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CONTROL IN TECHNICAL SYSTEMS

Control of the Search for Observation Objects from a Spatio-Temporal Poisson Flow in a Multi-Channel Search System

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Abstract—The article considers the problem of searching for objects of observation for the case, when the sequence of their appearance satisfies the laws of spatial and temporal Poisson flow. Its solution is obtained without taking into account the limitations associated with the significant excess of the search effort intensity over the intensity of the flow of observation objects. As a mathematical model, used for optimization of the search, the system of differential equations describing dynamics of changing of mathematical expectation of number of objects present in subdomains of the search system's field of view, but not yet detected. A procedure for optimizing the distribution of search effort intensity in search system channels for of dynamic and steady state search modes. Presented are examples.

Keywords: search for observation objects, spatiotemporal Poisson flows, the distribution of search effort intensities in search system channels, an infinite system of Kolmogorov differential equations

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1. INTRODUCTION

One of the important varieties of the general class of measurement process control problems [1, 2] is the class of search control problems [3–6]. A rational distribution of search effort in the search system (SAR) ensures the reduction of undetected observation objects (OO) in the SAR viewing area, reducing the search time, increasing the reliability of detection of the OO, and etc.

As a rule, the search task is considered on the assumption that there is one or more OOs in the SAR viewing area and their number does not change [3–6]. That is a quite rigid assumption, because for a number of practical problems the number of OOs is arbitrary, and the objects themselves can appear sequentially, for example, in accordance with the patterns of spatiotemporal random flow.

The works [7–9] are devoted to the study of the search problem in the latter case. Their common feature is the assumption about the significant excess of intensity of search effort of SAR over the intensity of Poisson flow of OO. The specified assumption is fulfilled for SAR in the conditions of high energy signal-to-noise ratio. It allows to approximate the mathematical model of evolution of probabilistic search characteristics in the form of infinite-dimensional system of Kolmogorov differential equations with mathematical model of discrete Markov process with two states, described with two-dimensional system of differential equations.

For a number of practical cases where the signal-to-noise energy ratio turns out to be insufficient, the assumption of a significant exceed of the intensity of SAR search effort over the intensity of Poisson's flow of OOs may not be fullfilled. This situation in particular arises when at a given intensity of OO in the field of view it is impossible to provide the necessary intensity search effort. For example, in the task of active radar search with a fixed probing signal power this may be due to the large distance between the observation area and the receiving antenna or the small values of the effective surfaces of scattering of objects of observation from the stream. The decrease of intensity of search effort for a given intensity of the Poisson flow of OOs leads to increasing of conditional probability of missing OO [3] and, as a consequence, an increase in the probability of the presence of not one, but several undetected OO, which makes application of the approaches considered in [7–9] inappropriate.

In this connection, the issue of determining the control of search effort distribution in multi-channel parallel-type SAR at intensities of the Poisson flow of the appearance of OO in area of the search system both comparable with intensity of search, as well as exceeding it in magnitude.

2. ANALYSIS OF THE STRUCTURE OF THE MATHEMATICAL SEARCH MODEL. PROBLEM STATEMENT

Let us denote by $X \in R^n$ $(0 < n \leq 3)$ the region of the search system with the Cartesian coordinate system $\{x_1, \ldots, x_n\}$. Suppose that the observation objects appear in X according to the patterns of the spatiotemporal Poisson flow φ [10–13].

Consider a multi-channel SAR, including I channels, each one serving its own part of the survey area X, which corresponds to a subset X_i, $i = \overline{1, I}$.

Each of the channels serves a different part of the overview area X, which corresponds to the subarea X_i, $i = \overline{1, I}$. Here we will assume that $X = \bigcup\limits_i X_i$, $X_i \bigcap X_j = \varnothing$, $i = \overline{1, I}$, $j = \overline{1, I}$, $i \neq j$. Then any two streams defined from φ as

$$\varphi_i(t) = \varphi(X_i, t), \quad \varphi_j(t) = \varphi(X_j, t), \quad i = \overline{1, I}, \quad j = \overline{1, I}, \quad i \neq j, \tag{2.1}$$

are Poisson and independent.

The temporal Poisson flow corresponding to X can be defined through (2.1) as

$$\varphi(t) = \varphi(X, t) = \sum_i \varphi(X_i, t). \tag{2.2}$$

Let us denote the intensity density of the spatiotemporal Poisson flow by $\nu(x, t)$, where $\nu(x, t)$ is a non-negative measurable on X function. Then the measures of intensities, or intensities $\xi_i(t)$, of temporal Poisson flows $\varphi_i(t) = \varphi(X_i, t)$, generated by the spatiotemporal Poisson flow φ in X_i, $i = \overline{1, I}$, can be defined through the integral on the Lebesgue measure of $\nu(x, t)$ [10–13]

$$\xi_i(t) = \int\limits_{X_i} \nu(x, t) dx, \quad i = \overline{1, I}. \tag{2.3}$$

For the integral flow $\varphi(t) = \varphi(X, t)$ respectively we get

$$\xi(t) = \int\limits_X \nu(x, t) dx = \sum_i \int\limits_{X_i} \nu(x, t) dx = \sum_i \xi_i(t). \tag{2.4}$$

Here $dx = \prod\limits_{q=1}^{n} dx_q$, $0 < n \leq 3$.

According to (2.3), (2.4) measures of intensities, or intensities $\xi_i(t)$, $i = \overline{1, I}$, $\xi(t)$, are known deterministic functions, in particular they can also be constants, and characterize temporal Poisson flows $\varphi_i(t)$, $i = \overline{1, I}$, $\varphi(t)$ as flows with variable parameters [14].

The probability of occurrence of another OO in the subregion X_i during time $[t, t + \Delta t]$ is defined as $\xi_i(t)\Delta t + o(\Delta t)$ [7–9], where $o(\Delta t)$ is a residual order of smallness higher than Δt.

Let ith SAR channel provide search intensity $\lambda_i(t) \geq 0$, $t \in [0, \bar{t}]$, where $\bar{t}$—is the length of the search time interval. This means that if X_i is present in k OO, then the probability that during time $[t, t + \Delta t]$ at least one of them will be found, is $k\lambda_i(t)\Delta t + o(\Delta t)$ [3, 7]. The search intensities $\lambda_i(t)$ in the subdomains X_i, $i = \overline{1, I}$ are assumed to be unknown deterministic functions to be determined by solving of the optimization problem.

The problem of the search system considering the subarea X_i can be interpreted as the problem of its serving for the OOs flow, forming a Poisson-type load with an intensity measure $\xi_i(t)$. To describe the problems of this class, as a rule, the mathematical apparatus of multiplication and death processes is used [10–12, 14–17]. As a mathematical model of such processes one can use Kolmogorov's system of linear ordinary differential equations with respect to the probabilities of each of possible states, forming a Cauchy problem with some initial conditions [14–16]:

$$\begin{aligned}
&\dot{P}_{i0} = -\xi_i(t)P_{i0} + \lambda_i(t)P_{i1},\\
&\dot{P}_{ik} = -(\xi_i(t) + k\lambda_i(t))P_{ik} + \xi_i(t)P_{ik-1} + (k+1)\lambda_i(t)P_{ik+1},\\
&k = 1, 2, \ldots, \quad P_{i0}(0) = 1, \quad P_{ik}(0) = 0, \quad i = \overline{1, I}, \quad t \in [0, \bar{t}],
\end{aligned} \tag{2.5}$$

where P_{i0} is the probability of absense of undetected OOs in X_i; P_{ik}—the probability of finding in X_i k of undetected OOs; the functions $\{\lambda_i(t), \xi_i(t), i = \overline{1, I}\}$ on the interval $[0, \bar{t}]$ are assumed continuous and bounded.

The Cauchy problem (2.5) reflects the physical meaning of the search problem performed sequentially in time as new OO appears in the region of view X. Due to the linear dependence of the coefficients at P_{ik} on k, the solution (2.5) according to [15, 17] exists, is unique, and satisfies the the regularity requirement $\sum_k P_{ik} = 1$.

It should be noted that the solution of the system (2.5) at $t \in [0, \bar{t}]$, including at $t = \bar{t}$, for each of $k = 1, 2, \ldots$ is determined by the values of the functions $\xi_i(t)$, $\lambda_i(t)$, $i = \overline{1, I}$ and can be obtained using the derivative function method [14, 15, 17]. In accordance with the general regularities of Poisson processes and processes generated by them [10–12, 14, 15], at $k \to \infty$ $P_{ik}(t) \to 0$, $i = \overline{1, I}$ $\forall t \in [0, \bar{t}]$.

Using mathematical models (2.5) to to find the control law of the search effort distribution in the SAR is difficult due to their infinite dimensionality.

Let us perform the transformation of the systems of differential equations Kolmogorov (2.5) (Appendix A). As a result we obtain

$$\dot{\mu} = -\lambda(t) \circ \mu + \xi(t), \quad \mu(0) = 0, \quad t \in [0, \bar{t}], \tag{2.6}$$

where μ, λ, $\xi \in R^I$; $\mu^T = [\mu_1 \ldots \mu_I]$; $\lambda(t)^T = [\lambda_1(t) \ldots \lambda_I(t)]$; $\xi(t)^T = [\xi_1(t) \ldots \xi_I(t)]$; $\mu_i = \sum_k kP_{ik}$—is the mathematical expectation of the number of undetected OOs located in X_i, $i = \overline{1, I}$; the letter T denotes the transpose operation; the operation $\circ$ denotes the product of Adamar.

From (2.6) it obviously follows that $\mu \geq 0$ $\forall t \in [0, \bar{t}]$. The vector equation (2.6) describes a mathematical model of a dummy dynamical system [1], characterizing the evolution in time of the mathematical expectations of the number of undetected OOs, located in subdomains X_i of the survey region X. Equation (2.6) for each value $i = \overline{1, I}$ is a one-dimensional convolution of the infinite dimensional system (2.5).

Let's define the quality criterion of the SAR operation by the ratio

$$\Upsilon = A^T\mu(\bar{t}) + B\int_0^{\bar{t}} \lambda(t)^T\lambda(t)dt \to \min_{\lambda}, \tag{2.7}$$

where $A^T = [a_1 \ldots a_I]$; $a_i > 0$, $i = \overline{1, I}$, $B \in R^1 > 0$ are weight coefficients.

The criterion (2.7) implies minimizing two components at the final moment of observation. The first component characterizes the weighted total mathematical expectation of the number of undetected OOs in the X review region, and the second—corresponds to the analogue of the energy cost of the SAR to conduct the search.

Let us set the problem to determine the distribution of search effort intensities $\lambda_i(t)$, $i = \overline{1, I}$, $t \in [0, \bar{t}]$ in (2.6) between channels of a multi-channel search system serving non-intersecting subareas X_i, $i = \overline{1, I}$ of the survey region X, which would satisfy the (2.7) criterion.

3. SYNTHESIS OF OO SEARCH CONTROL FROM SPATIO-TEMPORAL POISSON FLOW IN MULTICHANNEL SEARCH SYSTEM

To synthesize the control of the observation object search from spatiotemporal Poisson flow φ in multi-channel search system according to (2.6), (2.7), let us compose the Hamiltonian

$$H = \psi^T(-\lambda \circ \mu + \xi) + B\lambda^T\lambda, \tag{3.1}$$

where $\psi = \psi(t) \in R^I$—vector of conjugate variables.

From (3.1) we obtain a system of equations relative to $\psi(t)$:

$$\dot{\psi} = -\frac{\partial}{\partial \mu}\mathrm{H} = \psi \circ \lambda, \quad t \in [0, \bar{t}]. \tag{3.2}$$

Given (2.7), the boundary conditions for (3.2) can be represented as

$$\psi(\bar{t}) = \frac{\partial}{\partial \mu(\bar{t})} A^T \mu(\bar{t}) = A. \tag{3.3}$$

From the condition of the Hamiltonian minimum over λ we obtain

$$\frac{\partial}{\partial \lambda}\mathrm{H} = -\psi \circ \mu + 2B\lambda = 0. \tag{3.4}$$

Given (3.4), determine the structure of the optimal control

$$\lambda = \frac{1}{2B}\psi \circ \mu. \tag{3.5}$$

From (2.6), (3.2), (3.3) it obviously it follows that $\lambda \geq 0 \ \forall t \in [0, \bar{t}]$.

The set of relations (2.6), (3.2) (3.3), (3.5) form a two-point boundary problem, which is very difficult to solve. In this connection, we can use the Krylov–Chernousko method of successive approximations [1, 18] to determine the optimal search control.

Let at the qth step of the iterative procedure form the law of λ^q.

The method of successive approximations involves the following operations.

1. We solve equations (2.6) in forward time and the vector of variables μ^q, corresponding to the control λ^q.

2. The system of equations (3.2) is solved in inverse time with finite conditions (3.3) and the values of the vector of conjugate variables $\psi^q(t)$, $t \in [0, \bar{t}]$, corresponding to the control λ^q and the vector of variables μ^q.

3. Using the obtained values of vectors $\mu^q(t)$, $\psi^q(t)$ according to (3.5), the the intermediate value of the control vector, corresponding to $(q+1)$th step of the iteration procedure

$$\tilde{\lambda}^{q+1}(t) = \frac{1}{2B}\psi^q(t) \circ \mu^q(t), \quad t \in [0, \bar{t}]. \tag{3.6}$$

4. Based on the principle of partial control updating [1, 18] by (3.6) taking into account the values of the control vector λ^q obtained at the previous step, the calculation of its $(q+1)$th iteration

$$\{\lambda^{q+1}(t)\} = \{\tilde{\lambda}^{q+1}(t)\}_{\varepsilon^q} \cup \{\lambda^q(t)\}_{1-\varepsilon^q}, \quad t \in [0, \bar{t}], \tag{3.7}$$

where $\varepsilon^q \in (0, 1)$.

The parameter ε^q denotes the degree of updating of the search control law $\lambda^q(t)$, $t \in [0, \bar{t}]$. It is determined from the condition of the minimum of the criterion target function (2.7) at the corresponding step of the iteration procedure.

Next, the $(q+2)$th step is used as the initial control $\lambda^{q+1}(t)$, and the iterative procedure 1–4 is repeated.

Note that for the initial step of the iteration procedure $(q = 0)$ the initial control law λ^0 is chosen from the set of of admissible control laws given by the structure (3.5).

The optimal control is determined by the ratio

$$\lambda_{\rm op}(t) = \lim_{q \to \infty} \lambda^q(t), \quad t \in [0, \bar{t}]. \tag{3.8}$$

In practice, it is usually limited to a finite number of iterations $q \leq Q$, where Q—the number of step after which variations of the target criterion function (2.7) become insignificant. In this case, it is assumed that $\lambda_{\rm op}(t) \simeq \lambda^Q(t)$.

4. EXAMPLE OF SYNTHESIS OF THE CONTROL LAW FOR THE SEARCH OF OBJECTS OF SPATIOTEMPORAL POISSON FLOW FOR TWO-CHANNEL SEARCH SYSTEM

Let

$$I = 2, \quad A^T = [1 \;\; 1], \quad B = 0.1, \quad \bar{t} = 1, \quad \xi_1 = 2, \quad \xi_2 = 1. \tag{4.1}$$

Hereinafter, the variables are presented in dimensionless units. Different values of the flux intensities ξ_1 and ξ_2 in the subdomains X_1, X_2 of the survey area X, corresponding to the channels of the two-channel search system, are chosen to illustrate the influence of their values on the structure of the control law of the search control law.

Considering (4.1), we specify the mathematical models (2.5). As a result, we obtain

$$\begin{aligned}
&\dot{P}_{10} = -2P_{10} + \lambda_1(t)P_{11},\\
&\dot{P}_{1k} = -(2 + k\lambda_1(t))P_{1k} + 2P_{1k-1} + (k+1)\lambda_1(t)P_{1k+1},\\
&\dot{P}_{20} = -P_{20} + \lambda_2(t)P_{21},\\
&\dot{P}_{2k} = -(1 + k\lambda_2(t))P_{2k} + P_{2k-1} + (k+1)\lambda_2(t)P_{2k+1},\\
&k = 1, 2, \ldots, \quad P_{10}(0) = P_{20}(0) = 1, \quad P_{1k}(0) = P_{2k}(0) = 0, \quad t \in [0, 1].
\end{aligned} \tag{4.2}$$

According to (2.6) from (4.2), the equations of the time evolution of the mathematical expectations of the number of undetected OOs in subdomains X_1, X_2 of the survey region X:

$$\begin{aligned}
&\dot{\mu}_1 = -\lambda_1(t)\mu_1 + 2, \quad \mu_1(0) = 0,\\
&\dot{\mu}_2 = -\lambda_2(t)\mu_2 + 1, \quad \mu_2(0) = 0, \quad t \in [0, 1].
\end{aligned} \tag{4.3}$$

Concretizing the quality criterion (2.7), we get

$$\Upsilon = \mu_1(\bar{t}) + \mu_2(\bar{t}) + B\int_0^{\bar{t}} (\lambda_1^2(t) + \lambda_2^2(t))dt \to \min_{\lambda_1, \lambda_2}. \tag{4.4}$$

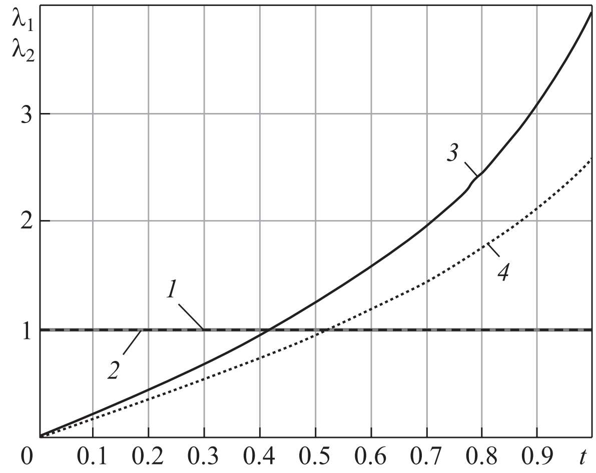

Fig. 1.

The Hamiltonian according to (3.1) is defined by by the relation

$$\mathrm{H} = \psi_1(-\lambda_1\mu_1 + 2) + \psi_2(-\lambda_2\mu_2 + 1) + B(\lambda_1^2 + \lambda_2^2). \tag{4.5}$$

From (3.2), (3.3) (4.3), (4.5) follow the equations for conjugate variables

$$\begin{aligned} &\dot{\psi}_1 = \psi_1\lambda_1, \quad \psi_1(\bar{t}) = 1, \\ &\dot{\psi}_2 = \psi_2\lambda_2, \quad \psi_2(\bar{t}) = 1, \quad t \in [0,1]. \end{aligned} \tag{4.6}$$

From (3.5) follows the structure of the control law of search

$$\lambda_1 = \frac{1}{2B}\psi_1\mu_1, \quad \lambda_2 = \frac{1}{2B}\psi_2\mu_2. \tag{4.7}$$

When implementing the procedure of the method of successive approximations as an initial approximation for the subdomains X_1 and X_2 of the SAR review area X was used as an initial approximation of search intensity on the interval [0, 1] (lines *1*, *2*, Fig. 1)

$$\lambda_1^0 = \lambda_2^0 = \begin{cases} f = 1, & t \in [0,1] \\ 0, & t \notin [0,1]. \end{cases} \tag{4.8}$$

At each step of the iterative procedure according to item 4 of the sequential approximation algorithm, the optimal value of the parameter $\varepsilon^q \in (0,1)$ associated with (3.7) with partial updating of the control.

The graph of the dependence of the target function $\Upsilon(q)$ of the criterion (4.4) on the number of iteration at $f = 1$ is shown in Fig. 2 (curve *1*).

The same figure shows the dependencies of the target function on the number of iterations at other initial approximations of the control law search, corresponding to $f = 0.75$ (curve *2*) and $f = 0.5$ (curve *3*).

The presented graphs have a piecewise linear structure and illustrate the greatest contribution of the first step of the iterative procedure in reduction of the target function Υ.

In all cases, variations in the values of Υ become insignificant already at $q > 2$. The corresponding $q = Q = 3$, $f = 1$ search control laws for the subdomains $X_1 - \lambda_1(t)$ and $X_2 - \lambda_2(t)$ of the review region X of the two-channel SAR is presented in Fig. 1 (respectively curves *3*, *4*). It

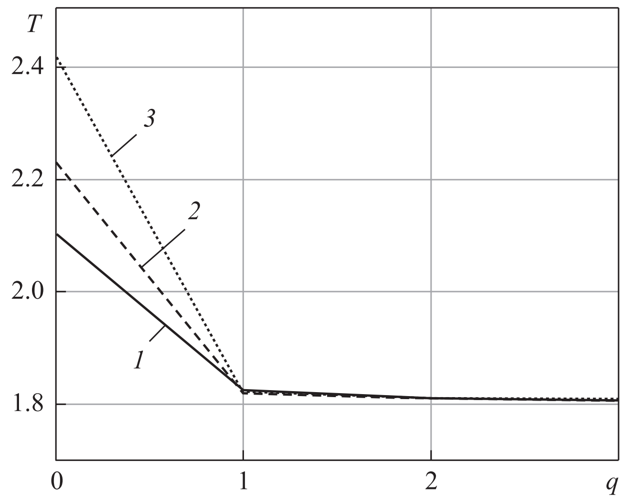

Fig. 2.

should be noted that similar control laws obtained at $f = 0.75$, $f = 0.5$ practically do not differ. It follows from the graphs that the intensity of of search effort for the X_1 subarea is higher than for the X_2. This is due to the fact that according to (4.1) the intensity measure ξ_1 of the Poisson flow $\varphi_1(t) = \varphi(X_1, t)$ is greater than the measure of the intensity ξ_2 of the Poisson flow $\varphi_2(t) = \varphi(X_2, t)$.

Note that in the conditions of the example the gain from the optimization of the search control, defined according to the relation $\delta = \frac{\Upsilon_0 - \Upsilon_{\text{op}}}{\Upsilon 0}$, is $\delta \simeq 0.14$ $(f = 1)$, $\delta \simeq 0.19$ $(f = 0.75)$, $\delta \simeq 0.25$ $(f = 0.5)$.

5. OPTIMIZATION OF DISTRIBUTION OF SEARCH EFFORT INTENSITIES AMONG CHANNELS OF MULTI-CHANNEL SEARCH SYSTEM IN STEADY STATE SEARCH MODE OF OBSERVATION OBJECTS FROM SPATIOTEMPORAL POISSON FLOW

When solving a number of practical problems, the functioning of the SAR in the search for objects of observation from the spatial-temporal Poisson flow is carried out in a steady-state mode. In particular, they can include the problem of search and detection of space debris, whose particle flux can be regarded as a stationary Poisson flow [19].

Let the intensity density of the spatiotemporal Poisson flow φ is independent of time $\nu(x,t) = \nu(x)$. Then for sublattices X_i, $i = \overline{1, I}$ of the review region X measures the intensities Poisson flows $\varphi_i(t) = \varphi(X_i, t)$, $i = \overline{1, I}$ will not depend on time:

$$\xi_i = \text{const}, \quad i = \overline{1, I}. \tag{5.1}$$

Let us define for the intensities of search efforts in the SAR channels the following constraints:

$$\lambda_i = \text{const}, \quad \lambda_i \geq 0, \quad i = \overline{1, I}, \tag{5.2}$$

$$\|\lambda\|_1 = \Lambda, \tag{5.3}$$

where $\|\cdot\|_1$ with (5.2) means l_1-norm of vector λ on C^I [20].

Then the mathematical model (2.6) for the mathematical expectation of the number of undetected objects in the steady-state ($\dot{\mu} = 0$) can be represented as

$$\lambda \circ \mu = \xi. \tag{5.4}$$

Let us define, taking into account the constraint (5.3), the criterion of the quality of search effort distribution in SAR channels

$$\Upsilon = A^T\mu + \eta(\|\lambda\|_1 - \Lambda) \to \min_{\lambda}, \tag{5.5}$$

where $\eta \in R^1$—indeterminate Lagrange multiplier.

Solving the optimization problem (5.4) (5.5) leads to the following result:

$$\lambda_{\text{op}} = sV, \tag{5.6}$$

where

$$s = \frac{\Lambda}{\sum_i \sqrt{a_i\xi_i}}, \quad V^T = [\sqrt{a_1\xi_1} \dots \sqrt{a_I\xi_I}].$$

Let us compare the obtained optimal distribution of the search effort (5.6) in the SAR with a uniform control law of search

$$\lambda_{\text{even}} = \frac{\Lambda}{I}E_I, \tag{5.7}$$

where $(E_I)^T = [1 \dots 1] \in R^I$—a unit vector.

Let $A = E_I$. Then the mathematical expectation of the number of undetected OOs in the survey area X, for (5.6)

$$\mu_{\text{op}} = \sum_i \mu_{\text{op}i} = \frac{\left(\sum_i \sqrt{\xi_i}\right)^2}{\Lambda}, \tag{5.8}$$

where

$$\mu_{\text{op}i} = \frac{\sqrt{\xi_i}\sum_i \sqrt{\xi_i}}{\Lambda}.$$

When the search effort is evenly distributed (5.7) for a similar characteristic respectively we get

$$\mu_{\text{even}} = \mu_{\text{even}i} = \frac{I\sum_i \xi_i}{\Lambda}, \tag{5.9}$$

where

$$\mu_{\text{even}i} = \frac{I\xi_i}{\Lambda}.$$

The relative gain is defined as $\delta = \frac{\mu_{\text{even}} - \mu_{\text{op}}}{\mu_{\text{even}}}$ or

$$\delta = 1 - \frac{\left(\sum_i \sqrt{\xi_i}\right)^2}{I\sum_i \xi_i}. \tag{5.10}$$

It can be shown (Appendix B) that for any $\xi_i > 0$, $i = \overline{1, I}$

$$\frac{\left(\sum\limits_i \sqrt{\xi_i}\right)^2}{I \sum\limits_i \xi_i} \leq 1, \tag{5.11}$$

where the equal sign obviously takes place for the same values of intensities $\xi_1 = \xi_2 = \ldots = \xi_I$.

Then, for example, in the exponential distribution of measures of intensities ξ_i of Poisson flows $\varphi_i(t)$ in X_i, $i = \overline{1, I}$, given by the relation $\xi_i = \exp\left\{-\frac{(i-m)^2}{D}\right\}$, where $m = 50$, $D = 500$, $I = 100$, the optimization gain is $\delta = 0.246$.

6. CONCLUSION

The considered approach to the control of the search for OO from spatiotemporal Poisson flow in a multichannel search system does not imply the use of restrictions on the significant excess of the intensity of the search effort of the SAR over the intensity of the Poisson flow of observation objects.

It is based on mathematical models of the time evolution of the probabilistic characteristics of Poisson flows in the subdomains of the survey area served by the SAR channels. Each of these models, which is an infinite system of Kolmogorov differential equations, is reduced to a scalar differential equation describing the dynamics of changes in the mathematical expectation of the number of undetected SN located in the corresponding subarea of the viewing area.

The dimensionality of the optimized dummy dynamic system is equal to the number of SAR channels. Due to the high complexity of the two-point boundary value problem arising during optimization, it is reasonable to determine the search control using the Krylov–Chernousko method of successive approximations with application of the partial control update principle.

The problem of optimization of search effort distribution in multi-channel SAR is significantly simplified if the search system operates in a steady-state mode. Its solution is reduced to the search of the conditional extremum argument by the uncertain Lagrange multipliers method.

The above examples illustrated the gain obtained from optimizing the distribution of search effort in a multi-channel SAR when searching for OO from a spatio-temporal Poisson flow.

APPENDIX A

Sequence of Transformation of Differential Kolmogorov Equations (2.5)

Let us multiply the kth equation of each ith $(i = \overline{1, I})$ of the system (2.5) by k $(k = 1, 2, \ldots)$ and sum them by k. As a result, we obtain

$$\sum_{k=1}^{\infty} \dot{P}_{ik} k = \lambda_i F + \xi_i G, \tag{A.1}$$

where $\sum_{k=1}^{\infty} \dot{P}_{ik} k = \dot{\mu}_i$—the rate of change of the mathematical expectation of the number of undetected OOs in X_i, $i = \overline{1, I}$;

$$G = -\sum_{k=1}^{\infty} P_{ik} k + \sum_{k=1}^{\infty} P_{ik-1} k, \tag{A.2}$$

$$F_i = -\sum_{k=1}^{\infty} P_{ik} k^2 + \sum_{k=1}^{\infty} P_{ik+1} k^2 + \sum_{k=1}^{\infty} P_{ik+1} k. \tag{A.3}$$

By revealing the sums in (A.2), it can be shown that

$$G = \sum_{k=0}^{\infty} P_{ik} = 1. \tag{A.4}$$

Carrying out a similar operation for (A.3), we obtain

$$F_i = -\sum_{k=1}^{\infty} P_{ik} k = -\mu_i. \tag{A.5}$$

From (A.1), (A.4), (A.5) it follows that

$$\dot{\mu}_i = -\lambda_i(t)\mu_i + \xi_i(t), \quad i = \overline{1, I}. \tag{A.6}$$

The description of the system of differential equations (A.6) corresponds to vector notation (2.6).

APPENDIX B

Rationale for Inequality (5.11)

From (5.11) it follows that

$$I \sum_{i=1}^{I} \xi_i \geq \sum_{i=1}^{I} \xi_i + 2 \sum_{j=1}^{I-1} \sum_{i=j+1}^{I} \sqrt{\xi_j \xi_i}. \tag{B.1}$$

Or

$$(I-1) \sum_{i=1}^{I} \xi_i \geq 2 \sum_{j=1}^{I-1} \sum_{i=j+1}^{I} \sqrt{\xi_j \xi_i}. \tag{B.2}$$

Let us write a system of inequalities

$$\begin{aligned} &\xi_1 + \xi_2 \geq 2\sqrt{\xi_1 \xi_2}, \\ &\xi_1 + \xi_3 \geq 2\sqrt{\xi_1 \xi_3}, \\ &\dots\dots\dots\dots\dots\dots \\ &\xi_{I-1} + \xi_I \geq 2\sqrt{\xi_{I-1} \xi_I}. \end{aligned} \tag{B.3}$$

Summing up the inequalities in (B.3), we obtain (B.2) and, respectively (B.1). The equation (5.11) is valid.

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