# Thrust Control for Aircraft Landing on a Carrier 

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#### Abstract

This paper considers aircraft landing on a carrier. We propose two schemes for calculating, first, the probability of a go-around due to disengaging the arresting gear and, second, the maximum descent of the aircraft's trajectory with respect to the deck level immediately after leaving the deck. The instant to increase the aircraft's thrust before touching the deck is a control parameter affecting these characteristics. The requirements imposed on the probability of a go-around and the maximum descent of the aircraft's trajectory allow determining an admissible range for the thrust increase instant. Numerical results are presented for a real aircraft landing on a real carrier.


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## 1. PROBLEM DESCRIPTION

As is well known, landing represents the most difficult and important stage of flight. The issues of accurate and safe aircraft landing, particularly on carriers, were considered by many domestic and foreign researchers; for example, see [1-13]. For a shipborne aircraft, additional complications arise because the landing surface is movable and limited in length. As a result, tougher requirements are applied for the accuracy of landing on carriers. This landing process has the following specifics: with the help of a hook, the aircraft should engage one of the arresting gear cables stretched across the landing section of the deck. If this engagement does not occur, the aircraft runs on the deck to take off and perform a go-around. ${ }^{1}$ The number of such go-arounds is quite large, constituting $1-2 \%$ of the total number of landings according to available statistics.

In the case of disengaging the arresting gear, due to the small time of running on the deck ( $1.5-2 \mathrm{~s}$ ), the pilot does not manage to reach the required velocity at the instant of turning off. As a result, immediately after leaving the deck, gravity prevails over the lifting force, and the initial section of the aircraft's trajectory has some descent with respect to the deck level. Under fixed angles of attack and pitch at the instant of leaving the deck and a given elevator control law, the maximum descent is uniquely determined by the aircraft's velocity at the instant of leaving the deck. To prevent water contact and ensure successful take-off, it is necessary to reduce the

[^0]maximum descent as much as possible. The remedy is to increase the aircraft's velocity at the instant of leaving the deck.

The increase in velocity can be achieved through an additional aircraft's thrust. Due to lagging, it takes some time. Therefore, in the case of disengaging the arresting gear, the pilot must increase the aircraft's thrust before the expected instant of touching the deck to increase the velocity significantly while the aircraft is moving on the deck. In this case, the following circumstance should be taken into consideration. If the aircraft's thrust is increased early, the velocity of leaving the deck will be high and the maximum descent will be small. But an early increase in the aircraft's thrust will appreciably raise the landing velocity, causing a tendency to fly over the arresting gear zone (and reduce the probability of a successful landing). If the increase in the aircraft's thrust occurs late, the aircraft will have an insufficient velocity of leaving the deck and an unacceptably high value of the maximum descent in the case of disengaging the arresting gear.

Thus, we face the issue of finding an admissible range of instants to increase the aircraft's thrust. On the one hand, it is necessary to ensure a sufficiently high probability of a successful landing. On the other hand, in the case of disengaging the arresting gear, it is necessary to reduce the maximum descent to avoid a water touch. This paper aims to provide an answer.

## 2. THE PROPOSED SOLUTION SCHEME

We consider only the longitudinal motion of the aircraft. A landing trajectory without thrust until the instant of touching the deck under no perturbations will be called the nominal trajectory. The nominal trajectory is a straight line; see the sloping dashed line in Fig. 1. In the absence of pitching, it intersects the deck at a given point $O$ (Fig. 1). Let $s_{\text {ag }}$ denote the length of the deck section occupied by the arresting gear. To describe the landing trajectory, we introduce the auxiliary planes $A-A, B-B, C-C, C_{1}-C_{1}, D-D$, and $E-E$ that are perpendicular to the vertical plane of the motions of the aircraft and ship. This plane coincides with that of Fig. 1. The lines $A-A, B-B, C-C, C_{1}-C_{1}, D-D$, and $E-E$ are the projections of the corresponding planes on the plane of Fig. 1. The plane $A-A$ is at a distance of a 2.5 -second flight along the nominal trajectory until touching the deck. In the ship-fixed coordinate system, this plane is stationary. The plane $B-B$ corresponds to the thrust increase instant: the pilot increases the engine thrust when the aircraft crosses the plane $B-B$. The planes $C-C$ and $C_{1}-C_{1}$ are associated with the ship and limit the deck section occupied by the arresting gear. The plane $D-D$ is also associated with the ship and corresponds to the instant of leaving the deck in the case of disengaging the arresting gear. In this paper, we assume that the arresting gear engagement does not occur only when flying over the zone $s_{\text {ag }}$. Otherwise, the arresting gear is supposed engaged with probability 1. Finally, the plane $E-E$ corresponds to the instant of reaching the maximum descent by the aircraft after leaving the deck.


Fig. 1.

The proposed solution scheme is as follows. The probability $P_{\mathrm{ag}}$ of flying over the arresting gear zone $s_{\text {ag }}$ (the probability of disengaging the arresting gear) is determined depending on the thrust increase instant $t_{t h}$ of the aircraft by the method ${ }^{2}$ described in [5, 13]; also, see the books $[2,9]$. Random perturbations are atmospheric turbulence and pitching. The analysis below deals with the trajectories corresponding to the factual overflight of the arresting gear zone. As it turns out, under the given thrust increase instant $t_{t h}$, the magnitude $v=|\mathbf{v}|$ of the aircraft's velocity vector $\mathbf{v}$ at the instant of leaving the deck and the maximum descent $\left|H_{\mathrm{ds}}\right|$ almost do not depend on the point of initial contact of the aircraft with the deck: they are uniquely determined by the instant $t_{t h}$ and the angle $\theta$ between the vector $\mathbf{v}$ and the horizontal plane at the instant of leaving the deck. Hence, the aircraft's motion on the deck and after leaving the deck can be simplified to a deterministic setting. In other words, the value $H_{\mathrm{ds}}$ as a function of the variables $t_{t h}$ and $\theta$ can be found by numerically integrating the motion equations.

As a result, we obtain two functions $P_{\mathrm{ag}}\left(t_{t h}\right)$ and $H_{\mathrm{ds}}\left(t_{t h}, \theta\right)$. There are conventional constraints on $P_{\mathrm{ag}}$ and $H_{\mathrm{ds}}$ (for the latter function, in the absence of pitching, i.e., $\theta=0$ ). Therefore, we make another simplification, neglecting pitching when the aircraft runs on the deck. The range of admissible instants $t_{t h}$ will be determined from satisfying these constraints on $P_{\mathrm{ag}}$ and $H_{\mathrm{ds}}$.

## 3. THE PROBABILITY OF DISENGAGING THE ARRESTING GEAR

Assume that the aircraft's thrust $F(t)$ grows exponentially starting from the thrust increase instant $t_{t h}$, i.e.,

$$
F(t)=\left\{\begin{array}{l}
F_{0} \text { for } t \leqslant t_{t h}  \tag{1}\\
F_{0}+\Delta F\left(1-\exp \left\{-\frac{t-t_{t h}}{\tau}\right\}\right) \text { for } t \geqslant t_{t h}
\end{array}\right.
$$

where the constant $\tau$ characterizes the engine acceleration time and $F_{0}$ and $\Delta F$ are some fixed values. The aircraft hits the ship's stern with a negligible probability. ${ }^{3}$ Hence, the probability of disengaging the arresting gear, supposed equal to the probability of flying over the arresting gear zone $s_{\mathrm{ag}}$, can be calculated as the difference $P_{\mathrm{ag}}=1-P$, where $P$ denotes the probability of landing on the deck section between the stern cut line and the line coinciding with the last arresting gear cable (the right end of the section $s_{\text {ag }}$ in Fig. 1).

To find the probability $P$, we use the method described in [5, 13]. This method consists in linearizing the aircraft's motion equations (equations (9) from the paper [5]) in a neighborhood of the nominal landing trajectory along which the aircraft moves in the absence of perturbations. The distinction from the papers $[5,13$ ] has a purely technical nature: the nominal landing trajectory below is a straight line only up to the thrust increase instant $t_{t h}$, deviating from the straight line after this instant due to the new thrust law (1); in [5, 13], the aircraft's thrust was considered constant until touching the deck.

The calculations were performed for $\Delta F=F_{0}$, i.e., the aircraft's thrust after the instant $t_{t h}$ asymptotically increased twice. We considered three values of the constant $\tau$ characterizing the engine acceleration time: $\tau=1 \mathrm{~s}, \tau=1.5 \mathrm{~s}$, and $\tau=2 \mathrm{~s}$. The same automatic feedback control law was applied to the same aircraft as in the papers [5, 13]. This aircraft is identical to the shipborne MiG-29K by performance characteristics. The resulting values of the probability $P_{\mathrm{ag}}$ are presented in Table 2. The value $\Delta t=t_{\text {land }}-t_{t h}$ on the horizontal axis of Fig. 2, determining the thrust increase instant $t_{t h}$, is the time between the instant $t_{t h}$ and the expected instant $t_{\text {land }}$ of touching the deck in the case of no thrust increase. The values of the probability $P_{\text {ag }}$ to plot the graphs in Fig. 2 are given in the Appendix; see Table 1.

[^1]

Fig. 2.

## 4. AIRCRAFT'S VELOCITY AT THE INSTANT OF TOUCHING THE DECK

This velocity is necessary to find the descent $H_{\text {ds }}$. Let the longitudinal motion of the aircraft in a turbulent atmosphere be described by system (9) from [5]. We find the velocity in a deterministic problem statement (no atmospheric turbulence and pitching). In the ship-fixed coordinate system $O x y$ (Fig. 3), the system (9) mentioned above yields

$$
\left\{\begin{array}{l}
m \frac{d v}{d t}=F(t) \cos \alpha-m g \sin (\vartheta-\alpha)-q S C_{x}  \tag{2}\\
m v \frac{d(\vartheta-\alpha)}{d t}=q S C_{y}+F(t) \sin \alpha-m g \cos (\vartheta-\alpha) \\
\frac{d \vartheta}{d t}=\omega_{z} \\
I_{z} \frac{d \omega_{z}}{d t}=q S b_{A} m_{z} \\
\frac{d y}{d t}=v \sin (\vartheta-\alpha) \\
\frac{d x}{d t}=v \cos (\vartheta-\alpha)-V .
\end{array}\right.
$$

All the notations used here were described in detail in [5, 13]. They are conventional for aircraft flight dynamics problems. In particular,

$$
\begin{gathered}
C_{y}=C_{y 0}+C_{y}^{\alpha} \alpha+C_{y}^{\delta} \delta, C_{x}=C_{x 0}+A C_{y}^{2} \\
m_{z}=m_{z 0}+m_{z}^{\alpha} \alpha+m_{z}^{\delta} \delta+\frac{b_{A}}{v_{0}} m_{z}^{\bar{w}_{z}} w_{z}, \quad q=\frac{\rho v^{2}}{2},
\end{gathered}
$$

where: $\delta$ is the deviation of the longitudinal control lever; $v$ is the magnitude of the aircraft's velocity vector in the stationary Earth-based coordinate system; $C_{y 0}, C_{y}^{\alpha}, C_{y}^{\delta}, A, C_{x 0}, m_{z}^{\alpha}, m_{z}^{\delta}$, $m_{z 0}$, and $m_{z}^{\bar{w}_{z}}$ are the aerodynamic coefficients of the aircraft; $\bar{w}_{z}=\frac{w_{z} b_{A}}{v_{0}} ; C_{x}$ is the dimensionless coefficient of the aerodynamic drag component $\mathbf{X}$ (Fig. 4): $C_{x}=|\mathbf{X}| / q S ; C_{y}$ is the dimensionless coefficient of the aerodynamic lift component $\mathbf{Y}: C_{y}=|\mathbf{Y}| / q S ; b_{A}$ is the mean aerodynamic wing chord; $m_{z}$ is the dimensionless coefficient of the longitudinal moment $M_{z}: M_{z}=m_{z} q S b_{A} ; I_{z}$ is the corresponding moment of inertia of the aircraft; $\vartheta$ is the pitch angle, i.e., the angle between the


Fig. 3.


Fig. 4.
fuselage axis and the horizontal plane; $S$ is the wing planform area; finally, $g=|\mathbf{g}|$, where $\mathbf{g}$ is the gravity vector.

Let the control law for $\delta$ be the same as in the papers $[5,13]$. With this control law, the system of equations (2) becomes closed and can be integrated numerically.

According to real observations, in the case of disengaging the arresting gear, the aircraft's velocity at the instant of leaving the deck is almost independent of the point of initial contact with the deck and is uniquely determined by the instant $t_{t h}$. Therefore, as the point of initial contact with the deck, we can choose the point $N$ (i.e., the aircraft touches the deck immediately after the last arresting gear cable, see Fig. 3). Furthermore, we can find the aircraft's velocity $v_{\text {tc }}$ at the instant of touching as $v_{\mathrm{tc}}=v\left(t_{\mathrm{tc}}\right)$, where the function $v(t)$ is obtained by integrating system (2) numerically and the instant $t_{\mathrm{tc}}$ is determined in advance from the condition $x\left(t_{\mathrm{tc}}\right)=s_{\mathrm{ag}}$ (Fig. 3).

For the three values of the engine acceleration time $\tau(\tau=1 \mathrm{~s}, \tau=1.5 \mathrm{~s}$, and $\tau=2 \mathrm{~s})$, the velocities $v_{\text {tc }}$ calculated depending on the instant $t_{t h}$ are presented in the Appendix, see Table 3.

## 5. AIRCRAFT'S VELOCITY AT THE INSTANT OF LEAVING THE DECK

The aircraft's motion on the deck is described by the equation

$$
\begin{equation*}
m \frac{d v}{d t}=F(t)-\frac{\rho v^{2}}{2} S C_{x}-f G \tag{3}
\end{equation*}
$$

where $G=m g, f$ denotes the friction coefficient, and $\rho$ is sea level atmospheric density. Let $\alpha \equiv 0$ and $\delta \equiv 0$ when the aircraft is on the deck. We represent $v(t)$ as the sum

$$
v(t)=v_{\mathrm{tc}}+\Delta v(t), \quad t_{\mathrm{tc}} \leqslant t \leqslant t_{\mathrm{lv}}
$$

where $t_{\mathrm{lv}}$ is the instant of leaving the deck. Note that $\Delta v(t) \ll v_{\mathrm{tc}}$ due to the small time $t_{\mathrm{lv}}-t_{\mathrm{tc}}$ of running on the deck. Therefore, we linearize equation (3) with respect to the function $\Delta v(t)$ to obtain

$$
\frac{d \Delta v}{d t}+\rho \frac{C_{x}}{m / S} v_{\mathrm{tc}} \Delta v=g\left(\frac{F_{0}}{G}-f-\frac{C_{x}}{m / S} \frac{\rho v_{\mathrm{tc}}^{2}}{2}\right)+g \frac{F(t)-F_{0}}{G},
$$

where $C_{x}=C_{x 0}+A C_{y 0}^{2}$ because $\alpha=\delta=0$, and $\Delta v\left(t_{\mathrm{tc}}\right)=0$. Solving this equation, we arrive at the explicit formula

$$
\begin{gathered}
\Delta v(t)=\frac{b_{1}+g \Delta F_{0} / G}{a_{1}}\left(1-\exp \left\{-a_{1}\left(t-t_{\mathrm{tc}}\right)\right\}\right) \\
+\frac{g \Delta F_{0}}{G} \frac{\tau}{1-a_{1} \tau} \exp \left\{-\frac{t_{\mathrm{tc}}-t_{p}}{\tau}\right\}\left(\exp \left\{-\frac{t-t_{\mathrm{tc}}}{\tau}\right\}-\exp \left\{-a_{1}\left(t-t_{\mathrm{tc}}\right)\right\}\right), \quad t_{\mathrm{tc}} \leqslant t \leqslant t_{\mathrm{lv}},
\end{gathered}
$$

where

$$
a_{1}=\rho \frac{C_{x} v_{\mathrm{tc}}}{m / S}, \quad b_{1}=g\left(\frac{F_{0}}{G}-f-\frac{C_{x}}{m / S} \frac{\rho v_{\mathrm{tc}}^{2}}{2}\right) .
$$

Let $T$ denote the aircraft's running time on the deck from the instant $t_{\mathrm{tc}}$ to the instant $t_{\mathrm{lv}}$, i.e., $T=t_{\mathrm{lv}}-t_{\mathrm{tc}}$, and let $L$ denote the length of the deck section between the points of touching and leaving by the aircraft (the instants $t_{\mathrm{tc}}$ and $t_{\mathrm{lv}}$ ). As has been emphasized in Section 4, the aircraft's velocity at the instant of leaving the deck is almost independent of the point of initial contact with the deck. Therefore, we assume that $L=N K$ (Fig. 3) and, consequently,

$$
\begin{gathered}
N K=\int_{t_{\mathrm{tc}}}^{t_{\mathrm{tc}}+T}\left(v_{\mathrm{tc}}+\Delta v(t)-V\right) d t=\left(v_{\mathrm{tc}}-V+\frac{b_{1}+g \Delta F_{0} / G}{a_{1}}\right) T \\
\quad+\frac{g \Delta F_{0}}{G} \frac{\tau^{2}}{1-a_{1} \tau} \exp \left\{-\frac{t_{\mathrm{tc}}-t_{p}}{\tau}\right\}\left(1-\exp \left\{-\frac{T}{\tau}\right\}\right) \\
-\left(\frac{b_{1}+g \Delta F_{0} / G}{a_{1}}+\frac{g \Delta F_{0}}{G} \frac{\tau}{1-a_{1} \tau} \exp \left\{-\frac{t_{\mathrm{tc}}-t_{p}}{\tau}\right\}\right) \frac{1-\exp \left\{-a_{1} T\right\}}{a_{1}} .
\end{gathered}
$$

Hence, $T$ satisfies the equation

$$
\begin{equation*}
T=\psi(T), \tag{4}
\end{equation*}
$$

where

$$
\begin{gathered}
\psi(T)=\tilde{\psi}(T) /\left(v_{\mathrm{tc}}-V+\frac{b_{1}+g \Delta F_{0} / G}{a_{1}}\right), \\
\tilde{\psi}(T)=N K+\left(\frac{b_{1}+g \Delta F_{0} / G}{a_{1}}+\frac{g \Delta F_{0}}{G} \frac{\tau}{1-a_{1} \tau} \exp \left\{-\frac{t_{\mathrm{tc}}-t_{p}}{\tau}\right\}\right) \\
\times \frac{1-\exp \left\{-a_{1} T\right\}}{a_{1}}-\frac{g \Delta F_{0}}{G} \frac{\tau^{2}}{1-a_{1} \tau} \exp \left\{-\frac{t_{\mathrm{tc}}-t_{p}}{\tau}\right\}\left(1-\exp \left\{-\frac{T}{\tau}\right\}\right) .
\end{gathered}
$$

The solution of (4) is found by the method of successive approximations:

$$
T=\lim _{n \rightarrow \infty} T_{n}, \quad T_{n}=\psi\left(T_{n-1}\right), n=1,2, \ldots,
$$

where $T_{0}$ is an initial approximation. The aircraft's velocity $v_{\mathrm{lv}}$ at the instant of leaving the deck is given by

$$
v_{\mathrm{lv}}=v_{\mathrm{tc}}+\Delta v\left(t_{\mathrm{tc}}+T\right),
$$

where $\Delta v\left(t_{\mathrm{tc}}+T\right)$ is the value of the function $\Delta v(t)$ for $t=t_{\mathrm{tc}}+T$. The numerical calculations were performed for the ship characteristics corresponding to the aircraft carrier Admiral Kuznetsov. The calculation results for the velocity $v_{\mathrm{lv}}$ as a function of the thrust increase instant $t_{t h}$ in the three cases (the engine acceleration times $\tau=1 \mathrm{~s}, \tau=1.5 \mathrm{~s}$, and $\tau=2 \mathrm{~s}$ ) are presented in the Appendix; see Table 4.

## 6. THE MAXIMUM DESCENT OF THE AIRCRAFT'S TRAJECTORY AFTER LEAVING THE DECK

After leaving the deck, the aircraft's motion is considered in the deterministic statement. In other words, it is described by system (2) with the following initial conditions:

$$
\begin{equation*}
v=v_{\mathrm{lv}}, \quad \alpha=0, \quad \vartheta=0, \quad \omega_{z}=0, \quad y=0, \quad x=s_{a g}+N K, \tag{5}
\end{equation*}
$$

where $N K$ is the length of the deck section between the points $N$ and $K$ (Fig. 3). By assumption, the deviation $\delta$ of the longitudinal control lever is constant and can be determined from the balancing condition

$$
m_{z 0}+m_{z}^{\alpha} \alpha+m_{z}^{\delta} \delta=0
$$

with $\alpha=\alpha_{\text {bal }}=15^{\circ}$.
The maximum descent $\left|H_{\mathrm{ds}}\right|$ (Fig. 1), representing the minimum value of the coordinate $y$, is obtained by integrating numerically system (2) with the initial conditions (5). The resulting graphs of $H_{\mathrm{ds}}$ are shown in Fig. 5. As in Fig. 2, the value $\Delta t=t_{\text {land }}-t_{t h}$ on the horizontal axis of Fig. 5, determining the thrust increase instant $t_{t h}$, is the time between the instant $t_{t h}$ and the expected instant $t_{\text {land }}$ of touching the deck in the case of no thrust increase. The values of the descent $H_{\mathrm{ds}}$ to plot the graphs in Fig. 5 are given in the Appendix; see Table 2.


Fig. 5.

## 7. ADMISSIBLE THRUST INCREASE INSTANTS

When choosing admissible thrust increase instants, a common technique is to consider constraints on the probability $P_{\mathrm{eb}}$ of landing on the emergency barrier instead of the probability $P_{\mathrm{ag}}$ of disengaging the arresting gear. The probability $P_{\mathrm{eb}}$ is set equal to the probability of disengaging the arresting gear after $n$ landing approaches, i.e., $P_{\mathrm{eb}}=P_{\mathrm{ag}}^{n}$. Due to limited fuel reserves, the value $n=3$ is often chosen, and consequently, $P_{\mathrm{eb}}=P_{\mathrm{ag}}^{3}$.

The aircraft can be seriously damaged during landing on the emergency barrier. According the existing standards [1], $P_{\mathrm{eb}}<10^{-4}$, which is equivalent to $P_{\mathrm{ag}}<0.0464$. Considering the above results for $P_{\mathrm{ag}}$, we arrive at the following constraints on the time $\Delta t$ :

$$
\Delta t<\Delta t_{\max }, \text { where } \Delta t_{\max }=\left\{\begin{array}{l}
1.40 \mathrm{~s} \text { for } \tau=1 \mathrm{~s} \\
1.65 \mathrm{~s} \text { for } \tau=1.5 \mathrm{~s} \\
1.85 \mathrm{~s} \text { for } \tau=2 \mathrm{~s}
\end{array}\right.
$$

In other words, we have the following picture for different engine acceleration times $\tau$ : for $\tau=1 \mathrm{~s}$, the thrust should be increased not earlier than 1.40 s before the expected instant of touching the deck; for $\tau=1.5 \mathrm{~s}$, not earlier than 1.65 s before the expected instant of touching the deck; for $\tau=2 \mathrm{~s}$, not earlier than 1.85 s before the expected instant of touching the deck.

Now we study the constraints imposed on $\Delta t$ due to the limited maximum descent of the aircraft's trajectory after leaving the deck. In the previous section, we have determined the descent without pitching. Clearly, in the presence of pitching, the maximum descent increases noticeably due to leaving the deck with negative angles $\theta$. Nevertheless, the descent is normalized in the absence of pitching under the assumption that it will increase with pitching. For example, according to the norms for US shipborne aircraft and carriers [1], the maximum descent in the absence of pitching must satisfy the condition $H_{\mathrm{ds}}>-3 \mathrm{~m}$. Let us follow this criterion for the example above, where the numerical values of the parameters characterize the aircraft and carrier under consideration. As it turns out, thrust may remain the same up to the instant of touching the deck, and the only constraint on the thrust increase instant is the condition $\Delta t<\Delta t_{\text {max }}$. However, this condition, among other factors, is determined by the carrier's height above the water surface. When a stricter condition is imposed on $H_{\mathrm{ds}}$, e.g., $H_{\mathrm{ds}}>-2 \mathrm{~m}$, we arrive at a bilateral constraint on $\Delta t$ :

$$
\Delta t_{\min }<\Delta t<\Delta t_{\max }, \text { where } \Delta t_{\min }=\left\{\begin{array}{l}
0.30 \mathrm{~s} \text { for } \tau=1 \mathrm{~s} \\
0.75 \mathrm{~s} \text { for } \tau=1.5 \mathrm{~s} \\
1.20 \mathrm{~s} \text { for } \tau=2 \mathrm{~s}
\end{array}\right.
$$

In other words, we have the following picture for different engine acceleration times $\tau$ : for $\tau=1 \mathrm{~s}$, the thrust should be increased not earlier than 1.40 s but not later than 0.3 s before the expected instant of touching the deck; for $\tau=1.5 \mathrm{~s}$, not earlier than 1.65 s but not later than 0.75 s before the expected instant of touching the deck; for $\tau=2 \mathrm{~s}$, not earlier than 1.85 s but not later than 1.20 s before the expected instant of touching the deck.

## 8. CONCLUSIONS

This paper has proposed an algorithm for finding an admissible range of the thrust increase instant for aircraft landing on a carrier. Based on the previously published results, we have proposed two schemes for calculating, first, the probability $P_{\mathrm{ag}}$ of disengaging the arresting gear and, second, the maximum descent $H_{\text {ds }}$ of the aircraft's trajectory with respect to the deck level immediately after leaving the deck (if the arresting gear is not engaged). The probability $P_{\mathrm{ag}}$ and the descent $H_{\mathrm{ds}}$ have been calculated numerically as functions of the thrust increase instant. As a result, the conventional constraints on $P_{\mathrm{ag}}$ and $H_{\mathrm{ds}}$ have been adopted to determine an admissible range for the thrust increase instant. The proposed scheme has been numerically implemented for the landing process of the real shipborne aircraft MiG-29K on the real carrier Admiral Kuznetsov.

## Table 1

| $\Delta t, \mathrm{~s}$ | $P_{\mathrm{ag}}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\tau=1 \mathrm{~s}$ | $\tau=1.5 \mathrm{~s}$ | $\tau=2 \mathrm{~s}$ |
| 0.2 | 0.0081 | 0.0079 | 0.0078 |
| 0.3 | 0.0086 | 0.0082 | 0.0079 |
| 0.4 | 0.0090 | 0.0085 | 0.0081 |
| 0.5 | 0.0096 | 0.0088 | 0.0083 |
| 1.0 | 0.0172 | 0.0129 | 0.0108 |
| 1.5 | 0.0545 | 0.0302 | 0.0206 |
| 1.7 | 0.1003 | 0.0498 | 0.0307 |
| 1.8 | 0.1385 | 0.0658 | 0.0389 |
| 1.9 | 0.1917 | 0.0885 | 0.0501 |
| 2.0 | 0.2635 | 0.1202 | 0.0658 |
| 2.1 | 0.3562 | 0.1642 | 0.0876 |
| 2.2 | 0.4689 | 0.2237 | 0.1178 |
| 2.3 | 0.5945 | 0.3015 | 0.1592 |
| 2.4 | 0.7205 | 0.3987 | 0.2148 |
| 2.5 | 0.8312 | 0.5123 | 0.2873 |

Table 2

| $\Delta t, \mathrm{~s}$ | $H_{\mathrm{ds}}, \mathrm{m}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\tau=1 \mathrm{~s}$ | $\tau=1.5 \mathrm{~s}$ | $\tau=2 \mathrm{~s}$ |
| 0.2 | -2.052 | -2.334 | -2.590 |
| 0.3 | -1.993 | -2.270 | -2.523 |
| 0.4 | -1.937 | -2.208 | -2.458 |
| 0.5 | -1.881 | -2.147 | -2.394 |
| 1.0 | -1.633 | -1.871 | -2.099 |
| 1.5 | -1.425 | -1.635 | -2.844 |
| 1.7 | -1.352 | -1.552 | -1.752 |
| 1.8 | -1.317 | -1.512 | -1.708 |
| 1.9 | -1.284 | -1.473 | -1.666 |
| 2.0 | -1.251 | -1.436 | -1.625 |
| 2.1 | -1.220 | -1.400 | -1.585 |
| 2.2 | -1.190 | -1.365 | -1.546 |
| 2.3 | -1.161 | -1.332 | -1.509 |
| 2.4 | -1.133 | -1.299 | -1.472 |
| 2.5 | -1.106 | -1.268 | -1.437 |

Table 3

| $\Delta t, \mathrm{~s}$ | $v_{\text {tc }}, \mathrm{m} / \mathrm{s}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\tau=1 \mathrm{~s}$ | $\tau=1.5 \mathrm{~s}$ | $\tau=2 \mathrm{~s}$ |
| 0.2 | 67.210 | 67.039 | 66.944 |
| 0.3 | 67.389 | 67.174 | 67.053 |
| 0.4 | 67.591 | 67.322 | 67.173 |
| 0.5 | 67.786 | 67.482 | 67.304 |
| 1.0 | 68.930 | 68.407 | 68.079 |
| 1.5 | 70.166 | 69.464 | 68.991 |
| 1.7 | 70.664 | 69.904 | 69.380 |
| 1.8 | 70.912 | 70.125 | 69.577 |
| 1.9 | 71.157 | 70.348 | 69.776 |
| 2.0 | 71.402 | 70.570 | 69.976 |
| 2.1 | 71.644 | 70.793 | 70.177 |
| 2.2 | 71.883 | 71.014 | 70.379 |
| 2.3 | 72.120 | 71.235 | 70.581 |
| 2.4 | 72.354 | 71.455 | 70.783 |
| 2.5 | 72.585 | 71.674 | 70.985 |

Table 4

| $\Delta t, \mathrm{~s}$ | $v_{\mathrm{lv}}, \mathrm{m} / \mathrm{s}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\tau=1 \mathrm{~s}$ | $\tau=1.5 \mathrm{~s}$ | $\tau=2 \mathrm{~s}$ |
| 0.2 | 76.941 | 75.952 | 75.208 |
| 0.3 | 77.198 | 76.192 | 75.430 |
| 0.4 | 77.453 | 76.433 | 75.652 |
| 0.5 | 77.705 | 76.674 | 75.876 |
| 1.0 | 78.929 | 77.859 | 76.991 |
| 1.5 | 80.077 | 78.997 | 78.084 |
| 1.7 | 80.153 | 79.436 | 78.511 |
| 1.8 | 80.727 | 79.651 | 78.722 |
| 1.9 | 80.935 | 79.865 | 78.931 |
| 2.0 | 81.142 | 80.075 | 79.137 |
| 2.1 | 81.345 | 80.283 | 79.342 |
| 2.2 | 81.543 | 80.487 | 79.545 |
| 2.3 | 81.739 | 80.689 | 79.746 |
| 2.4 | 81.931 | 80.888 | 79.944 |
| 2.5 | 82.120 | 81.084 | 80.140 |

## REFERENCES

1. Dekker, R.M., Tests to Determine the Suitability of Ship-based Aircraft for Take-off and Landing Operations, Central Aerohydrodynamic Institute Technical Report no. 12242, 1973.
2. Semakov, S.L., Vybrosy sluchainykh protsessov: prilozheniya v aviatsii (Bursts of Random Processes: Applications in Aviation), Moscow: Nauka, 2005.
3. Rife, J., Khanafseh, S., Pullen, S., et. al., Navigation, Interference Suppression, and Fault Monitoring in the Sea-based Joint Precision Approach and Landing System, Proc. of the IEEE, 2008, vol. 96, no. 12, pp. 1958-1975.
4. Isaacs, J.T., Ezal, K.O., and Hespanha, J.P., Local Carrier-based Precision Approach and Landing System, Proc. 2016 IEEE 55th Conference on Decision and Control (CDC-2016), Las Vegas, 2016, pp. 6284-6290.
5. Semakov, S.L. and Semakov, I.S., Estimating the Probability of Safe Landing for Aircrafts, Proc. 2019 IEEE 58th Conference on Decision and Control (CDC-2019), Nice, 2019, pp. 2568-2573.
6. Hess, R.A., Analysis of the Aircraft Carrier Landing Task, Pilot + Augmentation/Automation, IFACPapersOnLine, 2019, vol. 51, no. 34, pp. 359-365.
7. Sidar, M. and Doolin, B., On the Feasibility of Real-Time Prediction of Aircraft Carrier Motion at Sea, IEEE Trans. Autom. Control, 1983, vol. 28, no. 3, pp. 350-356.
8. Bem, L.A., Gurov, V.F., Kabachinskii, V.V., et al., Patent no. RU2042583C1, 1991.
9. Semakov, S.L., Crossings Problems in Random Processes Theory and Their Applications in Aviation, Newcastle: Cambridge Scholars Publishing, 2019.
10. Semakov, S.L., Aircraft Landing Strategy, Proc. 2021 IEEE 7th International Conference on Control, Automation and Robotics (ICCAR-2021), Singapore, 2021, pp. 184-188.
11. Barratt, S.T., Kochenderfe, M.J., and Boyd, S.P., Learning Probabilistic Trajectory Models of Aircraft in Terminal Airspace from Position Data, IEEE Trans. Intell. Transp. Syst., 2019, vol. 20, no. 9, pp. 35363545.
12. Zhen, Z., Yu, C., Jiang, S., and Jiang, J., Adaptive Super-twisting Control for Automatic Carrier Landing of Aircraft, IEEE Trans. Aerosp. Electron. Syst., 2020, vol. 56, no. 2, pp. 984-997.
13. Semakov, S.L. and Semakov, I.S., Method of Calculating the Probability of a Safe Landing for Ship-Based Aircraft, IEEE Trans. Aerosp. Electron. Syst., 2022, vol. 58, no. 6, pp. 5425-5442.
14. Semakov, S.L., First Arrival of a Stochastic Process at the Boundary, Autom. Remote Control, 1988, vol. 49, no. 6, pp. 757-764.
15. Semakov, S.L., The Probability of the First Hitting of a Level by a Component of a Multidimensional Process on a Prescribed Interval under Restrictions on the Remaining Components, Theor. Prob. App., 1989, vol. 34, no. 2, pp. 357-361.
16. Semakov, S.L. and Semakov, I.S., Estimating the Probability that a Random Process First Reaches the Boundary of a Region on a Given Time Interval, Proc. 2018 IEEE 57 th Conference on Decision and Control (CDC-2018), Miami Beach, 2018, pp. 256-261.

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[^0]:    ${ }^{1}$ Of course, the go-around maneuver can be decided before the expected instant of touching the deck if it becomes clear that, for one reason or another, the probability of a successful landing is not high enough. This aspect of landing on carriers was discussed in detail in [10].

[^1]:    ${ }^{2}$ This method is based on the results originally obtained in [14] and subsequently refined in $[15,16]$.
    ${ }^{3}$ It is less than $10^{-5}$ under moderate sea state; see [1].

