} \quad \sum_{k=1}^{K} \delta_{\tilde{i},\tilde{j}+1,k} = 1.$$

Since there can be no more than J phases for movement, we introduce constraint

$$\sum_{i=1}^{I} \sum_{k=1}^{K} \delta_{i,J+1,k} = 0. \tag{4}$$

Since the cargo is indivisible, at any phase (including the first one) we can use a maximum of one transportation

$$\sum_{k=1}^{K} \delta_{i,1,k} \leqslant 1, \quad i = \overline{1, I}. \tag{5}$$

If carriage is performed, it must be carried out from corresponding departure vertex

$$\sum_{k=1}^{K} \delta_{i,1,k} v_k^{\text{beg.}} = v_i^{\text{dep.}} \sum_{k=1}^{K} \delta_{i,1,k}, \quad i = \overline{1, I}. \tag{6}$$

The cargo may not be sent if the time of readiness for departure, plus the maximum duration of time spent at the departure vertex, is outside the planning horizon. Otherwise, we need to send the cargo no later than the maximum duration of time at the departure vertex from the moment of readiness. Therefore, we impose constraints

$$\sum_{k=1}^{K}\delta_{i,1,k}t_k^{\text{beg.}} + \left(1-\sum_{k=1}^{K}\delta_{i,1,k}\right)T_{\max.} \leqslant t_i^{\text{dep.}} + d_i, \quad i=\overline{1,I}. \tag{7}$$

We will introduce constraints in order to send the cargo not earlier than the moment of readiness

$$t_i^{\text{dep.}} \leqslant \sum_{k=1}^{K}\delta_{i,1,k}t_k^{\text{beg.}} + \left(1-\sum_{k=1}^{K}\delta_{i,1,k}\right)T_{\max.}, \quad i=\overline{1,I}. \tag{8}$$

Further we prohibit the cargo to leave the vertex and enter the vertex more than once

$$\sum_{j=1}^{J+1}\sum_{k:v_k^{\text{beg.}}=m,1\leqslant k\leqslant K}\delta_{i,j,k} \leqslant 1, \quad i=\overline{1,I}, \quad m=\overline{1,M}, \tag{9}$$

$$\sum_{j=1}^{J+1}\sum_{k:v_k^{\text{end}}=m,1\leqslant k\leqslant K}\delta_{i,j,k} \leqslant 1, \quad i=\overline{1,I}, \quad m=\overline{1,M}. \tag{10}$$

Departure at intermediate vertices of the route should not occur before arrival at these vertices. Therefore, taking into account the restrictions on the minimum and maximum parking time, we have

$$\sum_{k=1}^{K}\delta_{i,j,k}(t_k^{\text{end}} + t_{i,k}^{\text{st. min.}}) \leqslant \sum_{k=1}^{K}\delta_{i,j+1,k}t_k^{\text{beg.}} + \left(1-\sum_{k=1}^{K}\delta_{i,j+1,k}\right)\underline{T}, \quad i=\overline{1,I}, \quad j=\overline{1,J-1}, \tag{11}$$

where

$$\underline{T} = \max_{i\in\{1,\ldots,I\},k\in\{1,\ldots,K\}} t_k^{\text{end}} + t_{i,k}^{\text{st. min.}},$$

$$\sum_{k:1\leqslant k\leqslant K,v_k^{\text{end}}\neq v_i^{\text{arr.}}}\delta_{i,j,k}(t_k^{\text{end}} + t_{i,k}^{\text{st. max.}}) \geqslant \sum_{k=1}^{K}\delta_{i,j+1,k}t_k^{\text{beg.}}, \quad i=\overline{1,I}, \quad j=\overline{1,J-1}. \tag{12}$$

Constraints (11) are identical to constraints from [15, 16], however, it has been replaced $2T_{\max.}$ for $\underline{T}$. This replacement is required due to the fact that in the transportation model under consideration for some $k \in \{1,\ldots,K\}$ it may be $t_k^{\text{end}} > T_{\max.}$. Constraints (12) are identical to ones from [15, 16] taking into account the fact that parking time control is required only at intermediate vertices.

[15, 16] did not take into account the fact that the cargo is not obliged to arrive at the destination vertex within the planning horizon. Therefore, if it is supposed that the cargo will be at some intermediate vertex at the end of the planning horizon, it is necessary to guarantee the admissibility of such a parking

$$\sum_{k:1\leqslant k\leqslant K,v_k^{\text{end}}\neq v_i^{\text{arr.}}}\delta_{i,j,k}\left(t_k^{\text{end}} + t_{i,k}^{\text{st. max.}} - T_{\max.}\right) + T_{\max.}\sum_{k=1}^{K}\delta_{i,j+1,k} \geqslant 0, \quad i=\overline{1,I}, \quad j=\overline{1,J}. \tag{13}$$

It is also necessary to prohibit the further movement of cargo after arrival at the destination. To this end, we introduce restrictions

$$\sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}}=v_i^{\text{arr.}}} \delta_{i,j,k} \leqslant 2\left(1-\sum_{k=1}^{K}\delta_{i,j+1,k}\right), \quad i=\overline{1,I}, \quad j=\overline{1,J}. \tag{14}$$

Let $\hat{T}_{i,j}$ be amount of time spent cargo with number i in jth (in series) intermediate vertex of its route within the planning horizon

$$\hat{T}_{i,j}=\sum_{k=1}^{K}\delta_{i,j+1,k}\left(t_k^{\text{beg.}}-T_{\text{max.}}\right) + \sum_{k:v_k^{\text{end}}\neq v_i^{\text{arr.}}, t_k^{\text{end}}<T_{\text{max.}}, 1\leqslant k\leqslant K}\delta_{i,j,k}\left(T_{\text{max.}}-t_k^{\text{end}}\right), \quad i=\overline{1,I}, \quad j=\overline{1,J}. \tag{15}$$

For the convenience of formulating the mathematical model, we also set $\hat{T}_{i,J+1}=0$.

Let us introduce new variables $\mathcal{F}_i$. These variables characterize the expected amount of time, required before arrival at the destination vertex for cargo with number i after the end of the planning horizon

$$\mathcal{F}_i \stackrel{\text{def}}{=} \tau_{v_i^{\text{dep.}}, v_i^{\text{arr.}}} + \sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{i,j,k}\left(\tau_{v_k^{\text{end}}, v_i^{\text{arr.}}} - \tau_{v_k^{\text{beg.}}, v_i^{\text{arr.}}}\right) + \sum_{j=1}^{J}\sum_{k:t_k^{\text{end}}\geqslant T_{\text{max.}}, 1\leqslant k\leqslant K}\delta_{i,j,k}\left(t_k^{\text{end}}-T_{\text{max.}}\right), \quad i=\overline{1,I}. \tag{16}$$

Movement of cargoes must be carried out taking into account the expected time before arrival at the destination. For example, long stays at intermediate vertices in a route should be possible when such a strategy does not lead to the fact that the time spent by the cargo in the transport system will be exceeded. In this regard, we introduce constraints

$$\mathcal{F}_i + \sum_{j=1}^{J}\sum_{k:t_k^{\text{end}}<T_{\text{max.}}, v_k^{\text{end}}=v_i^{\text{arr.}}, 1\leqslant k\leqslant K}\delta_{i,j,k}\left(t_k^{\text{end}}-T_{\text{max.}}\right) + \sum_{k=1}^{K}\delta_{i,1,k}\left(T_{\text{max.}}-t_k^{\text{beg.}}\right) \leqslant T_i + \left(1-\sum_{k=1}^{K}\delta_{i,1,k}\right)\eta_{v_i^{\text{dep.}}, v_i^{\text{arr.}}}, \quad i=\overline{1,I}. \tag{17}$$

If there is a need to specify rigidly the set of vertices intersected by the cargo, one can add the appropriate constraints from [15].

Now we introduce variables ω_i, characterizing whether the cargo with number i arrived to a destination within the horizon planning: 0 — arrived, 1 — not arrived

$$\omega_i = 1-\sum_{j=1}^{J}\sum_{k:t_k^{\text{end}}<T_{\text{max.}}, v_k^{\text{end}}=v_i^{\text{arr.}}, 1\leqslant k\leqslant K}\delta_{i,j,k}, \quad i=\overline{1,I}. \tag{18}$$

Let us comment introduced variables. For this purpose we consider cargo with number $i^* \in \{1,\ldots,I\}$.

At first, we will comment (13)–(15). To do this, we first note that several cases are possible (we choose $j^* \in \{1,\ldots,J\}$):

1) j^*th phase is not used for movement of cargo with number i^*;
2) j^*th phase is used for movement of cargo with number i^*, the cargo did not arrive at the destination, j^*+1th phase is not used;
3) j^*th phase is used for movement of cargo with number i^*, the cargo did not arrive at the destination, j^*+1th phase is used;
4) j^*th phase is used for movement of cargo with number i^*, at the end of this phase the cargo arrived within the planning horizon or will arrive after the end of the planning horizon at the destination.

In case 1) $\sum\limits_{k=1}^{K} \delta_{i^*,j^*,k} = 0$. Due to constraints (1)

$$\sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} \neq v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = \sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} = v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 0,$$

and because of constraints (2), (3) it is true that $\sum\limits_{k=1}^{K} \delta_{i^*,j^*+1,k} = 0$. Therefore (13), (14) are satisfied. Value $\hat{T}_{i^*,j^*}$ will be equal to zero.

In case 2)

$$\sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} \neq v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 1, \qquad \sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} = v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 0,$$

$$\sum_{k=1}^{K} \delta_{i^*,j^*+1,k} = 0.$$

Constraint (14) is satisfied, since its left side will be equal to zero, and the right — to two. If the cargo, having not arrived at its destination, is in movement at the end of the planning horizon, then constraint (13) will be satisfied automatically. This fact is connected with ending time of a transportation will not be less than $T_{\max.}$. At the same time value $\hat{T}_{i^*,j^*}$ will be equal to zero, that is reasonable, since the stop (if this one will be) in j^*th intermediate (in series) vertex will occur after the end of the planning horizon. If the cargo, having not arrived at its destination, stays at the end of the planning horizon, then constraint (13) will ensure the admissibility of such stay at least until the end of the planning horizon. Variable $\hat{T}_{i^*,j^*}$ will be equal to the parking time in j^*th intermediate (in series) vertex during the planning horizon.

In case 3)

$$\sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} \neq v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 1, \qquad \sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} = v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 0,$$

$$\sum_{k=1}^{K} \delta_{i^*,j^*+1,k} = 1.$$

Constraint (14) is satisfied, since its left and right side will be equal to zero. Constraint (13) will be satisfied by definition, due to for all $i=\overline{1,I}$, $k=\overline{1,K}$ by basic assumptions $t_k^{\text{end}} + t_{i,k}^{\text{st. max.}} \geqslant 0$. Value $\hat{T}_{i^*,j^*}$ will be equal to the difference between departure and arrival time in j^*th intermediate (in series) vertex.

In case 4)

$$\sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} \neq v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 0, \qquad \sum_{k:1\leqslant k\leqslant K, v_k^{\text{end}} = v_{i^*}^{\text{arr.}}} \delta_{i^*,j^*,k} = 1.$$

Due to constraints (2), (3) $\sum\limits_{k=1}^{K} \delta_{i^*,j^*+1,k}$ can be equal to zero or one. Constraints (13) are satisfied in any of mentioned variants. If $\sum\limits_{k=1}^{K} \delta_{i^*,j^*+1,k}$ is equal to zero, then constraint (14) is satisfied. Value $\hat{T}_{i^*,j^*}$ will be equal to zero that corresponds to the sense of the introduced variable, since the cargo will have only $j^* - 1$ intermediate vertices. If $\sum\limits_{k=1}^{K} \delta_{i^*,j^*+1,k}$ is equal to one, then constraint (14) is not satisfied. So this variant is not allowable. It is meaningful, because in case of arriving in the destination there is no reason in the further carriage.

Now we discuss (16)–(18). If cargo with number i^* is not departed from the departure vertex, then $\sum\limits_{k=1}^{K} \delta_{i^*,1,k} = 0$. Consequently, from constraints (2), (3) we obtain $\sum\limits_{k=1}^{K} \delta_{i^*,j,k} = 0$, $j = \overline{2, J}$. It means, value $\mathcal{F}_{i^*}$ will be equal to the expected duration of the carriage from the departure vertex to the destination vertex. If precisely $\overline{j} \in \{1, \ldots, J\}$ transportation will be used, then summation of the first and second component $\mathcal{F}_{i^*}$ will give $\tau_{v_{k^*}^{\text{end}}, v_{i^*}^{\text{arr.}}}$, where k^* is the number of $\overline{j}$th (in series) transportation of cargo with number i^*. In other words, we will get the expected duration that is needed for the cargo to arrive in the vertex with index $v_{i^*}^{\text{arr.}}$ from the vertex with index $v_{k^*}^{\text{end}}$. If the cargo after the end of the planning horizon is in movement, then the third component of $\mathcal{F}_{i^*}$ will be non-zero and will characterize amount of time for the arriving of the cargo in vertex with index $v_{k^*}^{\text{end}}$ after the end of the planning horizon.

If the cargo has not departed within the planning horizon, then the second and third components of the left side of inequality (17) would be equal to zero. In this case, it is required that the expected duration of carriage from the departure vertex to the destination vertex would be no greater than the allowable time spent on the graph in sum with the expected time from the moment of readiness to the moment of departure. If the cargo is departed and delivered within the planning horizon, then $\mathcal{F}_{i^*} = 0$, and summation of the second and third components of the left side of the inequality will produce duration between the moment of departing and the arrival moment. If the cargo is departed but not delivered, then the second component of inequality (17) will be equal to zero, and value $\mathcal{F}_{i^*}$ will be summed with duration of being on the transport network after departing from the departure vertex.

If at some phase the cargo with number i^* has arrived in the destination, and the phase was finished before the end of the planning horizon, then $\omega_i^* = 1$. Otherwise, $\omega_i^* = 0$.

The need in not exceeding of the maximum allowable mass, when transportation with number k is used, imply restrictions

$$\sum_{i=1}^{I} \sum_{j=1}^{J+1} \delta_{i,j,k} w_i \leqslant W_k, \quad k = \overline{1, K}. \tag{19}$$

Let us note, that constraints (1)–(19) may be inconsistent, which means that the set of admissible strategies given by these restrictions is empty. Such a case may arise, for example, when transportation between vertices is too slow. To ensure consistency of constraints (1)–(19), one can demand conditions

$$t_i^{\text{dep.}} + d_i \geqslant T_{\max.}, \quad \tau_{v_i^{\text{dep.}}, v_i^{\text{arr.}}} \leqslant T_i + \eta_{v_i^{\text{dep.}}, v_i^{\text{arr.}}}, \quad i = \overline{1, I}. \tag{20}$$

If these conditions are true, then each cargo has the opportunity to stay at the departure vertex within the planning horizon. However, even if the conditions (20) are violated, this does not mean that the constraints (1)–(19) are necessarily inconsistent.

4. THE CRITERION FOR CARGOES SCHEDULING

Let us construct the criterion for searching of the optimal schedule

$$
\begin{aligned}
& c_1 \underbrace{\sum_{i=1}^{I}\sum_{j=1}^{J+1}\sum_{k=1}^{K}\delta_{i,j,k}\left(\min\{t_k^{\text{end}}, T_{\max.}\} - t_k^{\text{beg.}}\right)}_{\substack{\text{the total time in movement}\\ \text{during the planning horizon}}} + c_2 \underbrace{\sum_{i=1}^{I}\sum_{j=1}^{J+1}\hat{T}_{i,j}}_{\substack{\text{the total}\\ \text{parking time}\\ \text{in interme-}\\ \text{diate vertices}}} \\
& + c_3 \underbrace{\sum_{i=1}^{I}\left(\sum_{k=1}^{K}\delta_{i,1,k}t_k^{\text{beg.}} + \left(1 - \sum_{k=1}^{K}\delta_{i,1,k}\right)T_{\max.} - t_i^{\text{dep.}}\right)}_{\substack{\text{the total parking time in departure vertices}\\ \text{from the time of readiness for departure}}} \\
& + c_4 \underbrace{\sum_{i=1}^{I}\sum_{j=1}^{J+1}\sum_{k=1}^{K}\delta_{i,j,k}w_i C_k}_{\substack{\text{the total cost}\\ \text{of transportation}}} + c_5 \underbrace{\sum_{i=1}^{I}\mathcal{F}_i}_{\substack{\text{the total}\\ \text{expected}\\ \text{time till}\\ \text{delivery}}} + c_6 \underbrace{\sum_{i=1}^{I}\omega_i}_{\substack{\text{the total}\\ \text{quantity of}\\ \text{undelivered}\\ \text{cargoes during}\\ \text{the planning}\\ \text{horizon}}} \\
& \to \min_{\delta_{i,j,k},\hat{T}_{i,j}\geqslant 0,\hat{T}_{i,J+1}=0,\mathcal{F}_i\geqslant 0,\omega_i\in\{0,1\},i=\overline{1,I},j=\overline{1,J+1},k=\overline{1,K}},
\end{aligned}
\tag{21}
$$

with subject to constraints (1)–(19), where c_1, c_2, c_3, c_4, c_5, c_6 are some non-negative values. Note that, on the one hand, the requirement that variables $\mathcal{F}_i$, $\hat{T}_{i,j}$ are non-negative is unnecessarily, since these variables are non-negative by definition, $i = \overline{1,I}$, $j = \overline{1,J}$. Furthermore, due to constraints (16), (17) these variables are unnecessary itself. They can be substituted for other variables taking part in the optimization and more, such replacement will lead to an integer linear programming problem. However, on the other hand, the direct presence of these variables and the condition on their non-negativity makes it possible in some problems to speed up the search for the optimal solution. And, in addition, the use of these variables allows us to make the form of the criterial function more concise and understandable.

We will mean the multiplier of c_r in (21) as rth *criterion component*, $r = \overline{1,6}$.

We obtain the palette of various applied problems using various values of $c_1, c_2, c_3, c_4, c_5, c_6$. We have the problem to minimize the quantity of undelivered goods during the planning horizon or, in other words, the problem to maximize the quantity of delivered goods, when $c_1 = \ldots = c_5 = 0$, $c_6 = 1$. We get the problem to minimize the total cost of transportation when $c_1 = c_2 = c_3 = c_5 = c_6 = 0$, $c_4 = 1$. We obtain the problem to minimize the total expected time till delivery to respective destination vertices when $c_1 = c_2 = c_3 = c_4 = c_6 = 0,\ c_5 = 1$. We get the problem to minimize total elapsed and expected remaining travel time in case of $c_1 = c_2 = c_3 = c_5 = 1,\ c_4 = c_6 = 0$. We have the problem to minimize the total time spent in movement and stays during the planning horizon when $c_1 = c_2 = c_3 = 1$, $c_4 = c_5 = c_6 = 0$. It is possible to use other combinations of numbers $c_1, \ldots, c_6$ [15]. It should be noted that not all criterion components have the same dimension and order of values: the first, second, third, fifth components are measured in minutes, the fourth is in cost units, the sixth is in pieces. To take into account the different nature of the criterion components, for example, additional restrictions on their values can be introduced. For example, the number of delivered cargoes must be at least 10. Some restrictions arise from the logic of a particular problem. For example, the budget constraint — the total cost of transportation should not exceed the available budget. It can be introduced additionally.

It also should be noted that the introduced mathematical model of carriages is primarily intended for the problem to transport certain goods (parcels, containers) by various vehicles (airplanes, trains, etc.). Namely, the presented model allows "binding" this or that cargo to this or that vehicle and assigning the time of transportation to the cargo. At the same time, the introduced model can also be used to form a schedule for carriages of vehicles themselves. So, for example it is possible to generate, using the proposed model, a schedule for carriages of freight trains: in the terminology of this article, a freight train will become a cargo, a locomotive will become a vehicle, and the concept of transportation will be synonymous with "subthread". The mass of the train will be set to one, as well as the maximum mass for a particular transportation. At the same time, the proposed model can be used for other transport sectors, adding, if necessary, certain restrictions from a specific subject area. Hence, we summarize that the proposed mathematical model of carriages along the graph and the criterion form a universal problem statement of cargo carriages.

5. THE ALGORITHM TO FIND THE INITIAL/APPROXIMATE SOLUTION

In view of the possible high dimension of the problem (21) with subject to constraints (1)–(19) we propose the following algorithm for finding the initial/approximate solution of the problem. The algorithm is based on the sequential solution of the problem (21) with constraints (1)–(19) for some subset of cargoes

1. Set of cargoes numbers is split into S non-overlapping subsets $\mathcal{I}_s$, i.e. $\{i \in \mathbb{N} : i \leqslant I\} = \bigcup\limits_{s=1}^{S} \mathcal{I}_s$, and besides $\forall s_1 \in \overline{1,S}$, $s_2 \in \overline{1,S}$: $s_1 \neq s_2$ $\mathcal{I}_{s_1} \bigcap \mathcal{I}_{s_2} = \varnothing$.
2. Parameter s is initialized by 1.
3. The problem

$$\begin{gathered} c_1 \sum_{i\in\mathcal{I}_s} \sum_{j=1}^{J+1} \sum_{k=1}^{K} \delta_{i,j,k} \left(\min\{t_k^{\text{end}}, T_{\max.}\} - t_k^{\text{beg.}}\right) + c_2 \sum_{i\in\mathcal{I}_s} \sum_{j=1}^{J+1} \hat{T}_{i,j} \\ + c_3 \sum_{i\in\mathcal{I}_s} \left(\sum_{k=1}^{K} \delta_{i,1,k} t_k^{\text{beg.}} + \left(1 - \sum_{k=1}^{K} \delta_{i,1,k}\right) T_{\max.} - t_i^{\text{dep.}} \right) \\ + c_4 \sum_{i\in\mathcal{I}_s} \sum_{j=1}^{J+1} \sum_{k=1}^{K} \delta_{i,j,k} w_i C_k + c_5 \sum_{i\in\mathcal{I}_s} \mathcal{F}_i \\ + c_6 \sum_{i\in\mathcal{I}_s} \omega_i \to \min_{\delta_{i,j,k}, \hat{T}_{i,j} \geqslant 0, \hat{T}_{i,J+1}=0, \mathcal{F}_i \geqslant 0, \omega_i \in \{0,1\}, i\in\mathcal{I}_s, j=\overline{1,J+1}, k=\overline{1,K}}, \end{gathered} \tag{22}$$

with subject to constraints (1)–(18) and constraints

$$\sum_{i\in\mathcal{I}_s} \sum_{j=1}^{J+1} \delta_{i,j,k} w_i \leqslant W_k - \sum_{i\in\bigcup\limits_{p=1}^{s-1} \mathcal{I}_p} \sum_{j=1}^{J+1} \tilde{\delta}_{i,j,k} w_i, \quad k = \overline{1,K} \tag{23}$$

is solved. If solution of this problem exists, then values $\tilde{\delta}_{i,j,k}$ are set to one, if cargo with number i at the jth phase uses transportation with number k, and to zero, otherwise, $i \in \mathcal{I}_s$, $j = \overline{1, J+1}$, $k = \overline{1,K}$. Go to step 4. If solution of problem (22) with subject to constraints (1)–(18), (23) does not exist, then the procedure to find the approximate solution completes unsuccessfully.

4. If $s < S$, then s is increased by one and there is a jump to step 3. If $s = S$, then the procedure to find the approximate solution completes successfully.

If $S = 1$, then the proposed algorithm will allow to find the exact solution, since in this case the problem (21) with subject to constraints (1)–(19) is solved directly. If $S > 1$, then there is no guarantee for the solution, obtained by algorithm, be optimal in problem (21) with subject to constraints (1)–(19). It may turn out in some cases, that the direct solution of problem (21) with constraints (1)–(19) will be found faster than an approximate solution by algorithm. However, as it will be shown later in the example, the use of the algorithm in some cases makes it possible to quickly find a feasible solution in the problem (21) with subject to constraints (1)–(19) with an acceptable criterion value (of the order 5–10% increase in the value of the criterion relative to the criterion on the optimal solution).

The example will also demonstrate the case when the algorithm presented above fails, i.e. feasible solution to the problem (21) subject to constraints (1)–(19) was not found during the algorithm work. In the general case, the unsuccessful completion of the algorithm can be caused both by the initial inconsistency of the constraints (1)–(19) and by the specifics of constructing an approximate solution, when the schedule is searched iteratively for cargo groups. For guaranteed successful completion of the algorithm, one can require the fulfillment of the conditions (20). Another way to get a solution on sending at least some amount of cargoes is the use of another algorithm that differs from the one presented above in that at step 3, in the absence of a solution, the algorithm does not stop its work. Let us describe two methods for constructing such an algorithm. In the first method, in the absence of a solution at step 3, the cargoes that failed to build a schedule are deleted from the carriages list, i.e. there is a denial to carry for these cargoes. A similar approach has been used, for example, in [4, 7]. The second method is in the use of values T_i, d_i in problem (22) with subject to constraints (1)–(18), (23) not as fixed, but as optimization variables, $i \in \mathcal{I}_s$. There will be at least one solution in this case — to stay at the departure vertex within the planning horizon. Note that the proposed modifications, generally speaking, require a change in the initial data, so they are not used in the example below.

We propose several variants of the proposed algorithm by analogy with [15, 16]. We will name as *algorithm by direction* (algorithm 1) such a version of the algorithm when the split at the first step is based on the principle of being in sets $\mathcal{I}_s$ of cargoes numbers with the same departure vertices and the same arrival vertices, $s = \overline{1, S}$. The fewer elements in the set, the smaller the number of this set. We will name as *algorithm by minimal/maximal time* (algorithm 2.1/algorithm 2.2) such a version of the algorithm when at the first step the split is carried out in ascending and descending order of cargoes readiness moments for departure. Namely, set $\mathcal{I}_1$ will consist of cargo number with the earliest/latest time of readiness for departure, set $\mathcal{I}_2$ — with the second/penultimate and so on.

6. THE EXAMPLE

Let us consider the problem of forming freight carriages on the section of the railway network from [15, 16]. To presentation brevity we present only values of parameters from the present statement of the problem, which were not used in constructing the model of carriages along the graph in [15, 16] in view of the large amount of initial data: $C_k = 2$, $W_k = 1$, $w_i = 1$, $i = \overline{1, 62}$, $k = \overline{1, 1249}$. We set values τ_{m_1,m_2} as minimal travel time from vertex with index m_1 to the vertex with index m_2 when departure occurs after 360 minutes from the point of reference (in fact, at 6.00 AM), $m_1, m_2 = \overline{1, 42}$. If there is no available transportation, then τ_{m_1,m_2} is set equal to 4000, $m_1, m_2 = \overline{1, 42}$. We will suppose for simplicity that $\eta_{m_1,m_2} = 0$, $m_1, m_2 = \overline{1, 42}$. [15, 16] introduced only 1, 2, 3 criterion components. Therefore we set $c_1 = c_2 = c_3 = 1$, $c_4 = c_5 = c_6 = 0$ for results comparability of present problem statement and [15, 16]. Criterial functions in (21) and [15, 16] coincide when $c_1, \ldots, c_6$ are set in the above manner. Note that although values W_k, w_i, $i = \overline{1, 62}$, $k = \overline{1, 1249}$ in [15, 16] were not used formally, but with such values of these parameters, constraints for the maximum weight (19) are identical to constraints for maximum quantity of trains using the same "subthread" from [15, 16]. Furthermore, values C_k do not participate in optimization in this case, $k = \overline{1, 1249}$. Thus, the main difference between the present problem statement and its analogue in [15, 16] is in the fact that arrival in destinations in [15, 16] must occur obligatorily during the planning horizon, while in the present study this may not happen. Conditions (20) are broken.

According to the results of the numerical experiment, it turned out that algorithms 1 and 2.1 was completed successfully unlike algorithm 2.2. The optimal value of criterial function in (21) for solution obtained by means of algorithm 1/algorithm 2.1 is 26 951/27 790. The value of criterion [16] for solutions by algorithms 1 and 2 from this paper that are analogous to algorithms 1 and 2.1 respectively — 26 951 and 27 755. Both solutions by algorithm 1, and by algorithm 2.1 are such that every train arrives in the respective destinations during the planning horizon despite of it was not obligatory. Train routes for algorithms from the present paper and for its analogues from [16] coincide partially. Some deterioration in the value of criterion (21) for solution by means of algorithm 2.1 with respect to the value of criterion in [16] by means of algorithm 2 is caused by the fact that there is a control of time till arrival in the destination in every vertex, traversed by cargo, in the present problem statement. But the more important reason is the absence of solution uniqueness of problem (22) subject to constraints. (1)–(18), (23). At the same time the choice of specific solution at the sth step influences on cargo carriages with the schedule only to be found next — at $s + 1$th, $s + 2$th, $\ldots$, Sth steps of any variant of the algorithm. Thus, additional optimization should be carried out among the optimal solutions in the problem (22) with subject to constraints (1)–(18), (23). However, the formalization of the criterion for such optimization is non-trivial and is of separate scientific interest.

Note that if we consider not even all cargoes, but part of them, for example, the first 15 cargoes (i.e. cargoes with numbers 1, 2, $\ldots$, 15) from the carriage list, then optimal solution in problem (21) with subject to constraints (1)–(19) for 2 hours of computation is not found. In this regard, for this example, it is unfortunately impossible to draw a conclusion about the accuracy of the obtained approximate solution.

The computation time to search for the solution by algorithm 1/algorithm 2.1 is 45.5/9.5 minutes. It should be noted the degradation of the solution search time by algorithm 1 in comparison with [15, 16]. This is due to the fact that within the framework of the problem, additional opportunities have appeared: not to depart to the destination, not to arrive at the destination at the end of the planning horizon. At the same time, for algorithm 2.1, the computation time increased insignificantly.

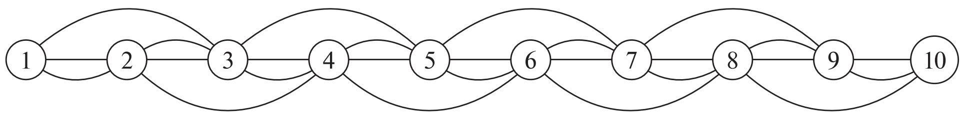
Let us analyze computation time for algorithm 2.1 with respect to the quantity of cargoes planned to be carried. We choose 5, 10, 15, $\ldots$, 55, 60 first cargoes from the carriage list and calculate for each the computation time of algorithm 2.1 in minutes.

Table 1. The computation time of algorithm 2.1 in minutes

Quantity of cargoes to be carried	5	10	15	20	25	30	35	40	45	50	55	60
The computation time of algorithm	0.71	1.51	2.65	3.13	4.16	4.8	6.97	6.73	7.28	8	8.5	9.1

As follows from the presented results, an increase in the number of cargoes does not necessarily result to an increase in the computation time. It is caused by the fact that adding each new cargo to the carriage list starts a chain reaction and leads to a complete recalculation of the schedule due to the fact that, according to algorithm 2.1, the schedule is primarily searched for the cargo with the minimum time of readiness for departure, and not for the cargo with the minimum number. The initial carriage list is not sorted by increasing time of readiness for departure. At the same time, adding every 5 new cargoes for calculation, as a rule, leads to an increase in the algorithm running time by 0.5-1.5 min.

Let's consider another example, which is a model one. Let the transport network has the following form



Multigraph G of the transport network.

The numbering of tracks in the figure is omitted. If two adjacent vertices are connected by two edges, i.e. two tracks, then edge, represented by a straight line has number 1, otherwise — 2.

Let $T_{\text{max.}} = 1440$ minutes. Let us choose some point of reference. Starting from this point of reference:

- every 60 minutes at the vertex with index 1, 10 cargoes of the same mass in 1 unit appear, these cargoes need to be transported to the vertex with index 10;
- every 60 minutes some vehicle is departed from the vertex with index m to the vertex with index $m+1$ using track with number 2; this vehicle can transport no more than 5 units of mass, transportation cost is 3 per unit of mass, duration of transportation — 120 minutes, $m = \overline{1,9}$.
- every 60 minutes some vehicle is departed from the vertex with index m to the vertex with index $m+1$ using track with number 1; this vehicle can transport no more than 5 units of mass, transportation cost is 9 per unit of mass, duration of transportation — 60 minutes, $m = \overline{1,9}$.
- every 60 minutes some vehicle is departed from the vertex with index m to the vertex with index $m+2$; this vehicle can transport no more than 5 units of mass, transportation cost is 81 per unit of mass, duration of transportation — 60 minutes, $m = \overline{1,8}$.

This choice of transportation costs was caused that faster transportation in the same direction should cost more expensive. The cargo travel time — 1 day.

Let $\tau_{m_1,m_2} = 60|m_2 - m_1|$, $m_1 = \overline{1,10}$, $m_2 = \overline{1,10}$. Such choice is caused by the fact that the transportation duration between adjacent vertices of the multigraph G can be 60 minutes, $m_1, m_2 = \overline{1,10}$. Let $\eta_{m_1,m_2} = 0$, $m_1, m_2 = \overline{1,10}$.

Considering the above, it turns out $I = 240$, $K = 624$. Let $d_i = 180$, $t_{i,k}^{\text{st. min.}} = 0$ and $t_{i,k}^{\text{st. max.}} = 120$, $i = \overline{1,I}$, $k = \overline{1,K}$. Since the maximum number of transportation when there is a need to move from the vertex with index 1 to the vertex with index 10, is equal to 9, then we set $J = 9$. Conditions (20) are broken.

Let's illustrate the work of algorithms 1, 2.1 and 2.2 for various values of c_1, c_2, c_3, c_4, c_5, c_6. Let us preliminarily note that algorithm 1 produces an exact solution, since in the problem under study, all cargoes have the same departure vertices and destination vertices.

In Table 2 values of the criterion function in (21) on one or another solution are highlighted by the bold font.

As follows from Table 2, sometimes the search for an exact solution takes about the same time as the search for an approximate one. But in some cases ($c_1 = c_2 = c_3 = c_5 = c_6 = 0$, $c_4 = 1$; $c_1 = c_2 = c_3 = c_4 = c_6 = 0$, $c_5 = 1$; $c_1 = \ldots = c_5 = 0$, $c_6 = 1$) the approximate solution is found many times faster. At the same time the search for an approximate solution stably takes 6–7 minutes for any of the considered variants of numbers c_1, c_2, c_3, c_4, c_5, c_6. As a rule, algorithm 2.2 is more accurate than algorithm 2.1. The error of the best of the approximate solutions is on the order of 5–10% relative to the exact one. Since every cheap transportation is long in time, there is a large total time of transportation and stops in the problem to minimize the total cost of transportation.

Table 2. Results of the numerical experiment

Solution	Criterion parameters						Criterion components						Time to find the solution, minutes
	c_1	c_2	c_3	c_4	c_5	c_6	1	2	3	4	5	6	
Exact	1	1	1	0	0	0	61 440	2460	2100	65 151	22 140	50	6.75
Algorithm 2.1							64 200	6000	6600	54 510	29 700	60	7
Algorithm 2.2							63 900	1800	6000	57 990	27 300	55	6,5
Exact	1	1	1	0	1	0	66 000	0	0	73 260	10 800	50	6.85
Algorithm 2.1							71 100	4800	2100	63 645	15 300	65	7
Algorithm 2.2							71 400	0	0	67 365	11 700	55	6.5
Exact	0	0	0	1	0	0	82 740	62 760	33 060	**3615**	83 160	235	183.5
Algorithm 2.1							76 200	66 300	37 500	**3855**	84 600	240	6.5
Algorithm 2.2							116 700	34 500	9300	**5955**	60 000	190	7
Exact	0	0	0	0	1	0	86 940	15 180	1620	60 165	**10 800**	55	73.15
Algorithm 2.1							87 600	42 600	12 600	47 355	**23 700**	130	6
Algorithm 2.2							81 300	17 100	0	60 105	**11 700**	60	6.5
Exact	0	0	0	0	0	1	80 460	13 860	5700	49 209	25 140	**50**	250.3
Algorithm 2.1							73 500	41 700	30 000	31 200	52 500	**125**	5.65
Algorithm 2.2							73 800	18 000	6300	49 815	27 900	**55**	6.25

And, on the contrary, the cost of transportation increases in the problem of maximizing the number of delivered cargoes. The problem of minimizing the expected remaining time before arriving at destinations expectedly leads to the fact that most of cargoes (for solutions according to algorithms 1 and 2.2) arrive at the destination vertices within the planning horizon.

Separately, it should be noted that the computation time of the algorithms significantly depends not only on the size of the constraints matrix, but also on its content. So, for example, sometimes a certain set of constraints can be deleted from the constraints matrix due to their redundancy. In this case, the dimension of the problem will be decreased. However, due to the large number of input data, it is difficult to make an a priori assumption about the number of such exceptions and the (small) large computation time of the algorithms.

All numerical experiments were carried out using the ILOG CPLEX 12.5.1 mathematical package on a personal computer (Intel Core i5 4690, 3.5 GHz, 8 GB DDR3 RAM).

7. CONCLUSION

In this paper, we formulated the scheduling problem in the general statement. For this purpose, a new mathematical model of carriages along the multigraph of the transport network was proposed, which is given in the form of a system of linear constraints. A universal optimization criterion was proposed, which makes it possible to obtain important applied problems for various values of the parameters, for example, the problem of minimizing the total cost of transportation or the problem of maximizing the quantity of cargoes delivered within the planning horizon. In the work, algorithms were proposed for finding an approximate solution to the problem. The main idea of these algorithms is the decomposition of the problem by searching for a schedule sequentially for some groups of cargoes. The use of such a decomposition does not always lead to an exact or even acceptable solution. However, due to decomposition in some cases with the order of 1–1.5 million binary variables, it turned out to be possible in a relatively short time to find an acceptable solution with an accuracy of 5–10%. A distinctive feature of the proposed model of carriages along the multigraph of the transport network is that the arrival of goods is allowed even after the planning horizon, if it is predicted that the delivery will be completed on time. This circumstance opens the way to another decomposition — by splitting the planning horizon. Namely, the planning horizon can be split into several parts and search for the schedule for the cargo carriages can be performed sequentially on each part of the planning horizon. Such a decomposition may allow the formation of a schedule for even more cargoes/transportations at a fixed count time.

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