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An Interval Observer-Based Method to Diagnose Discrete-Time Systems

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Abstract—This paper proposes a method for diagnosing linear dynamic systems described by discrete-time models with exogenous disturbances based on interval observers. Formulas are derived to construct an interval observer producing two values of the residual as follows: if zero is between these values, then the system has no faults to be detected by the observer. The case where zero does not belong to the interval between these values is qualified as the occurrence of a fault. The theoretical results are illustrated by an example.

Keywords: linear systems, discrete-time models, interval observers, diagnosis, faults

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1. INTRODUCTION AND PROBLEM STATEMENT

This paper further develops the works [1, 2], devoted to the design of interval observers for systems described by linear models with exogenous disturbances. The corresponding problem has been actively studied in recent years. Overviews of the current results can be found in [3, 4]; the solutions for various classes of systems and related applications, in [5–10]. Characteristic features of the cited research are as follows. First, the interval observer has a dimension coinciding with that of the original system; second, the set of admissible values of the full state vector is estimated. At the same time, it may be of theoretical and practical interest to obtain such an estimate only for a given linear function of the state vector. The corresponding interval observer may be considerably simpler than the full-dimensional observer and the resulting interval may have an appreciably smaller width.

In [11–16], interval observers were used to perform functional diagnosis. The authors [11–13, 15] designed the observer based on the original system, which led to cumbersome constructs and complicated methods for minimizing the influence of exogenous disturbances on the diagnosis process. The paper [14] considered the diagnosis problem in a family of coupled subsystems: for each subsystem, a particular interval observer of full dimension was constructed. In [16], a practical problem was solved based on a special-form interval observer.

As is known, an adaptive threshold is traditionally used to reduce the probabilities of false alarms and fault omissions during diagnosis. This threshold sets lower and upper bounds for the residual in the absence of faults. Although the concept of an adaptive threshold appeared more than 30 years ago, it has been developed for various classes of systems up to the present time; for example, see [17, 18]. In these works, the residual was generated by a diagnostic observer whereas the adaptive threshold was formed separately. Such an approach leads to rather complicated diagnostic schemes.

In contrast, due to its specifics, an interval diagnostic observer produces only two values of the residual, which significantly simplifies the scheme. In addition, the residuals are formed so that in the absence of faults, the values of one residual are nonpositive and those of the other are nonnegative. In other words, if zero lies between these values, then the system has no faults to be detected by the observer. The case where zero does not belong to the interval between these values is qualified as the occurrence of a fault. In addition, unlike traditional adaptive threshold schemes, the values of residuals produced by the interval observer are independent of the control and output signals of the diagnosed system. This property also simplifies the decision process based on the diagnosis results.

In this paper, we construct minimal-dimension interval observers for discrete time-invariant systems described by linear dynamic models operating under exogenous disturbances in order to solve the problems of functional diagnosis (fault detection and isolation). In [1, 2], interval observers were used to estimate the values of a given linear function of the state vector of the original system. In contrast to [1, 2], in accordance with the diagnosis task, we change the observer structure and also consider several related issues: methods to maximize sensitivity to faults and isolate them.

Consider a class of systems with the linear discrete-time model

$$\begin{aligned} x(t+1) &= Fx(t) + Gu(t) + Dd(t) + L\rho(t), \\ y(t) &= Hx(t), \end{aligned} \tag{1.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^l$ denote the state, control, and output vectors, respectively; F, G, H, L, and D are given constant matrices; $\rho(t) \in \mathbb{R}^q$ is an unknown bounded time-varying function describing exogenous disturbances of the system, i.e., $\underline{\rho} \leqslant \rho(t) \leqslant \overline{\rho}$ with given values $\underline{\rho}$ and $\overline{\rho}$. In many cases, system faults occur due to unacceptable changes in system parameters. Therefore, we assume that the variations of the function $d(t) \in \mathbb{R}^p$ within $\underline{d} \leqslant d(t) \leqslant \overline{d}$ with given values $\underline{d}$ and $\overline{d}$ are admissible, being not treated as a fault; leaving the interval $[\underline{d}, \overline{d}]$ is qualified as a fault to be detected. As in the paper [3], for arbitrary vectors x^1, x^2 and matrices A^1, A^2, the relations $x^1 \leqslant x^2$ and $A^1 \leqslant A^2$ are understood elementwise.

2. THE MAIN RESULT

The problem under consideration will be solved using the minimal-dimension model of system (1.1). In the general case, this model is described by the equation

$$\begin{aligned} x_*(t+1) &= F_*x_*(t) + G_*u(t) + J_*y(t) + D_*d(t) + L_*\rho(t), \\ y_*(t) &= H_*x_*(t), \end{aligned} \tag{2.1}$$

where $x_*(t) \in \mathbb{R}^k$ and $k < n$ denotes the model dimension; $y_* \in \mathbb{R}$; F_*, G_*, J_*, H_*, D_*, and L_* are the matrices to be determined. By assumption, the relations $x_*(t) = \Phi x(t)$ and $y_*(t) = R_*y(t)$ with some matrices Φ and R_* hold in the absence of faults and exogenous disturbances. The rules for building this model are presented in Section 3.

According to [1, 2], the model matrices satisfy the conditions

$$\begin{aligned} &\Phi F = F_*\Phi + J_*H, \quad R_*H = H_*\Phi, \\ &\Phi G = G_*, \quad \Phi D = D_*, \quad \Phi L = L_*. \end{aligned} \tag{2.2}$$

As was demonstrated in [1], the matrices F_* and H_* can be written in the canonical form

$$F_* = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}, \quad H_* = (\ 1 \ \ 0 \ \ 0 \ \ \dots \ \ 0\ ). \tag{2.3}$$

From the standpoint of the problem solved, this form seems ideal since the matrix F_* is stable for discrete systems and nonnegative (a necessary property to construct an interval observer) and the matrix H_* is nonnegative (a property simplifying the observer's form).

The desired interval observer is constructed based on model (2.1). By analogy with [12, 13], we find it in the form

$$\begin{aligned} \underline{x}_*(t+1) &= F_*\underline{x}_*(t) + G_*u(t) + J_*y(t) + D_*^+\underline{d} - D_*^-\overline{d} + L_*^+\underline{\rho} - L_*^-\overline{\rho}, \\ \overline{x}_*(t+1) &= F_*\overline{x}_*(t) + G_*u(t) + J_*y(t) + D_*^+\overline{d} - D_*^-\underline{d} + L_*^+\overline{\rho} - L_*^-\underline{\rho}, \\ \underline{y}_*(t) &= H_*\underline{x}_*(t), \\ \overline{y}_*(t) &= H_*\overline{x}_*(t), \\ \underline{r}(t) &= R_*y(t) - \overline{y}_*(t), \\ \overline{r}(t) &= R_*y(t) - \underline{y}_*(t), \end{aligned} \tag{2.4}$$

where $A^+ = \max\{0, A\}$ and $A^- = A^+ - A$ for an arbitrary matrix A. Obviously, $A^+ \geqslant 0$ and $A^- \geqslant 0$.

Theorem 1. *If $\underline{x}_*(0) \leqslant x_*(0) \leqslant \overline{x}_*(0)$, then the relation $0 \in [\underline{r}(t), \overline{r}(t)]$ holds for all $t \geqslant 0$ in the absence of faults. The case $0 \notin [\underline{r}(t), \overline{r}(t)]$ for some $t > 0$ is qualified as the occurrence of a fault.*

Proof. We introduce the errors $\underline{e}(t) = x_*(t) - \underline{x}_*(t)$ and $\overline{e}(t) = \overline{x}_*(t) - x_*(t)$. In view of the expressions (1.1), (2.1), and (2.2), the equation for the first error can be written and transformed as follows:

$$\begin{aligned} \underline{e}(t+1) &= x_*(t+1) - \underline{x}_*(t+1) \\ &= F_*x_*(t) + G_*u(t) + J_*y(t) + D_*d(t) + L_*\rho(t) \\ &\quad - (F_*\underline{x}_*(t) + G_*u(t) + J_*y(t) + D_*^+\underline{d} - D_*^-\overline{d} + L_*^+\underline{\rho} - L_*^-\overline{\rho}) \\ &= F_*(\underline{e}(t) + \underline{x}_*(t)) - F_*\underline{x}_*(t) + D_*d(t) - (D_*^+\underline{d} - D_*^-\overline{d}) \\ &\quad + L_*\rho(t) - (L_*^+\underline{\rho} - L_*^-\overline{\rho}) \\ &= F_*\underline{e}(t) + D_*d(t) - (D_*^+\underline{d} - D_*^-\overline{d}) + L_*\rho(t) - (L_*^+\underline{\rho} - L_*^-\overline{\rho}). \end{aligned} \tag{2.5}$$

Since $D_* = D_*^+ - D_*^-$,

$$\begin{aligned} D_*d(t) - (D_*^+\underline{d} - D_*^-\overline{d}) &= D_*^+d(t) - D_*^-d(t) - (D_*^+\underline{d} - D_*^-\overline{d}) \\ &= D_*^+(d(t) - \underline{d}) + D_*^-(\overline{d} - d(t)). \end{aligned}$$

In the absence of faults, we have $\underline{d} \leqslant d(t) \leqslant \overline{d}$ and, in addition, $D_*^+ \geqslant 0$ and $D_*^- \geqslant 0$. Consequently,

$$D_*d(t) - (D_*^+\underline{d} - D_*^-\overline{d}) \geqslant 0.$$

Similar considerations are adopted to show that

$$L_*\rho(t) - (L_*^+\underline{\rho} - L_*^-\overline{\rho}) \geqslant 0.$$

Recall that, by assumption, $\underline{e}(0) = x_*(0) - \underline{x}_*(0) \geqslant 0$ and $F_* \geqslant 0$. From (2.5) it therefore follows that $\underline{e}(1) \geqslant 0$. By induction we establish the inequality $\underline{e}(t) \geqslant 0$ for all $t \geqslant 0$. The second inequality $\overline{e}(t) \geqslant 0$ is proved by analogy.

Considering (2.2) and $H_* \geqslant 0$, formula (2.4) implies

$$\begin{aligned}\underline{r}(t) &= R_*y(t) - \overline{y}_*(t) = R_*Hx(t) - H_*\overline{x}_*(t)\\ &= H_*\Phi x(t) - H_*(\overline{e}(t) + x_*(t))\\ &= H_*x_*(t) - H_*(\overline{e}(t) + x_*(t))\\ &= -H_*\overline{e}(t) \leqslant 0\end{aligned}$$

for all $t \geqslant 0$. Similar considerations yield $\overline{r}(t) = R_*y(t) - \underline{y}_*(t) \geqslant 0$. The last two inequalities are equivalent to the required result, which can be written as the implication

$$d(t) \in [\underline{d}, \overline{d}] \Rightarrow 0 \in [\underline{r}(t), \overline{r}(t)]$$

for all $t \geqslant 0$. Then, under the condition $0 \notin [\underline{r}(t), \overline{r}(t)]$ for some $t > 0$, applying the negation operation to this implication gives

$$0 \notin [\underline{r}(t), \overline{r}(t)] \Rightarrow d(t) \notin [\underline{d}, \overline{d}],$$

which corresponds to the occurrence of a fault. The maximal sensitivity to faults is ensured by choosing appropriate matrices of the observer; see Section 3. The proof of Theorem 1 is complete.

Remark 1. In principle, the condition $\underline{x}_*(0) \leqslant x_*(0) \leqslant \overline{x}_*(0)$ can be omitted: due to observer's stability, the requirement $0 \in [\underline{r}(t), \overline{r}(t)]$ will hold for all $t \geqslant t_0$ with some finite time instant t_0.

Remark 2. Since the matrix F_* is stable by construction, the observer (2.4) is stable as well. It seems natural to assume that the original system is also stable and the control action $u(t)$ is finite; in this case, the variables $y(t)$, $\underline{y}_*(t)$, $\overline{y}_*(t)$ and the residuals $\underline{r}(t)$ and $\overline{r}(t)$ will be finite as well.

Thus, the built observer produces the interval $[\underline{r}(t), \overline{r}(t)]$. If zero belongs to this interval, the decision about no system faults is made (see Section 1); otherwise, the occurrence of a fault is concluded. In view of the observer's equations (2.4), the width of the interval $[\underline{r}(t), \overline{r}(t)]$ depends on exogenous disturbances and the admissible range of the variable $d(t)$. The smaller this width is, the more reliably the faults will be detected.

In terms of diagnosis quality, in particular, sensitivity to faults, the best interval observer is the one with the minimal width $[\underline{r}(t), \overline{r}(t)]$. According to (2.4), the corresponding case is when the model has insensitivity to the disturbance, i.e., $L_* = \Phi L = 0$. The method for building such a model was developed in [1, 2]. We briefly describe it below.

3. MODEL BUILDING

3.1. Main Relations

Due to the canonical form (2.3), equations (2.2) can be written as

$$\Phi_1 = R_*H, \quad \Phi_iF = \Phi_{i+1} + J_{*i}H, \quad i = 1, \ldots, k-1, \quad \Phi_kF = J_{*k}H, \tag{3.1}$$

where Φ_i and J_{*i} indicate the ith rows of the matrices Φ and J_*, respectively, $i = 1, \ldots, k$, and k is the dimension of model (2.1). These equations are reduced [1, 2] into the single one

$$(\ R_* \quad -J_{*1} \quad -J_{*2} \quad \ldots \quad -J_{*k}\ )V^{(k)} = 0, \tag{3.2}$$

where

$$V^{(k)} = \begin{pmatrix} HF^k \\ HF^{k-1} \\ \dots \\ H \end{pmatrix}.$$

The condition of insensitivity to disturbances ($\Phi L = 0$) can be represented as

$$\begin{pmatrix} R_* & -J_{*1} & -J_{*2} & \dots & -J_{*k} \end{pmatrix} L^{(k)} = 0, \tag{3.3}$$

where

$$L^{(k)} = \begin{pmatrix} HL & HFL & HF^2L & \dots & HF^{k-1}L \\ 0 & HL & HFL & \dots & HF^{k-2}L \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}.$$

Since the row $(R_* \ \ -J_{*1} \ \ -J_{*2} \ \ \dots \ \ -J_{*k})$ satisfies (3.2), from (3.2) and (3.3) we obtain

$$\begin{pmatrix} R_* & -J_{*1} & -J_{*2} & \dots & -J_{*k} \end{pmatrix} (V^{(k)} \ L^{(k)}) = 0. \tag{3.4}$$

Equation (3.4) has a nontrivial solution if

$$\operatorname{rank} (V^{(k)} \ L^{(k)}) < l(k+1).$$

This condition serves to determine the minimal dimension $k \geqslant 1$ under which equation (3.4) is solvable. Then, it is necessary to find the solution of (3.4), obtain the rows of the matrix Φ from (3.1), and let $G_* := \Phi G$ and $D_* := \Phi D$.

3.2. Maximizing Sensitivity to Faults

If equation (3.4) with the minimal dimension k has several solutions, it is possible to choose the one with the maximal contribution of faults to the observer (consequently, the maximal sensitivity to faults, estimated by the norm of the matrix $D_* = \Phi D$). This can be done more efficiently as follows. By analogy with the analysis of the contribution made by exogenous disturbances, we introduce the matrix

$$D^{(k)} = \begin{pmatrix} HD & HFD & HF^2D & \dots & HF^{k-1}D \\ 0 & HD & HFD & \dots & HF^{k-2}D \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}.$$

Due to (3.1), it can be demonstrated that

$$\|D_*\| = \|(R_* \ - J_1 \ - J_2 \ \dots \ - J_k) D^{(k)}\|.$$

Then the contribution of faults is maximized by maximizing the norm

$$\|(R_* \ - J_1 \ - J_2 \ \dots \ - J_k) D^{(k)}\|$$

subject to condition (3.4).

Here, the idea is to find the minimal dimension k under which equation (3.4) has at least two linearly independent solutions of the form $(R_* \ \ -J_{*1} \ \ -J_{*2} \ \ \dots \ \ -J_{*k})$. All these solutions are combined in a matrix W so that each row represents some solution of equation (3.4). According to the aforesaid, another solution is an arbitrary linear combination of the rows of this matrix with a weight vector $w = (w_1, \ldots, w_N)$, where N specifies the number of rows in the matrix W. The problem is to determine the vector w maximizing the norm $\|wWD^{(k)}\|$.

To solve this problem, we calculate the singular-value decomposition of the matrix product $WD^{(k)}$. In other words, the matrix $WD^{(k)}$ is represented as

$$WD^{(k)} = U_D \Sigma_D V_D,$$

where U_D and V_D are orthogonal matrices and the matrix Σ_D has the form

$$\Sigma_D = (\mathrm{diag}(\sigma_1, \ldots, \sigma_s)\ 0) \quad \text{or} \quad \Sigma_D = \begin{pmatrix} \mathrm{diag}(\sigma_1, \ldots, \sigma_s) \\ 0 \end{pmatrix}$$

depending on the number of rows and columns of the matrix $WD^{(k)}$, where $s = \min(N, kp)$ and $0 \leqslant \sigma_1 \leqslant \ldots \leqslant \sigma_s$ denote the singular values of the matrix $WD^{(k)}$ [19, 20]. Choosing the ith transposed column of the matrix U_D as the weight vector $w = (w_1, \ldots, w_N)$ yields $\|wWD^{(k)}\| = \sigma_i$ [19, 20]. In view of the considerations above, the appropriate vector $w = (w_1, \ldots, w_N)$ is the transposed column of the matrix U_D that corresponds to the maximal singular value and $(R\ -J_{*1}\ -J_{*2}\ \ldots\ -J_{*k}) := wW$. Finally, it is necessary to obtain the rows of the matrix Φ from (3.1) and let $G_* := \Phi G$ and $D_* := \Phi D$.

Note that this solution is optimal for the chosen dimension k; increasing the dimension further may give a better solution in terms of the maximum norm of the matrix $(R\ -J_{*1}\ -J_{*2}\ \ldots\ -J_{*k})D^{(k)}$.

3.3. Minimizing the Contribution of Exogenous Disturbances

If for all $k < n$ equation (3.4) is unsolvable, we cannot build the model insensitive to exogenous disturbances. Then it is necessary to employ robust methods minimizing the contribution of exogenous disturbances to the model [19]. Based on the analysis above, this problem obviously reduces to minimizing the norm $\|(R_*\ -J_1\ -J_2\ \ldots\ -J_k)L^{(k)}\|$ subject to condition (3.2).

By analogy with the considerations above, the idea is to find the minimal dimension k under which equation (3.2) has at least two linearly independent solutions of the form $(\,R_*\ \ -J_{*1}\ \ -J_{*2}\ \ \ldots\ \ -J_{*k}\,)$. All these M solutions, are combined in a matrix V so that each row represents some solution of equation (3.2). The problem is to determine a weight vector $v = (v_1, \ldots, v_M)$ mimizing the norm $\|vVL^{(k)}\|$.

Next, we calculate the singular-value decomposition of the matrix product $VL^{(k)}$, i.e., $VL^{(k)} = U_L \Sigma_L V_L$, and take the first transposed column of the matrix U_L as the weight vector $v = (v_1, \ldots, v_M)$. According to the aforesaid, the linear combination of the solutions corresponding to the rows of the matrix V with the weights $v_1, \ldots, v_M$ gives the optimal solution $(\,R_*\ \ -J_{*1}\ \ -J_{*2}\ \ \ldots\ \ -J_{*k}\,) = vV$. Finally, it is necessary to obtain the rows of the matrix Φ from (3.1) and let $G_* := \Phi G$ and $D_* := \Phi D$. Thus, the robust model has been designed.

Other methods for building robust models, particularly the ones considering the contribution of faults, were discussed in [19].

4. ISOLATING FAULTS

The observer constructed above allows detecting the set of faults defined by the condition $D_* := \Phi D \neq 0$. To isolate faults, i.e., determine where they occur, it is necessary to design a bank of observers in which each observer will be sensitive to a particular set of faults and insensitive to the others. Such a bank can be constructed as follows. Let the set of possible faults in (1.1) be defined by the sum $\sum_{i=1}^{s} D_i d_i(t)$ instead of the term $Dd(t)$. A model insensitive to the first fault is built by solving the equation

$$(R_* \quad -J_{*1} \quad -J_{*2} \quad \dots \quad -J_{*k})(V^{(k)} \ D_1^{(k)}) = 0, \tag{4.1}$$

where

$$D_1^{(k)} = \begin{pmatrix} HD_1 & HFD_1 & HF^2D_1 & \dots & HF^{k-1}D_1 \\ 0 & HD_1 & HFD_1 & \dots & HF^{k-2}D_1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}.$$

Its minimal dimension k is determined starting from $k = 1$. Next, the rows of the matrix Φ are obtained, $G_* := \Phi G$ is assigned, and an interval observer is constructed according to the above rules. It will be insensitive to several other faults, particularly to those for which $D_j = D_1 N$ with some matrix N, and sensitive to the faults for which $\Phi D_j \neq 0$. Choosing the first fault among them, we build a model and an observer insensitive to it by analogy. The procedure continues until the consideration of all faults.

The information about the sensitivity and insensitivity of each observer is reflected by the syndrome matrix S, where the rows correspond to observers and the columns to faults. In this matrix, $S(i, j) = 0$ if the ith observer is insensitive to the jth fault, and $S(i, j) = 1$ otherwise. The syndrome matrix may have two or more identical columns, meaning that some system faults are indistinguishable from each other by the described procedure. Therefore, it is necessary to apply more sophisticated approaches.

For fault isolation, the most convenient matrices are

$$S^1 = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}, \quad S^2 = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix}.$$

The first matrix allows isolating faults of arbitrary multiplicity, but it is rarely implementable in applications due to the very strict requirement of insensitivity to many faults. From this point of view, the second matrix seems more practical, but it is not always implementable as well. The matter is that the elements of the matrix S may have certain relations due to the peculiarities and faults of system (1.1), which make their choice nonarbitrary.

5. A PRACTICAL EXAMPLE

Consider an electric drive whose open circuit is described by the following model with viscous friction:

$$\begin{aligned} x_1(t+1) &= \gamma_1 x_2(t) + x_1(t), \\ x_2(t+1) &= \gamma_2 x_2(t) + \gamma_3 x_3(t) + \rho(t), \\ x_3(t+1) &= \gamma_4 x_2(t) + \gamma_5 x_3(t) + \gamma_6 u(t) + d(t), \\ y_1(t) &= x_2(t), \quad y_2(t) = x_3(t). \end{aligned} \tag{5.1}$$

Here, x_1 is the rotation angle of the gearbox output shaft, x_2 is the angular velocity of the electric motor shaft, and x_3 is the electric motor current. The coefficients γ_1–γ_6 depend on the drive parameters and the sampling interval; in particular, viscous friction is specified by the coefficient γ_2.

These coefficients are given by

$$\gamma_1 = \frac{\Delta t}{i_r}, \quad \gamma_2 = -\frac{\Delta t k_b}{J} + 1, \quad \gamma_3 = \frac{\Delta t k_m}{J},$$
$$\gamma_4 = -\frac{\Delta t k_\omega}{L_m}, \quad \gamma_5 = -\frac{\Delta t R_m}{L_m} + 1, \quad \gamma_6 = \frac{\Delta t k_u}{L_m}$$

with the following notations: Δt is the sampling interval; i_r is is the gear ratio; k_b is the viscous friction coefficient; k_m is the torque coefficient; J is the moment of inertia of the motor rotor and rotating parts of the gearbox reduced to this rotor; k_ω is the counter-emf coefficient; R_m is the rated active resistance of the armature circuit; L_m is the armature circuit inductance; k_u is the power amplifier gain; finally, $u(t)$ is the drive input voltage.

The electric drive is described by the matrices

$$F = \begin{pmatrix} 1 & \gamma_1 & 0 \\ 0 & \gamma_2 & \gamma_3 \\ 0 & \gamma_4 & \gamma_5 \end{pmatrix}, \ G = \begin{pmatrix} 0 \\ 0 \\ \gamma_5 \end{pmatrix}, \ H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \ D = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \ L = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

We build a model insensitive to the disturbance. Letting $k = 1$, we calculate the matrices $V^{(1)}$ and $B^{(1)}$:

$$V^{(1)} = \begin{pmatrix} 1 & \gamma_1 & 0 \\ 0 & \gamma_4 & \gamma_5 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Since $\operatorname{rank}(V^{(1)}, B^{(1)}) = 3 < 2(1+1) = 4$, equation (3.4) has a solution with the matrices

$$R_* = (\gamma_4 \quad -\gamma_1), \quad J_* = (\gamma_4 \quad -\gamma_1\gamma_5).$$

As a result, $\Phi = (\gamma_4 \ 0 \ -\gamma_1)$, $G_* = -\gamma_1\gamma_6$, and $D_* = -\gamma_1$; model (2.1) takes the form

$$\begin{aligned} x_*(t+1) &= \gamma_4 y_1(t) - \gamma_1\gamma_5 y_2(t) - \gamma_1\gamma_6 u(t) - d(t), \\ y_*(t) &= x_*(t), \end{aligned} \tag{5.2}$$

where $x_* = \gamma_4 x_1 - \gamma_1 x_3$. Obviously, $D_*^+ = 0$ and $D_*^- = \gamma_1$.

According to (2.4) and (5.2), the interval observer is described by the equations

$$\begin{aligned} \underline{x}_*(t+1) &= \gamma_4 y_1(t) - \gamma_1\gamma_5 y_2(t) - \gamma_1\gamma_6 u(t) - \gamma_1 \overline{d}, \\ \overline{x}_*(t+1) &= \gamma_4 y_1(t) - \gamma_1\gamma_5 y_2(t) - \gamma_1\gamma_6 u(t) - \gamma_1 \underline{d}, \\ \underline{y}_*(t) &= \underline{x}_*(t), \quad \overline{y}_*(t) = \overline{x}_*(t), \\ \underline{r}(t) &= \gamma_4 y_1 - \gamma_1 y_3 - \overline{y}_*(t), \quad \overline{r}(t) = \gamma_4 y_1 - \gamma_1 y_3 - \underline{y}_*(t). \end{aligned}$$

For the sake of simplicity in simulation, we choose $\gamma_1 = \gamma_3 = \gamma_6 = 1$, $\gamma_2 = \gamma_4 = \gamma_5 = -1$, and $u(t) = 2 + \sin(t)$; the disturbance $\rho(t)$ is represented by a random variable with the uniform distribution on the interval $[-0.2, 0.2]$; finally, the admissible variations of the function $d(t)$ belong to the interval $[\underline{d}, \overline{d}] = [-0.05, 0.05]$. Figures 1 and 2 show the simulation results for the observer with the initial conditions $x_1(0) = x_2(0) = x_3(0) = 0$, $\underline{x}_*(0) = -0.2$, and $\overline{x}_*(0) = 0.2$.

In Fig. 1, $d(t) = 0$ for $t < 40$ s, and $d(t) = 0.04$ for $t \geqslant 40$ s. Since the value $d(t) = 0.04$ lies within the admissible interval, we have $0 \in [\underline{d}, \overline{d}]$, which is qualified as no faults. In Fig. 2, the function $d(t)$ is represented by a random variable with the uniform distribution on the interval

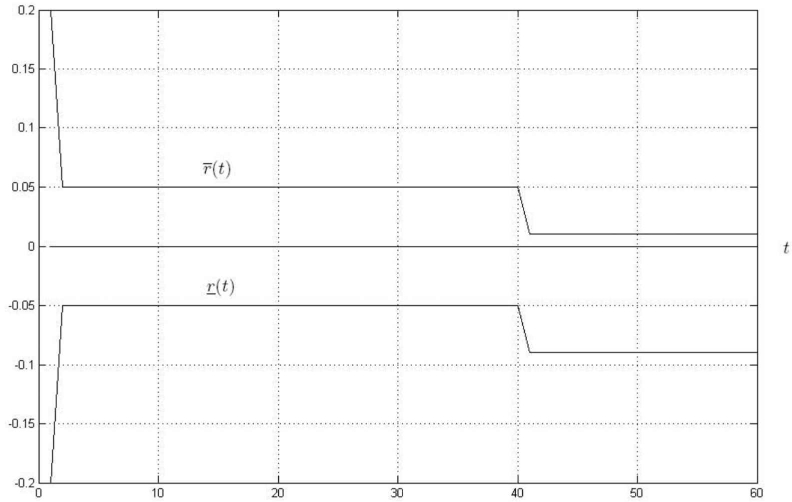

Fig. 1. The residuals $\underline{r}$ and $\overline{r}$ without faults.

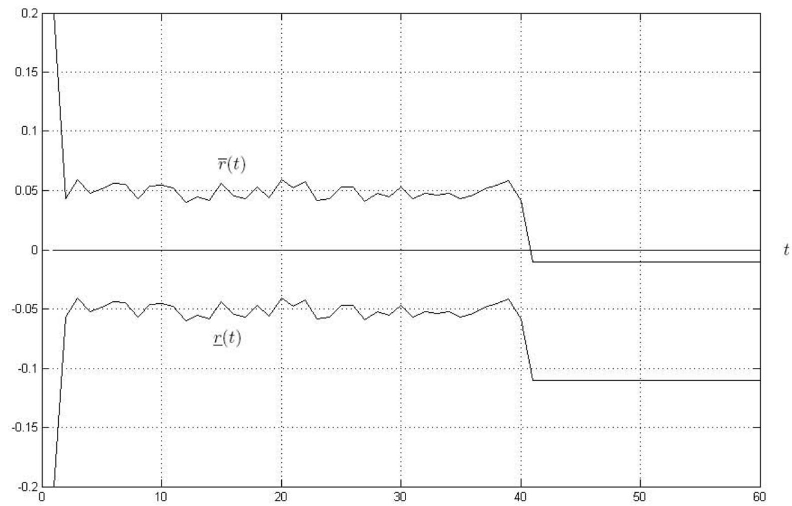

Fig. 2. The residuals $\underline{r}$ and $\overline{r}$ under fault occurrence.

$[-0.01, 0.01]$ for $t < 40$ s, and $d(t) = 0.06$ for $t \geqslant 40$ s. Now, $0 \notin [\underline{r}(t), \overline{r}(t)]$ for $t > 40$ s, and the occurrence of a fault is concluded.

According to Fig. 2, the random variable $d(t)$ affects the behavior of the functions $\underline{r}(t)$ and $\overline{r}(t)$. The disturbance $\rho(t)$ is also represented by a random variable, but the functions $\underline{r}(t)$ and $\overline{r}(t)$ in Fig. 1 are constant (except for a jump due to the change in the function $d(t)$). Therefore, the disturbance has no impact on the result.

6. CONCLUSIONS

This paper has considered interval observer-based functional diagnosis for linear dynamic systems described by discrete-time models with exogenous disturbances. Formulas have been derived to construct an interval observer that is insensitive to disturbances and sensitive to a limited extent. Such an observer produces two values of the residual as follows: if zero is between these values, then the system has no faults to be detected by the observer. The case where zero does not belong to the interval between these values is qualified as the occurrence of a fault. The theoretical results have been illustrated by an example of observer design for a real technical system. The simulation results of this example have confirmed the correctness of theoretical constructs related to fault detection.

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