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NONLINEAR SYSTEMS

Aircraft Cruise Altitude and Speed Profile Optimization in a Real Atmosphere

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Abstract—This paper considers subsonic turbojet aircraft fuel consumption minimization problem during cruise phase, assuming fixed time of arrival. The problem takes into account real atmosphere data. We utilize tailwind/headwind component values at various flight levels, as well as air temperatures and atmospheric pressures at various altitudes. The solution to the altitude and speed flight profile optimization problem is through constrained coordinate descent method. The paper considers optimizing the fuel consumption of a medium-haul aircraft during the cruise phase using sample data set on temperature, pressure, and wind speed. The proposed approach achieves a decrease in fuel consumption of 1.2% when optimizing with regard to real atmosphere.

Keywords: optimization, subsonic turbojet aircraft, fuel consumption, air temperature, wind speed, flight modeling

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1. INTRODUCTION

The urgency of fuel consumption minimization problem raises due to flight cost-efficiency requirements getting stricter, as well as the importance of CO_2 emission reduction. Therefore, solutions providing even proportions of percent efficiency increase are of clear practical interest. The paper [1] considers aircraft fuel consumption minimization problem during cruise phase in case of standard atmosphere and lack of wind. It shows the possibility of decreasing the fuel consumption by 0.4% for a 5000 km long flight of a medium-range aircraft. In one respect, it confirms the conclusion of paper [2] stating that speed and altitude maneuvering doesn't provide any noticeable fuel conservation when travelling for 5000 km or less. Conversely, saving 0.4% of fuel is, arguably, a meaningful result.

When considering the problem of arriving at the destination at the target time, it is obvious that an appropriate solution requires accounting for the wind speed along the flight route [3, 4]. The flight schedule may account for the predominant winds at typical flight levels. However, every flight requires to analyse current forecast data. Among the related recent publications, let us mention [5–7]. The paper [5] deals with optimizing the speed flight profile considering the tailwind or headwind during a constant altitude flight while assuming that the fuel consumption approximated as a function of the mass and speed of the aircraft is known. The study [6] analyses the influence of wind profile on optimal altitude and speed flight profile. The authors of [7] consider the optimization of direct operational costs that include fuel consumption and travel time in case of variable winds along the flight route.

The paper [8] shows that the temperature of the air substantially influences the results of optimization of the flight program. It considers temperature distributions typical for various climate zones that differ from the standard atmosphere. The study [9] considers estimation of atmospheric parameter values in the current flight path way point based on forecast values for specific way points. It emphasizes the importance of using atmospheric data for flight path computation.

This paper studies the problem of altitude and speed cruise flight profile optimization using data on free-air temperature, tailwind and headwind at various altitudes along the flight path. Meteorological services can provide the necessary data. So [9] uses the data from the global forecast system at the $0.25^\circ \times 0.25^\circ$ latitude and longitude intervals for 27 fixed isobaric altitude levels. Such forecasts are issued each 12 h at 3 h intervals. The paper [10] uses similar global spectral model data with 6 h refresh rate. Unlike these studies, this article describes the required data set without binding to any specific meteorological service data structure. It consists of 7 sections including introduction. Section 2 describes the structure of the air temperature and wind speed data used in this paper. It also provides calculation formulas for other atmospheric parameters. Section 3 states the fuel consumption optimization problem in consideration. The value of the optimization goal—fuel consumption during cruise phase—is found as a numerical solution of a system of nonlinear equations describing the movement of aircraft center of gravity. Section 4 considers this system. Section 5 deals with the developed optimization procedure. Section 6 contains a case study of optimization of subsonic turbojet passenger aircraft fuel consumption. Section 7 concludes the paper.

2. ATMOSPHERE PARAMETERS

It has been established that atmospheric pressure P at altitude h_t is defined as

$$P = P_0 \exp\left(-\int\limits_{h_0}^{h_t} \frac{g}{RK(h)} dh \right), \tag{1}$$

where P_0 is an atmospheric pressure value at altitude h_0, g is acceleration of gravity, R is an air gas constant, $K(h)$ is temperature in kelvins as a function of altitude.

We assume that a set of forecast data for multiple way points $[L_0, L_1, \ldots, L_{n_L}]$ describes the state of the real atmosphere. Here L_i is the distance from the starting point of the cruise flight route that we aim to optimize. For convenience, we assume that the forecast data are available for the starting and end points of the route, i.e., $L_0 = 0$ and $L_{n_L} = r_{\mathrm{cr}}$, where r_{cr} is the length of the planned cruise flight. The data on temperature $K(h)$ at various altitudes $\left[h_{K_0}, h_{K_1}, \ldots, h_{K_{n_h}}\right]$ and the pressure value P_0 at altitude h_{K_0} are required for each point L_i, $(i = 0, \ldots, n_L)$. Here $h_{K_{n_h}}$ should be not lower than the maximum altitude of the cruise flight. For simplicity, assume that the values of these altitudes are the same for all way points L_i. The number and values of altitudes should allow adequate numerical estimation of the integral in (1) for $h_{K_i} \leqslant h_t < h_{K_{i+1}}$ by means of the sum

$$\int\limits_{h_{K_0}}^{h_t} \frac{g}{RK(h,L)} dh \approx \sum_{j=1}^{i} \frac{g(h_{K_j} - h_{K_{j-1}})}{R\frac{K(h_{K_{j-1}},L)+K(h_{K_j},L)}{2}} + \frac{g(h_t - h_{K_i})}{R\frac{K(h_t,L)+K(h_{K_i},L)}{2}}, \tag{2}$$

where the values $K(h_t, L)$ at altitudes $h_t \neq h_i$ and $K(h, L)$ for an arbitrary way point $L \neq L_i$ are obtained with the linear interpolation method. It should be noted that picking temperature data altitudes has to account for the tropopause.

Thus, it is possible to compute the estimate of the pressure value $P(h, L)$ for any way point using these data. This estimation is of real importance since a cruise flight is performed at barometric altitude, i.e., level flight is flight at a specified atmospheric pressure value. Additionally, these data can provide estimations of speed of sound a and air density ρ for any way point with the help of standard formulas:

$$a(h, L) = \sqrt{\kappa R K(h, L)}, \tag{3}$$

$$\rho(h, L) = \frac{P(h, L)}{R K(h, L)}, \tag{4}$$

where $\kappa = 1.4$ is heat capacity ratio of air.

Similarly, the array of headwind/tailwind values should be specified for multiple way points $\left[L_{\text{wind}_0}, L_{\text{wind}_1}, \dots, L_{\text{wind}_{nw}}\right]$ that may not match the way points with the available temperature data. Wind speed should be specified for all possible cruise flight levels. Wind speed in a current way point should be determined using linear interpolation. Therefore, atmospheric parameters appear as the following set of values and forecast data:

$$\begin{gathered}
L_K = [L_0, L_1, \dots, L_{n_L}]\,, h_K = \left[h_{K_0}, h_{K_1}, \dots, h_{K_{n_h}}\right], \\
K_{\text{data}} = \begin{bmatrix} K(h_{K_{n_h}}, L_0) & \dots & K(h_{K_{n_h}}, L_{n_L}) \\ \dots & \dots & \dots \\ K(h_{K_0}, L_0) & \dots & K(h_{K_0}, L_{n_L}) \end{bmatrix}, \quad P_{0_{\text{data}}} = [P_0(L_0), \dots, P_0(L_{n_L})]\,, \\
L_{\text{wind}} = \left[L_{\text{wind}_0}, L_{\text{wind}_1}, \dots, L_{\text{wind}_{nw}}\right], h_{\text{wind}} = \left[h_{\text{wind}_1}, \dots, h_{\text{wind}_{n_{FL}}}\right], \\
V_{\text{wind}_{\text{data}}} = \begin{bmatrix} V_{\text{wind}}(h_{\text{wind}_{n_{FL}}}, L_{\text{wind}_0}) & \dots & V_{\text{wind}}(h_{\text{wind}_{n_{FL}}}, L_{\text{wind}_{nw}}) \\ \dots & \dots & \dots \\ V_{\text{wind}}(h_{\text{wind}_1}, L_{\text{wind}_0}) & \dots & V_{\text{wind}}(h_{\text{wind}_1}, L_{\text{wind}_{nw}}) \end{bmatrix}.
\end{gathered} \tag{5}$$

These data enable flight modeling that accounts for forecast data of the real atmosphere. Account must be taken of that the modeling accuracy rely on data properly reflecting all variations of wind speed values and atmospheric parameters along the flight path, as values of wind speed and atmospheric parameters for a current way point are determined using linear interpolation of data (5).

3. PROBLEM STATEMENT

The problem of traversing the specified distance r_{cr} in the specified time t_{cr} for a cruise flight with consideration of data on wind speed and temperature, in its most simple case, can be solved as a level flight with constant Mach number which can be calculated as

$$M = \left(\frac{r_{\text{cr}}}{t_{\text{cr}}} - V_{\text{wind}_{\text{av}}}\right) \Big/ a_{\text{av}}, \tag{6}$$

where $V_{\text{wind}_{\text{av}}}$ is average wind speed and a_{av} is an average speed of sound value at the chosen flight level. The choice of this flight level should follow the minimization of fuel consumption with respect not only to air density that determines lift and drag forces but also to fuel required for additional climbing, as well as to different flight levels having different wind speeds.

If the cruise flight distance is sufficiently long for a level change to be allowed, then we can complicate the problem by forming the altitude flight profile in the following way: for specified number N of altitude sections and specified minimal flight time without level change $t_{FL_{\min}}$ find optimal flight level values h_{FL_i} out of a set of allowed flight levels

$$h_{FL_i} \in \{h_1, \ldots, h_{n_{FL}}\} \tag{7}$$

in each of N sections and in time t_{FL_i} of each section on condition that

$$t_{FL_i} \geqslant t_{FL_{\min}}, \tag{8}$$

$$\sum_{i=1}^{N} t_{FL_i} = t_{\text{cr}}. \tag{9}$$

Aircraft mass reduction due to fuel consumption, different air density in different flight sections defined by the temperature and pressure forecast, and different wind speeds can lead to changing flight speeds in different sections, allowing fuel efficiency to be improved even when the aircraft is required to arrive at a specified time. For this reason the presented study, as well as [1], suggests forming the speed profile of the cruise flight in the following way: split the cruise phase into n sections of equal length $r = r_{\text{cr}}/n$ and find optimal travel time t_{V_i} for each of them on condition that

$$\sum_{i=1}^{n} t_{V_i} = t_{\text{cr}}. \tag{10}$$

The flight in each speed section will have Mach number defined similarly to (5):

$$M_i = \left(\frac{r}{t_{V_i}} - V_{\text{wind}_{\text{av}_i}}\right) \Big/ a_{\text{av}_i}, \tag{11}$$

where $V_{\text{wind}_{\text{av}_i}}$ and a_{av_i} are the average wind speed and speed of sound values in the ith section.

Then let us state the optimization problem.

Problem 1. For the cruise phase, for the specified aircraft parameters, the phase initial conditions, the distance r_{cr} and the time t_{cr} of the cruise flight, the number of speed sections n and altitude sections N, the allowed flight levels and the atmospheric parameters forecast values (5), find the values of the vector

$$x = [t_{V_1}, \ldots, t_{V_n}, h_{FL_1}, \ldots, h_{FL_N}, t_{FL_1}, \ldots, t_{FL_N}], \tag{12}$$

that satisfy the constraints (7)–(10) *and minimize fuel consumption*

$$q_{\text{cr}}(x) = \int_0^{t_{\text{cr}}} q_{\text{c}}(t)dt, \tag{13}$$

where $q_{\text{c}}(t)$ *is fuel flow.*

It should be noted that the relation between the specified distance r_{cr} and time t_{cr} of the cruise flight has to be reachable for the minimum and maximum allowed Mach numbers considering the speed of the wind.

4. FUEL CONSUMPTION MODELING

The fuel flow in (13) can be expressed in a simplified way as

$$q_{\text{c}} = \eta(M, T, h)T, \tag{14}$$

where T is the current value of total engine thrust and $\eta(M, T, h)$ is the thrust specific fuel consumption that depends on the current Mach number, thrust, flight altitude and other parameters values. Obtaining the $\eta(M, T, h)$ value is generally achieved with approximating formulas corresponding to different operating regimes of the engines of the aircraft in question [11]. The static model allows us to estimate the thrust value based on the current speed and mass of the aircraft, assuming that the airspeed and flight altitude are constant [1].

The special feature of the problem in consideration accounting for real atmospheric data is that transients emerge not only due to infrequent changes in target speed and altitude values according to the planned altitude and speed flight profile, but also to changing air temperature at current way point, which leads to change of speed of sound, and consequently, to change air speed required to maintain the specified Mach number. The change in atmospheric pressure leads to geometric altitude maneuvering in order to maintain constant barometric altitude level flight. For this reason, in order to estimate values of thrust and specific fuel consumption, we suggest numerical modeling of the system of differential and algebraic equations that describe the movement of the center of gravity of the aircraft. This system can be logically split into three parts. The first one reflects the laws of physics [1, 12, 13]:

$$m\dot{V} = T\cos(\alpha + \phi) - \frac{1}{2}c_{\mathrm{x}}\rho S V^2 - mg\sin\Theta + Vq_{\mathrm{c}}, \tag{15}$$

$$mV\dot{\Theta} = T\sin(\alpha + \phi) - \frac{1}{2}c_{\mathrm{y}}\rho S V^2 - mg\cos\Theta, \tag{16}$$

$$\dot{h} = V\sin\Theta, \tag{17}$$

$$\dot{L} = V\cos\Theta + V_{\mathrm{wind}}, \tag{18}$$

$$\dot{m} = -q_{\mathrm{c}}, \tag{19}$$

where m is aircraft mass, V is airspeed, T is total engine thrust, α is angle of attack, ϕ is engine installation angle, $c_{\mathrm{x}}, c_{\mathrm{y}}$ are aerodynamic drag and lift coefficients, ρ is air density, S is wing area, g is acceleration of gravity, Θ is flight path angle, h is flight altitude, L is travelled distance, V_{wind} is tailwind or headwind.

The second part is models of thrust and pitch dynamics. For fuel consumption modeling purposes, they can be simplified to first-order differential equations. Pitch is an angle between an aircraft axis and the horizon, it is controlled with an altitude control. We find the angle of attack value based on the values of pitch and flight path angle:

$$\dot{T} = -k_1 T + k_2\delta_T, \tag{20}$$

$$\dot{\theta} = -k_3\theta + k_4\delta_\theta, \tag{21}$$

$$\alpha = \theta - \Theta, \tag{22}$$

where θ is pitch, δ_T, δ_θ are values of control signals, k_1, k_2, k_3, k_4 are model coefficients. Equation (14) for the calculation of fuel flow also belongs to this part.

The third part is the modeling of the control system. We assume that thrust control is formed by a PID controller that regulates speed expressed by a Mach number:

$$\delta_T(t) = \mathrm{PID}(M_i - M(t)), \tag{23}$$

where M_i is a value derived from (11) for a current speed section, $V(t)$ in $M(t) = V(t)/a(t)$ is derived from (15), $a(t)$ is found with (3) using real atmospheric data (5) for the current way point. For the purpose of this study, we do not require an exact recreation of the behavior of the control system. A simplified model is sufficient to acquire transients that are close to the real ones in order to estimate the fuel flow. For this reason, we may assume that $k_2 = k_1$ in (20), then the proportional control factor (23) should be adjusted so that the value of $\delta_T(t)$ would be equal to the required thrust. In this case, the value of $\delta_T(t)$ is constrained by the thrust values in idle mode and the maximal thrust available at the current flight speed and altitude. These values are typically not reached during a cruise flight. We can model the pitch control more simply as a barometric altitude PI controller:

$$\delta_\theta(t) = \mathrm{PI}(P_i - P(t)), \tag{24}$$

where P_i is an atmospheric pressure value corresponding to the chosen flight level for the current altitude section, $P(t)$ is a current atmospheric pressure value at the altitude $h(t)$ derived from (17). $P(t)$ is defined by formulas (1) and (2) using real atmospheric data (5). We can also assume $k_4 = k_3$ in (21), then the proportional control factor (24) should be tuned so that the value $\delta_\theta(t)$ would be equal to the required pitch. The constraint for $\delta_\theta(t)$ is the sum of the current angle of attack and the maximal cruise flight path angle value.

In order to solve the systems (14)–(24) we must define constants ϕ and S in (15) and (16), equation coefficients of (20) and (21), controllers coefficients (23) and (24), arrays or approximating functions of aerodynamic coefficients c_{x} and c_{y} in (15) and (16) that are functions of an angle of attack and a Mach number, as well as an approximating function for a thrust specific fuel consumption value $\eta(M, T, h)$ in (14). We assume that the acceleration of gravity is $g_0 = 9.80665$. The real atmospheric data (5) provide air density ρ for (15), (16) and wind speed V_{wind} for (18).

Thus, we find the value of the objective function (13) for the chosen vector (12) and specified initial condition by solving the equation system (14)–(24) for the given time t_{cr}. To achieve acceptable accuracy while dealing with constrained computational complexity, we suggest numerically solving this system using the first-order Euler method with a time step of one second. At the same time, the difference in modeling results compared to using the fourth-order Runge-Kutta method is negligible [1]. We may consider such time step sufficiently short for the modeled system but increasing it can lead to an incorrect control system modeling. Moreover, we will not use simplified static equations for consumption computation in sections with constant airspeed and flight altitude suggested in [1], as in case of real atmosphere modeling, such sections are small or absent.

In order to correctly compare the values of the objective function obtained for the variants of vector (12) with different end altitudes h_{FL_N}, we will specify, in addition to initial condition, the required flight level value at the end of the cruise phase $h_{FL_{\mathrm{final}}}$ and isolate the fixed sufficient additional time t_{add} for reaching this flight level from any allowed flight level h_{FL_N}. Then the flight modeling for a time $t_{\mathrm{cr}} + t_{\mathrm{add}}$ for the chosen vector (12) would yield a fuel consumption value for the same end altitude $h_{FL_{\mathrm{final}}}$.

5. OPTIMIZATION PROCEDURE

Solving the considered problem of fuel consumption optimization accounting for real atmospheric data does not require changing the optimization procedure suggested in [1]. So, let us recount only the core concept.

Due to the value of the objective function (13) being found using modeling, the analytical gradient function does not exist. Therefore, we suggest using the deterministic gradient-free search optimization method based on coordinate descent [14] with auxiliary candidate points and accounting for the constraints.

Variable vector (12) consists of three groups:

$x = [x^1, x^2, x^3]$, where $x^1 = [t_{V_1}, \ldots, t_{V_n}, h_{FL_1}]$ are time values for each n speed section, $x^2 = [h_{FL_1}, \ldots, h_{FL_N}]$ are flight level values for each of the N altitude sections and $x^3 = [t_{FL_1}, \ldots, t_{FL_N}]$

are duration values for each N sections. The feature of the second group of variables x^2 is that its elements belong to the specified finite set of allowed flight levels $h_{FL_i} \in \{h_1, \ldots, h_{n_{FL}}\}$, moreover, the number of elements n_{FL} of this set is not large. For this reason, every allowed flight level will be a candidate point. The variables in the groups x^1 and x^3 have to satisfy the constraints (10) and (9), respectively. We may ensure this in the following way: when a selected component rises, then every other component should be reduced in such a way that the respective equation would remain correct. Beyond that, the group x^3 must satisfy the constraint (8). We should note that the Mach number constraints

$$M_i \in [M_{\min}, M_{\max}] \tag{25}$$

when accounting for the wind speed cannot be accurately rearranged into constraints for the components of group x^1. Thus, we will use the widest interval possible:

$$t_{V_i} \in \left[\frac{r}{M_{\max} a_{\max} + V_{\text{wind}_{\max}}}, \frac{r}{M_{\min} a_{\min} + V_{\text{wind}_{\min}}} \right],$$

and verify that obtaining a resulting Mach number value that falls beyond the scope of the allowable range would lead to the respective variant of the variable vector being declined. We implemented this check in the objective function value computation procedure. The procedure aborts the computation of the fuel consumption and sets the maximal objective function value if the constraints (25) are not satisfied.

Therefore, the optimization procedure is organized as a coordinate search for the minimum of the objective function until it meets the condition that iterating through all coordinates finds a new objective function minimum while accounting for the chosen threshold. In addition, we implemented a limitation on the maximum number of search steps.

6. CASE STUDY

In order to benchmark the proposed procedures for fuel consumption modeling and optimization, we developed software that uses the specified coefficients of equations (20) and (21), approximating functions for aerodynamic coefficients and specific fuel consumption; also, it contains an implementation of the model control system (23) and (24) that allows the behavior of the system to correspond to a real aircraft. Table 1 presents the initial values, constraints, and other parameters for the modeled medium-haul passenger aircraft.

Table 1. Modeling parameters

Name	Denotation	Value	Commentary
Initial mass	$m(0)$	75 000 kg	75 t
Initial altitude	h_{FL_0}	9144 m	FL300
Desired end altitude	$h_{FL_{\text{final}}}$	9144 m	FL300
Initial speed	M_0	0.77	
Modeled distance	r_{cr}	5 000 000 m	5000 km
Cruise flight time	t_{cr}	21 600 s	6 h
Minimal speed	$M_{\min}$	0.6	
Maximal speed	$M_{\max}$	0.85	
Maximal flight path angle	$\Theta_{\max}$	1 degree	
Number of speed sections	n	10	
Number of altitude sections	N	4	

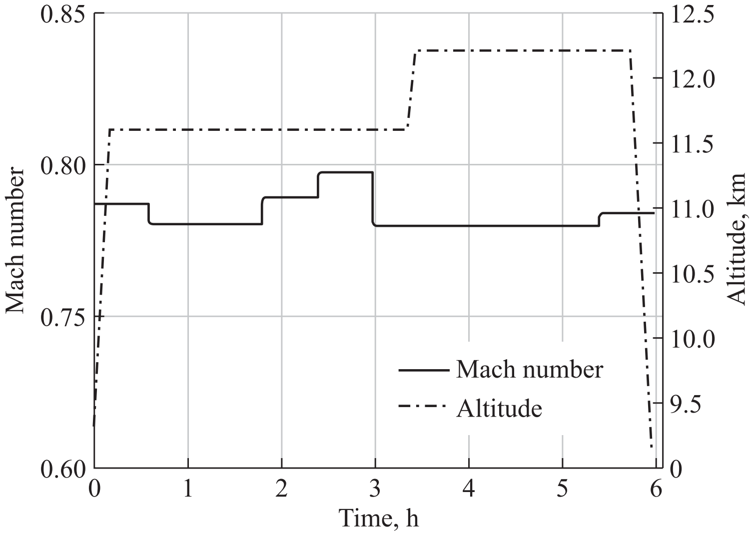

Fig. 1. Speed expressed through Mach number, and flight altitude in the standard atmosphere.

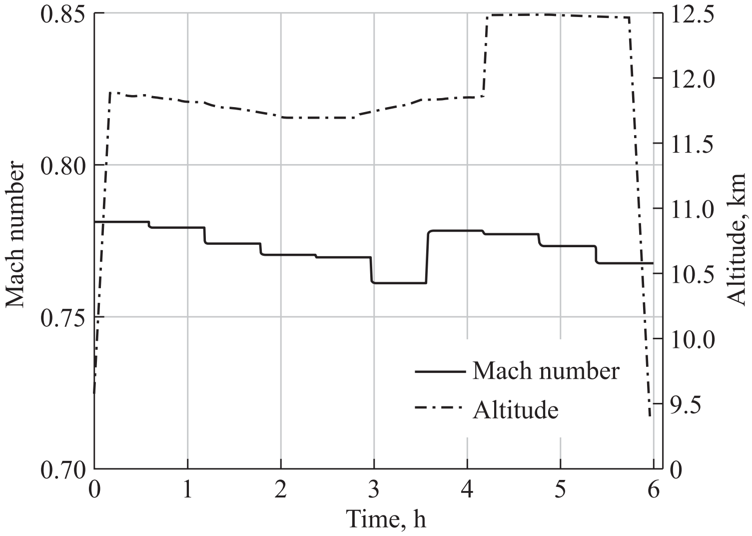

Fig. 2. Speed expressed through Mach number, and flight altitude in the real atmosphere based on Table 2 data.

Figure 1 presents the altitude and speed profile for the standard atmosphere [15] without wind acquired using these data and the optimization procedure described above. The fuel consumption estimate is 12 385 kg. The flight profile and fuel consumption obtained differ from the ones in [1] due to the use of a more accurate approximation of the aerodynamic coefficients, as well as the control system utilizing a Mach number regulator instead of a true air speed regulator.

Table 2 presents sample data on the temperature and pressure along the flight path that differ from the standard atmosphere. For comparison, the right-hand column, titled ISA, contains the respective standard atmosphere values.

Modeling the cruise flight with the altitude and speed profile obtained for the standard atmosphere, with Table 2 data and Section 2 atmospheric parameter formulas, yields a fuel consumption of 12 445 which is a higher value than the one obtained for the standard atmosphere, and also the arrival time appears to be 4.5 minutes earlier, which does not satisfy the fixed arrival time condition.

Table 2. Forecast data on temperature and atmospheric pressure

Way points, km	0	400	900	1750	2250	3000	4000	5000	ISA
Pressure, hPa, (2 m altitude)	1019	1014	1013	1008	1002	997	999	1000	1013
Temperature, °C, at altitude: 2 m	30	30	25	14	16	23	32	27	15
500 m	24	24	19	10	12	20	32	30	11.75
1000 m	21	19	14	4	6	16	28	27	8.5
1500 m	15	15	10	2	4	14	23	21	5.25
2000 m	11	12	9	1	1	11	19	17	2
2500 m	10	8	6	–1	–3	10	14	12	–1.25
3000 m	6	5	4	–2	–5	6	9	6	–4.5
3600 m	2	1	–2	–4	–6	1	4	2	–8.4
4200 m	–2	–3	–6	–7	–10	–3	–1	0	–12.3
5500 m	–11	–12	–14	–16	–20	–11	–9	–9	–20.75
9000 m	–42	–42	–42	–44	–40	–38	–37	–38	–43.5
11 000 m	–54	–53	–50	–45	–40	–48	–57	–58	–56.5
12 000 m	–54	–53	–50	–45	–40	–48	–57	–58	–56.5

Table 3. Tailwind speed, m/s

Way points, km / Flight level	0	400	900	1750	2250	3000	4000	5000
FL300	21	31	22	25	48	29	38	33
FL320	20	31	22	26	46	29	40	32
FL340	19	32	23	26	43	30	42	31
FL360	18	30	23	27	39	29	41	32
FL380	17	26	24	29	32	27	40	33
FL400	17	24	24	30	30	26	40	33

Now let us search for the optimal altitude and speed profile for the same base data but with the real atmospheric temperature and pressure values from Table 2. In this case, we obtain the fuel consumption of 12 296 kg, that is, a decrease of 1.2% while satisfying the fixed arrival time condition. The absolute decrease value of 149 kg obtained in this case study is an economically significant result. Figure 2 shows the obtained altitude and speed profile. Note that the geometric altitude h shown in the figure is not constant during flight at the same level and changes accordingly to variations in atmospheric pressure along the flight path.

Let us add wind speed data to this route. Table 3 presents tailwind speed values. “FL300” denotes a flight level corresponding to the flight altitude of 30 000 feet in the standard atmosphere. Negative values are used for the headwind speed. Consider a case of positive tailwind on the entire route. In this case, obviously, using the profile obtained for the standard atmosphere without considering the wind speed would result in the arrival time being shorter than the targeted one. In the case study considered, the time of a 5000 km flight is 5 h 15 min and its fuel consumption is 11 157 kg. The profile shown in Fig. 3 is obtained with optimization accounting for Table 2 real atmosphere data and Table 3 wind speeds. At the same time, fuel consumption is 11 112 kg, i.e. an additional 0.4% saving occurs while satisfying the fixed arrival time condition. In Fig. 3 one can see that flying at high levels appears to be uneconomical in case of low airspeed due to tailwind.

Let us use Table 3 data with an opposite sign as a case of a flight with headwind. Note that in this case, the constraint $M < 0.85$ for the desired cruise flight distance and time is impossible to satisfy. The schedule usually accounts for the prevailing wind direction of a route. If the forecast is non-standard, then, obviously, the flight time should be lengthened. Let us optimize for

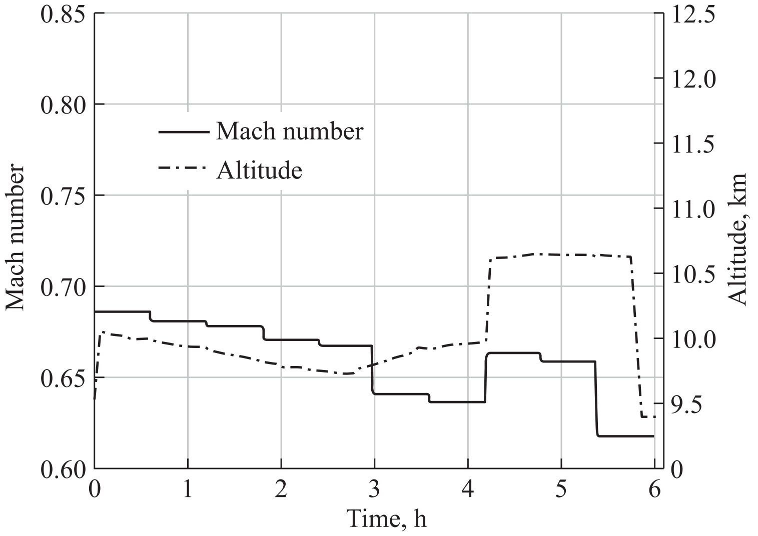

Fig. 3. Speed expressed through Mach number, and flight altitude in the real atmosphere with tailwind based on Tables 2 and 3 data.

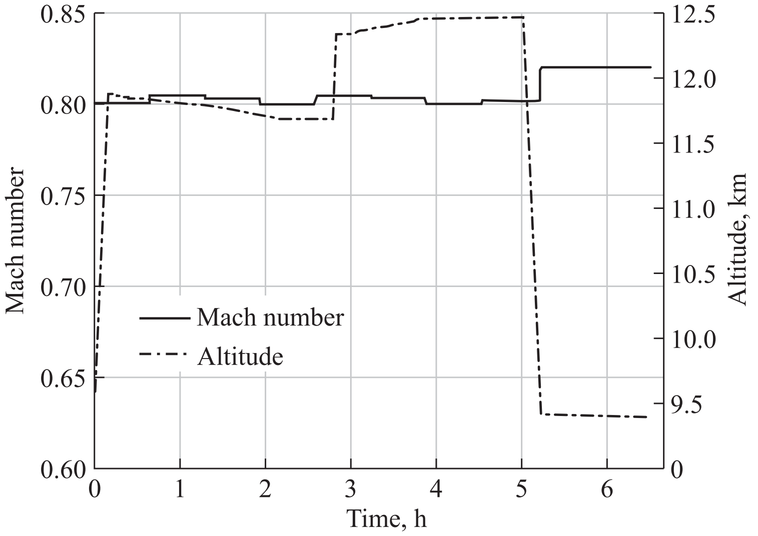

Fig. 4. Speed expressed through Mach number, and flight altitude in the real atmosphere with headwind based on Tables 2 and 3 data.

$t_{\text{cr}} = 23\,400$ s (6.5 h). We obtained fuel consumption of 15 720 kg. Figure 4 shows the altitude and speed profile. It is important to note that the optimization procedure accounts for differences in wind speed at different flight levels.

The case studies mentioned above used a middle-end PC with an Intel Core i5 2.8 GHz CPU. It takes around 1 s to execute the objective function computation procedure that solves a system of differential equations (14)–(24) with the first-order Euler method with a time step of 1 s while modeling a flight of 6 h. The optimization procedure for the values $n = 10$, $N = 4$ presented in Table 1 $n = 10$, $N = 4$, which results in 18 variables in vector (12), converges in 100–150 iterations. So, one of the cases required 135 iterations; in the process, the objective function was computed 703 times. Therefore, it took about 12 min for the optimization procedure to be completed, which is acceptable for practical purposes. It is worth noting that for a shorter distance flight not only the objective function computation takes less time due to shorter flight time, but also the number of optimization procedure iterations gets significantly lower because shorter distance requires lower values of numbers of speed n and altitude N sections, and therefore less variables. When dealing with long-distance flights, we can implement step-by-step optimization, when initially specified values of numbers of sections n and N are low, which allows to quickly obtain a tentative solution. Besides, in the real world a definitive optimization would also use lower values of n and N due to the fact that, e.g. a 5000 km flight typically allows to change the flight level not more than once, that is, $N = 2$. This paper uses higher values for its case studies in order to find possible optimal solutions. At the same time, the optimal solution has 2 flight level changes only in the case, the altitude curve of which is shown on Fig. 4. Other cases with $N = 4$ yielded similar flight level values for the first two and the last two sections, i.e. using $N = 2$ would yield the same solutions.

7. CONCLUSION

We obtain the value of fuel consumption for the specified base data as a result of modeling a flight throughout the considered phase. Current atmospheric parameters—the values of atmospheric pressure, air density, and speed of sound in each way point are an important part of the model. To determine these values, we used data on atmospheric pressure along the route and air temperature on an altitude grid spanning from a base altitude, for which the atmospheric pressure is determined, to a maximum possible flight altitude. We note that these temperature data account for the tropopause. Optimization of the altitude and speed cruise flight profile using these real atmosphere data along the route allows a notable decrease in fuel consumption (over 1%) for a subsonic turbojet passenger aircraft.

The data on wind speed at different flight levels along the flight route allow us not only to choose a speed profile that provides arrival at the specified time but also to achieve additional fuel savings. The consideration of wind speed substantially influences the choice of a fuel-efficient flight level, among other things, by considering the dependence of wind speed on altitude.

Therefore, to optimize a flight it is crucial to have maximal quantity and accuracy of current and forecast data on the real atmosphere state: atmospheric pressure, air temperature, and wind speeds along the entire route.

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