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Stability Analysis of Mechanical Systems with Highly Nonlinear Positional Forces under Distributed Delay

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Abstract—This paper considers mechanical systems with linear velocity forces and highly nonlinear positional forces containing distributed-delay terms. Asymptotic stability conditions of system equilibria are proved using Lyapunov's direct method and the decomposition method. The developed approaches are applied to the monoaxial stabilization of a solid body. The theoretical outcomes are confirmed by computer simulation results.

Keywords: mechanical systems, distributed delay, stability, decomposition, Lyapunov–Krasovskii functional, monoaxial stabilization

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1. INTRODUCTION

Stability analysis of mechanical systems under given forces and their stabilization by available control forces are topical problems in the modern theory of control [1–5]. If the forces acting on a system are highly nonlinear, i.e., their power series expansions in terms of the generalized coordinates or velocities contain no linear terms, Lyapunov's direct method is a basic stability analysis tool. In the case of time-delay systems, this method is based either on the Razumikhin approach or Lyapunov–Krasovskii functionals [6–8]. However, note that for highly nonlinear systems, the construction of Lyapunov functions and Lyapunov–Krasovskii functionals becomes considerably more complicated.

The decomposition method is an effective way for solving this problem. The method investigates the properties of solutions of high-dimensional differential systems by analyzing the properties of reduced-dimension subsystems extracted from an original system [1, 9–11]. It is widely and successfully applied to analyze stability and stabilize mechanical systems. For example, this method was used to design control laws in electromechanical and robotic systems [12–16], track the trajectories of mechanical systems [17], design angular orientation control laws for spacecraft [18], and transfer a controlled Lagrangian system from an arbitrary initial state to a given terminal state in a finite time [2]. The monograph [9] considered a mechanical system described by linear autonomous differential equations of the second order; as demonstrated therein, its stability can be analyzed by studying the stability of two isolated subsystems described by first-order differential equations. This result was further developed in the publications [19–22].

Time-delay systems are an important and widespread class of dynamical systems found in mechanics due to nonlinear hysteresis-type effects, material shape memory effects, and finite signal propagation in control systems [6–8, 23]. Moreover, delay (in particular, distributed delay) can be deliberately introduced into controlled mechanical systems as the integral part of the PID controller to improve their transient characteristics [4, 24, 25].

Control systems with distributed delays were considered in [4, 24–29]; stability conditions based on Lyapunov–Krasovskii functionals were obtained, with both negative definite and sign-constant derivatives. The paper [30] investigated the stability of linear controlled mechanical systems with a distributed delay in positional forces using the decomposition method. The stability analysis involved the assumption of a large parametric coefficient at the vector of generalized velocities-dependent forces.

The novelty of this paper consists both in the problem statement and in the solution approaches. We consider a mechanical system under positional forces with a highly nonlinear dependence on the generalized coordinates with distributed-delay terms and other forces with a linear dependence on the generalized velocities. We adopt special Lyapunov–Krasovskii functionals of the full type [6] and the decomposition method to prove asymptotic stability conditions of system equilibria. As established below, for the system with highly nonlinear positional forces, stability conditions can be presented in a more simple and constructive form compared to the linear case [30].

2. PROBLEM STATEMENT

Consider a dynamic mechanical system described by the equations

$$A\ddot{q}(t) + B\dot{q}(t) + Q(q(t)) + \int\limits_{t-\tau}^{t} D(q(\xi))d\xi = 0, \tag{1}$$

with the following notations: $q(t)$ and $\dot{q}(t)$ are the n-dimensional vectors of generalized coordinates and generalized velocities, respectively; A and B are constant matrices of compatible dimensions; $Q(q)$ and $D(q)$ are continuous vector functions of the variable $q \in \mathbb{R}^n$; finally, τ is a fixed positive delay. Thus, the system has linear velocity forces and (generally speaking) nonlinear positional forces with distributed-delay terms.

Each solution $q(t, t_0, \chi)$, $t \geqslant t_0$, of system (1) is defined by an initial time instant $t_0 \geqslant 0$ and an initial function $\chi(\xi)$, where $\chi(\xi)$ belongs to the space $C^1([-\tau, 0], \mathbb{R}^n)$ of all continuously differentiable functions with the uniform norm $\|\chi\|_\tau = \max_{\xi\in[-\tau,0]} (\|\chi(\xi)\| + \|\dot{\chi}(\xi)\|)$ and $\|\cdot\|$ is the Euclidean vector norm. We denote by $q_t(t_0, \chi)$ the solution segment: $q_t(t_0, \chi) : \xi \to q(t + \xi, t_0, \chi)$, $\xi \in [-\tau, 0]$.

Assume that $Q(0) = D(0) = 0$. Hence, this system has the trivial equilibrium

$$q = \dot{q} = 0. \tag{2}$$

The goal of this paper is to obtain asymptotic stability conditions for this equilibrium.

Note that in the paper [30], such a problem was solved for the case of linear positional forces. Stability was analyzed using the decomposition approach to mechanical systems [9, 19–21]. However, the conditions derived in [30] involve some system of linear matrix inequalities (LMIs) and ensure the asymptotic stability of the equilibrium only under a large parametric coefficient at velocity forces.

This paper considers the case of highly nonlinear positional forces. In addition to the decomposition method, we employ an approach based on constructing a special Lyapunov–Krasovskii functional of the full type [6]. As shown below, in contrast to the linear case, the asymptotic stability of the system with highly nonlinear positional forces does not require a large parametric coefficient at velocity forces. Moreover, the stability conditions are written in a simpler and more constructive form. The results are applied to the monoaxial stabilization of a solid body.

3. CONSTRUCTION OF THE LYAPUNOV–KRASOVSKII FUNCTIONAL OF THE FULL TYPE

Let system (1) have the representation

$$A\ddot{q}(t) + B\dot{q}(t) + \frac{\partial \Pi(q(t))}{\partial q} + P(q(t))q(t) + \int\limits_{t-\tau}^{t} \frac{\partial \widetilde{\Pi}(q(\xi))}{\partial q} d\xi = 0, \quad q \in \mathbb{R}^n. \tag{3}$$

Here, A and B are symmetric and positive definite matrices, $\Pi(q)$ and $\widetilde{\Pi}(q)$ are continuously differentiable and homogeneous functions of order $\mu + 1 > 2$, and $P(q)$ is a continuous skew-symmetric matrix such that

$$\|P(q)\| \leqslant p_0 \|q\|^{\sigma-1}, \quad q \in \mathbb{R}^n, \tag{4}$$

where constants p_0 and σ satisfy the conditions $p_0 > 0$ and $\sigma > 1$.

Thus, in the case under consideration, A is a symmetric and positive definite matrix of inertial system characteristics, and the linear velocity forces are dissipative. In addition, according to the canonical structure theorem of force fields [31], the nonlinear positional forces are represented as the sums of potential and nonconservative components; for the vector function under the integration sign, the nonconservative component is zero.

As is known [21], if

$$\widetilde{\Pi}(q) \equiv 0, \tag{5}$$

the equilibrium (2) is asymptotically stable under the following sufficient conditions: the function $\Pi(q)$ is positive definite, whereas the parameter σ satisfies the inequality

$$2\sigma > \mu + 1. \tag{6}$$

We investigate the stability of this equilibrium provided that identity (5) fails.

Theorem 1. *If $\mu > 1$, the function $\Pi(q) + \tau\widetilde{\Pi}(q)$ is positive definite, and inequality* (6) *holds, then the equilibrium* (2) *of system* (3) *is asymptotically stable.*

The proofs of all theorems of this paper are given in the Appendix.

Remark 1. Theorem 1 provides a constructive way to stabilize the mechanical system by choosing positional forces with a distributed delay. It is important that the potential energy $\Pi(q)$ of the system may be non-positive definite, and the order of the nonconservative forces may be less than the homogeneity order of the vector functions $\partial\Pi(q)/\partial q$ and $\partial\widetilde{\Pi}(q)/\partial q$. Note that for linear systems, this stabilization process yields more rigid and less constructive conditions on the choice of the integrand term; for details, see [8, 25, 30].

The Lyapunov–Krasovskii functional (A.1) (see the proof of Theorem 1) serves for showing the asymptotic stability of the equilibrium of system (3) and, moreover, estimating the rate of convergence of the solutions to the equilibrium.

Indeed, due to (A.2) and (A.3), this functional satisfies the differential inequality

$$\dot{V} \leqslant -\tilde{c}\, V^{\frac{\mu+1}{2}}, \quad \tilde{c} = \text{const} > 0,$$

for $\|q_t\| < \delta$. Integrating it, we arrive at the following result.

Corollary 1. *Under the conditions of Theorem 1, there exist positive numbers $d_1, d_2, d_3,$ and $\tilde{\delta}$ such that if the initial data of the solution $q(t, t_0, \chi)$ of system* (3) *satisfy $t_0 \geqslant 0$ and $\|\chi\|_\tau < \tilde{\delta}$, then*

$$\|q(t, t_0, \chi)\| \leqslant d_1 \|\chi\|_\tau \left(1 + d_3 \|\chi\|_\tau^{\mu-1}(t - t_0)\right)^{-\frac{1}{\mu-1}},$$

$$\|\dot{q}(t, t_0, \chi)\| \leqslant d_2 \|\chi\|_\tau \left(1 + d_3 \|\chi\|_\tau^{\mu-1}(t - t_0)\right)^{-\frac{1}{\mu-1}}$$

for all $t \geqslant t_0$.

4. STABILITY ANALYSIS BASED ON DECOMPOSITION

Now consider the case where the matrices A and B in system (1) are constant without any special structure (in contrast to the previous section, they are not assumed symmetric and positive definite) and the vector functions $Q(q)$ and $D(q)$ are continuous for $q \in \mathbb{R}^n$ and homogeneous of an order $\mu > 1$. To analyze the stability of such a system, we apply the decomposition method as proposed in [9, 19–22].

We construct the isolated subsystems without delay:

$$A\dot{x}(t) = -Bx(t), \tag{7}$$

$$B\dot{y}(t) = -Q(y(t)) - \tau D(y(t)). \tag{8}$$

Theorem 2. *If $\mu > 1$ and the trivial solutions of subsystems* (7) *and* (8) *are asymptotically stable, then the trivial equilibrium* (2) *of system* (1) *is asymptotically stable as well.*

Remark 2. Like Theorem 1, this theorem provides a constructive way to stabilize the mechanical system by choosing the integrand function in the distributed-delay term. The Lyapunov–Krasovskii functional constructed in its proof also serves for estimating the rate of convergence of the solutions to the equilibrium.

5. MONOAXIAL STABILIZATION OF A SOLID BODY

The differential systems (1) and (3) have a typical structure for mathematical models describing a rich variety of mechanical systems. Nevertheless, many important applications lead to mathematical models in which the differential equations of motion are resolved with respect to the first derivatives. Despite this circumstance, the developed approaches can still be used for such models. Among them, we mention the dynamics of objects approximated by a solid body rotating relative to its center of mass. This section is devoted to one such problem: the developed approaches are successfully applied to the monoaxial stabilization of a solid body in space.

Consider a solid body rotating relative to its center of mass (point O) with an angular velocity ω. Let $Oxyz$ be the system of principal central axes of inertia rigidly connected with the body, and let $\Theta = \operatorname{diag}(J_1, J_2, J_3)$ be the tensor of inertia of the body in these axes. The dynamic Euler equations describing the body's rotational motion under the control moment M_u have the form

$$\Theta\,\dot{\omega}(t) + \omega(t) \times (\Theta\,\omega(t)) = M_u. \tag{9}$$

Passing to the monoaxial stabilization of the body, we introduce two unit vectors: r, stationary in the coordinate system $Oxyz$, and s, stationary in the inertial space. By the theorem on the total and local derivatives, we have the kinematic Poisson equation

$$\dot{s}(t) + \omega(t) \times s(t) = 0 \tag{10}$$

for the unit vector s. Together with equations (9), it forms a closed differential system. The monoaxial stabilization problem for the solid body [32] is to find the control moment M_u ensuring the existence and asymptotic stability of the solution

$$s = r, \quad \omega = 0 \tag{11}$$

for system (9), (10). This solution corresponds to the equilibrium of the body's axis.

As proved in [32], this problem can be solved using the control moment

$$M_u = -a\|s(t) - r\|^{\mu-1} s(t) \times r - F\omega(t).$$

Here, the first and second components are the restoring and dissipative moments, respectively. In addition, $a > 0$, $\mu \geqslant 1$, and F is a constant and positive definite matrix. A similar problem from the class of nonlinear dynamic problems [5, 33], however differing by the linear control moment and the noninertial base coordinate system, was considered in [36]. (Such coordinate systems are widely used in cosmodynamics [34, 35] along with inertial coordinate systems.) To smoothen the transients, a fundamentally important requirement for some satellite attitude control problems (in particular, under almost resonant conditions [34, 37]), the authors [36] applied a distributed-delay control law (the integral term), which proved to be effective.

Let us adopt the same approach to the current problem. For this purpose, we introduce the additional moment

$$M_\tau = b \int\limits_{t-\tau}^{t} \|s(\xi) - r\|^{\mu-1} s(\xi) \times r \, d\xi$$

with a fixed coefficient b and a fixed positive delay τ into the control system with the moment M_u. Then the Euler equations take the form

$$\begin{aligned} \Theta\dot{\omega}(t) + \omega(t) \times (\Theta\omega(t)) = -F\omega(t) - a\|s(t) - r\|^{\mu-1} s(t) \times r \\ + b \int\limits_{t-\tau}^{t} \|s(\xi) - r\|^{\mu-1} s(\xi) \times r \, d\xi. \end{aligned} \tag{12}$$

Assume that the initial functions $\chi(\xi)$ for system (10), (12) belong to the space $C([-\tau, 0], \mathbb{R}^6)$ of all continuous functions with the uniform norm $\|\chi\|_\tau = \max_{\xi\in[-\tau,0]} \|\chi(\xi)\|$. It is required to obtain asymptotic stability conditions of the equilibrium of system (10), (12). Such a problem was solved in [30] for the linear ($\mu = 1$) restoring moment and the linear moment M_τ. As proved therein, the monoaxial stabilization of the body can be ensured if

$$|b|\tau < a \tag{13}$$

and the dissipative component of the control moment has a sufficiently large and positive coefficient.

This section shows that with a highly nonlinear ($\mu > 1$) restoring moment and a highly nonlinear moment M_τ, the monoaxial stabilization of the solid body can be ensured under rigid constraints on the system parameters. In particular, the relaxation of requirements for the dissipative moment plays a crucial role in attitude stabilization problems for artificial Earth satellites: the creation of dissipative moments in space conditions is a difficult task.

Theorem 3. *Let $\mu > 1$. Then the inequality*

$$b\tau < a \tag{14}$$

ensures the asymptotic stability of the equilibrium (11) *of system* (10), (12).

Remark 3. Compared to condition (13), inequality (14) defines a wider range of admissible values of the system parameters, and Theorem 3 does not require a large parametric coefficient at the moment of dissipative forces. Moreover, in contrast to the linear case (see [30]), Theorem 3 leads to the following interesting result: with a highly nonlinear moment M_τ, the monoaxial stabilization of the body can be ensured for $a = 0$ and $b < 0$ (i.e., when no restoring moment acts on the body).

6. COMPUTER SIMULATION

Consider a solid body with the moments of inertia $J_1 = 5$, $J_2 = 6$, and $J_3 = 4$. Hereinafter, all physical quantities have units of measurement in the SI system. The problem consists in the monoaxial stabilization of the body in the inertial coordinate system in the equilibrium (11) with $r = \left(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right)^\top$. This problem can be solved using the control law described in the previous section (equations (10) and (12)).

Let $F = h \cdot \mathrm{diag}\,(1, 1, 1)$, $h = 0.7$, $a = 2$, $b = 2$, and $\tau = 0.9$. For $\mu = 1$ (the case of linear positional forces), we obtain the example considered in the paper [30]. By analogy with [30], assume that the solid body deviates from the equilibrium so that for $t \in [-\tau, 0]$, the roll, pitch, and yaw angles ("aircraft" angles) are $\varphi(t) = 0.5$, $\theta(t) = 0.6$, and $\psi(t) = -0.8$, respectively, and the projections of the body's angular velocity on the principal central axes of inertia are $\omega_x(t) = \omega_y(t) = \omega_z(t) = 1$. Figure 1 shows the monoaxial body stabilization process (the direction cosines of the stabilized axis relative to the base coordinate system).

Inequality (13) holds in this case. Similarly, inequality (13) will remain valid if we revert the sign of the parameter b. However, for $b = -2$ and the same values of all other parameters and initial conditions, the chosen control law fails to stabilize the body axis; see the numerical integration results in Fig. 2. Recall that the theorem proved in [30] ensures stabilization only for sufficiently large values of the parameter h.

Now we choose $\mu = 2$, which corresponds to the nonlinear restoring moment in the control law, and integrate equations (10) and (12) for $b = -2$ and the same values of all other parameters

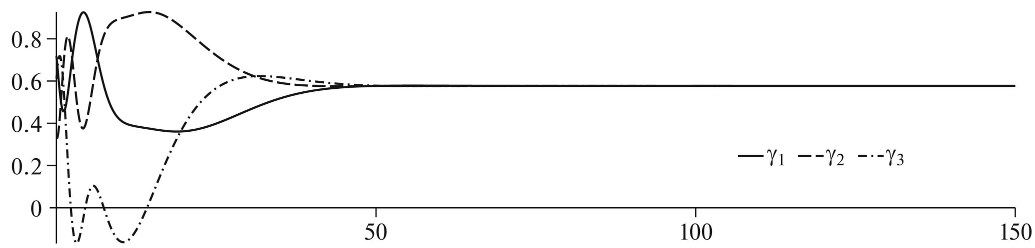

Fig. 1. The direction cosines of the stabilized axis: $b = 2$ and $\mu = 1$.

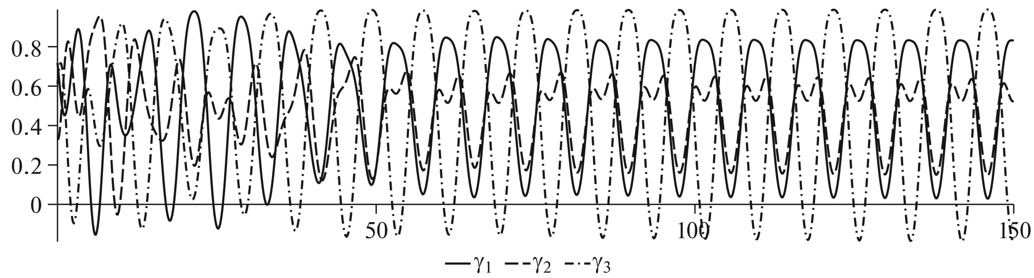

Fig. 2. The direction cosines of the stabilized axis: $b = -2$ and $\mu = 1$.

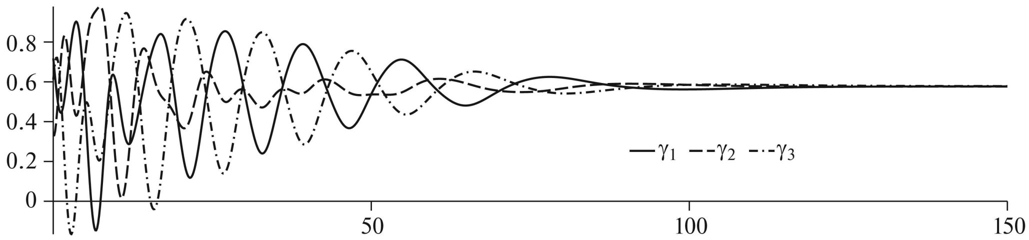

Fig. 3. The direction cosines of the stabilized axis: $b = -2$ and $\mu = 2$.

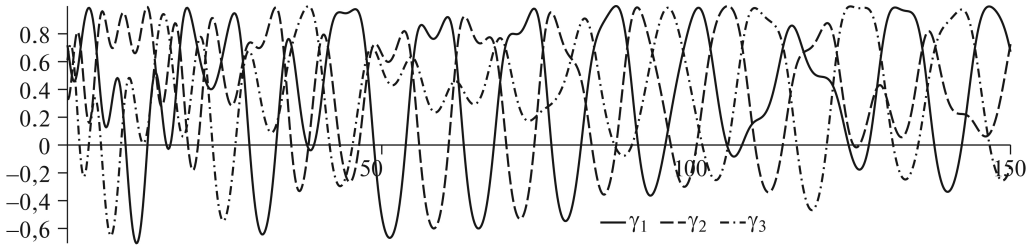

Fig. 4. The direction cosines of the stabilized axis: $a = 0.6$, $h = 0.2$, $b = -0.6$, and $\mu = 1$.

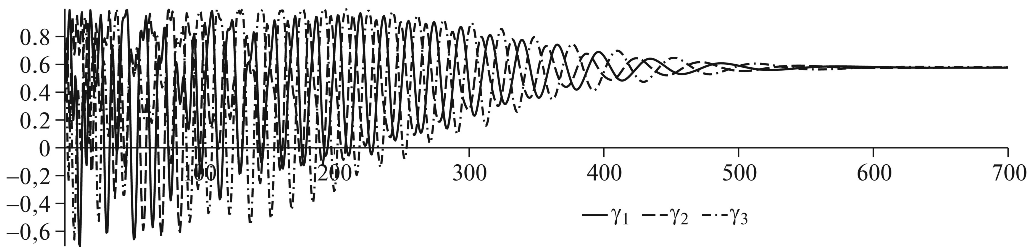

Fig. 5. The direction cosines of the stabilized axis: $a = 0.6$, $h = 0.2$, $b = -0.6$, and $\mu = 2$.

and initial conditions. According to Fig. 3, the resulting monoaxial body stabilization process is convergent, which fully agrees with Theorem 3.

Let us also illustrate Remark 3 to Theorem 3. To this end, we significantly reduce the coefficients at the restoring and dissipative moments by choosing $a = 0.6$ and $h = 0.2$ and let $b = -0.6$. First, we choose $\mu = 1$. In this case, inequality (13) fails, and the monoaxial body stabilization process does not converge. The chaotic process on a short time interval, shown for clarity in Fig. 4, continues in the same way on a one hundred times longer time interval as well.

Next, we choose $\mu = 2$, leaving all other parameters and initial conditions without change. In this case, inequality (14) holds. The corresponding computer simulation results are presented in Fig. 5.

Clearly, the monoaxial body stabilization process converges to the program position, which agrees with Theorem 3. The slow convergence of this process is due to the highly nonlinear control law and the small coefficients a and h. However, their smallness should not always be considered a drawback of the control system. In particular, in satellite attitude stabilization problems, this property can be a natural consequence of the operating conditions of the control system.

According to Remark 3, the coefficient a in Theorem 3 can be 0. Let us choose $a = 0$, $h = 1$, $\tau = 1$, $b = -1$, and $\mu = 2$, leaving the other parameters and initial conditions without change. The computer simulation results in Fig. 6 confirm the theoretical conclusion.

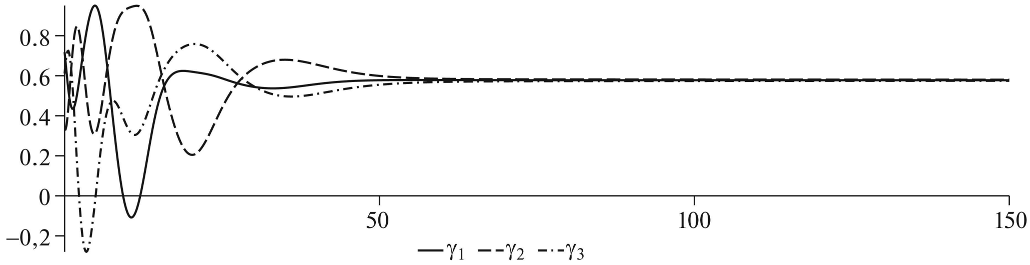

Fig. 6. The direction cosines of the stabilized axis: $a = 0$, $h = 1$, $b = -1$, and $\mu = 2$.

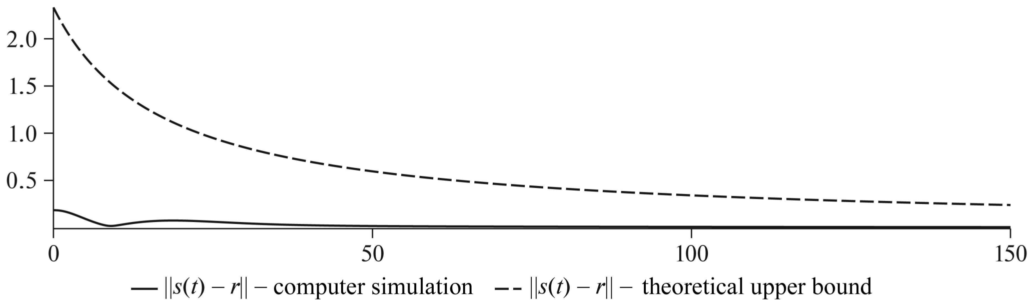

Fig. 7. Computer simulation results vs. theoretical outcomes.

In addition, for the case under consideration, the proof of Theorem 3 yields the following upper bounds for the domain of attraction and the rate of convergence of solutions:

$$\begin{gathered}\|s(t) - r\| \leqslant 9.21\|\chi\|_\tau \left(1 + 0.23\|\chi\|_\tau (t - t_0)\right)^{-1}, \\ \|\omega(t)\| \leqslant 7.44\|\chi\|_\tau \left(1 + 0.23\|\chi\|_\tau (t - t_0)\right)^{-1}\end{gathered} \tag{15}$$

for $\|\chi\|_\tau < 0.28$. We compare these theoretical outcomes with computer simulation results (Fig. 7). Here, the solid line corresponds to the values $\|s(t) - r\|$ obtained by numerical integration as a function of t and the dotted line to the upper bound (15).

The calculations involve the following initial functions: $\varphi(t) = 0.8$, $\theta(t) = 0.8$, $\psi(t) = -0.8$, and $\omega_x(t) = \omega_y(t) = \omega_z(t) = 0.1$ for $t \in [-\tau, 0]$. With this choice, the value $\|\chi\|_\tau$ is 0.25, and hence the inequality $\|\chi\|_\tau < 0.28$ holds.

7. CONCLUSIONS

This paper has considered the stability of solutions of differential systems describing the dynamics of mechanical systems (particularly in control systems) characterized by highly nonlinear positional forces and distributed delay. Special Lyapunov–Krasovskii functionals of the full type have been constructed, and the decomposition method has been applied to establish the following results: in contrast to the linear case, the asymptotic stability of a system with highly nonlinear positional forces does not require a large parametric coefficient at velocity forces; moreover, asymptotic stability conditions are written in a simpler and constructive form. We have proved two theorems on the stability of equilibria of such mechanical systems and one theorem on the monoaxial stabilization of a solid body in space by a positional forces-nonlinear control law with distributed delay. The computer simulation results have been presented to confirm the theoretical outcomes. Further research may consider the influence of delay on the upper bound for the domain of attraction.

APPENDIX

Proof of Theorem 1. Using the approaches from [20–22, 38], we construct the Lyapunov–Krasovskii functional

$$V(q_t) = \frac{1}{2}\lambda\dot{q}^\top(t)A\dot{q}(t) + \frac{1}{2}q^\top(t)Bq(t) + q^\top(t)A\dot{q}(t)$$
$$- q^\top(t)\int\limits_{t-\tau}^{t}(\xi - t + \tau)\frac{\partial\widetilde{\Pi}(q(\xi))}{\partial q}d\xi + \int\limits_{t-\tau}^{t}(\alpha + \beta(\xi - t + \tau))\|q(\xi)\|^{\mu+1}d\xi, \tag{A.1}$$

where λ, α, and β are positive parameters. Differentiating it along the trajectories of system (3) yields

$$\dot{V} = -\lambda\dot{q}^\top(t)B\dot{q}(t) + \dot{q}^\top(t)A\dot{q}(t) - \lambda\dot{q}^\top(t)\left(\frac{\partial\Pi(q(t))}{\partial q} + \int\limits_{t-\tau}^{t}\frac{\partial\widetilde{\Pi}(q(\xi))}{\partial q}d\xi + P(q(t))q(t)\right)$$
$$- q^\top(t)\left(\frac{\partial\Pi(q(t))}{\partial q} + \tau\frac{\partial\widetilde{\Pi}(q(t))}{\partial q}\right) - \dot{q}^\top(t)\int\limits_{t-\tau}^{t}(\xi - t + \tau)\frac{\partial\widetilde{\Pi}(q(\xi))}{\partial q}d\xi$$
$$- \beta\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi + (\alpha + \beta\tau)\|q(t)\|^{\mu+1} - \alpha\|q(t-\tau)\|^{\mu+1}.$$

Due to the properties of homogeneous functions [32], we obtain the upper bounds

$$\lambda c_1\|\dot{q}(t)\|^2 + c_2\|q(t)\|^2 - c_3\|q(t)\|\|\dot{q}(t)\| - c_4\tau\|q(t)\|\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu}d\xi + \alpha\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi \leqslant V(q_t)$$
$$\leqslant \lambda c_5\|\dot{q}(t)\|^2 + c_6\|q(t)\|^2 + c_3\|q(t)\|\|\dot{q}(t)\| + c_4\tau\|q(t)\|\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu}d\xi + (\alpha + \beta\tau)\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi,$$
$$\dot{V} \leqslant -(\lambda c_7 - c_8)\|\dot{q}(t)\|^2 + \lambda\|\dot{q}(t)\|\left(c_9\|q(t)\|^{\mu} + c_{10}\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu}d\xi + p_0\|q(t)\|^{\sigma}\right) - c_{11}\|q(t)\|^{\mu+1}$$
$$+ c_{12}\tau\|\dot{q}(t)\|\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu}d\xi - \beta\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi + (\alpha + \beta\tau)\|q(t)\|^{\mu+1} - \alpha\|q(t-\tau)\|^{\mu+1}.$$

Here, c_k are positive constants, $k = 1, \ldots, 12$.

By Young's inequality [7], for $\|q_t\|_\tau < \delta$, the positive numbers λ, α, β, and δ can be chosen so that

$$\frac{1}{2}\left(\lambda c_1\|\dot{q}(t)\|^2 + c_2\|q(t)\|^2 + \alpha\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi\right) \leqslant V(q_t)$$
$$\leqslant 2\left(\lambda c_5\|\dot{q}(t)\|^2 + c_6\|q(t)\|^2 + (\alpha + \beta\tau)\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi\right), \tag{A.2}$$
$$\dot{V} \leqslant -\frac{1}{2}\left(\lambda c_7\|\dot{q}(t)\|^2 + c_{11}\|q(t)\|^{\mu+1} + \beta\int\limits_{t-\tau}^{t}\|q(\xi)\|^{\mu+1}d\xi\right). \tag{A.3}$$

Hence, (A.1) is a Lyapunov–Krasovskii functional of the full type that satisfies the conditions of the asymptotic stability theorem [6, p. 22].

The proof of Theorem 1 is complete.

Proof of Theorem 2. Passing to the new variables $x(t) = \dot{q}(t)$, $y(t) = q(t) + B^{-1}A\dot{q}(t)$, we transform system (1) to

$$\begin{aligned} A\dot{x}(t) &= -Bx(t) - Q(y(t) - B^{-1}Ax(t)) - \int\limits_{t-\tau}^{t} D(y(\xi) - B^{-1}Ax(\xi))d\xi, \\ B\dot{y}(t) &= -Q(y(t) - B^{-1}Ax(t)) - \int\limits_{t-\tau}^{t} D(y(\xi) - B^{-1}Ax(\xi))d\xi. \end{aligned} \tag{A.4}$$

The trivial solutions of the isolated subsystems (7), (8) are asymptotically stable. Therefore, see [32, 39], for any numbers $\nu_1 \geqslant 2$ and $\nu_2 \geqslant 2$ there exist twice continuously differentiable Lyapunov functions $V_1(x)$ and $V_2(y)$ with homogeneity orders ν_1 and ν_2, respectively, such that for all $x, y \in \mathbb{R}^n$,

$$\begin{gathered} m_{11}\|x\|^{\nu_1} \leqslant V_1(x) \leqslant m_{12}\|x\|^{\nu_1}, \qquad m_{21}\|y\|^{\nu_2} \leqslant V_2(y) \leqslant m_{22}\|y\|^{\nu_2}, \\ \left\|\frac{\partial V_1(x)}{\partial x}\right\| \leqslant m_{13}\|x\|^{\nu_1-1}, \qquad \left\|\frac{\partial V_2(y)}{\partial y}\right\| \leqslant m_{23}\|y\|^{\nu_2-1}, \\ \left(\frac{\partial V_1(x)}{\partial x}\right)^{\top} A^{-1}Bx(t) \geqslant m_{14}\|x\|^{\nu_1}, \quad \left(\frac{\partial V_2(y)}{\partial y}\right)^{\top} B^{-1}(Q(y) + \tau D(y)) \geqslant m_{24}\|y\|^{\nu_2+\mu-1}. \end{gathered}$$

Here, m_{kj} are positive constants, $k = 1, 2$, $j = 1, 2, 3, 4$.

Consider the Lyapunov function

$$\widetilde{V}(x, y) = V_1(x) + V_2(y). \tag{A.5}$$

Calculating its derivative along the trajectories of system (A.4) and using the properties of homogeneous functions, we obtain the upper bound

$$\begin{aligned} \dot{\widetilde{V}} \leqslant{}& -m_{14}\|x(t)\|^{\nu_1} + c_1\|x(t)\|^{\nu_1-1}(\|x(t)\|^{\mu} + \|y(t)\|^{\mu}) \\ &+ c_2\|x(t)\|^{\nu_1-1}\int\limits_{t-\tau}^{t} (\|x(\xi)\|^{\mu} + \|y(\xi)\|^{\mu})\, d\xi - \left(\frac{\partial V_2(y(t))}{\partial y}\right)^{\top} B^{-1}\int\limits_{t-\tau}^{t} D(y(\xi))d\xi \\ &- \left(\frac{\partial V_2(y(t))}{\partial y}\right)^{\top} B^{-1}Q(y(t)) + c_3\|y(t)\|^{\nu_2-1}\|Q(y(t)) - Q(y(t) - B^{-1}Ax(t))\| \\ &+ c_4\|y(t)\|^{\nu_2-1}\int\limits_{t-\tau}^{t} \|D(y(\xi)) - D(y(\xi) - B^{-1}Ax(\xi))\|d\xi, \end{aligned}$$

where c_1, c_2, c_3, and c_4 are positive constants.

Note that for any numbers $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, it is possible to indicate $h_1 > 0$ and $h_2 > 0$ such that

$$\begin{aligned} \|Q(y) - Q(y - B^{-1}Ax)\| &\leqslant \varepsilon_1\|y\|^{\mu} + h_1\|x\|^{\mu}, \\ \|D(y) - D(y - B^{-1}Ax)\| &\leqslant \varepsilon_2\|y\|^{\mu} + h_2\|x\|^{\mu} \end{aligned}$$

for all $x, y \in \mathbb{R}^n$.

Now, we choose the Lyapunov–Krasovskii functional

$$V(x_t, y_t) = \widetilde{V}(x(t), y(t)) - \left(\frac{\partial V_2(y(t))}{\partial y}\right)^{\top} B^{-1} \int\limits_{t-\tau}^{t} (\xi + \tau - t) D(y(\xi)) d\xi$$

$$+ \int\limits_{t-\tau}^{t} (\alpha_1 + \beta_1(\xi + \tau - t)) \|x(\xi)\|^{\nu_1} d\xi + \int\limits_{t-\tau}^{t} (\alpha_2 + \beta_2(\xi + \tau - t)) \|y(\xi)\|^{\nu_2+\mu-1} d\xi,$$

where $\widetilde{V}(x, y)$ is the Lyapunov function given by (A.5) and $\alpha_1, \beta_1, \alpha_2$, and β_2 are positive parameters. As a result, we have

$$c_5\|x(t)\|^{\nu_1} + c_6\|y(t)\|^{\nu_2} - c_7\tau\|y(t)\|^{\nu_2-1} \int\limits_{t-\tau}^{t} \|y(\xi)\|^{\mu} d\xi$$

$$+ \alpha_1 \int\limits_{t-\tau}^{t} \|x(\xi)\|^{\nu_1} d\xi + \alpha_2 \int\limits_{t-\tau}^{t} \|y(\xi)\|^{\nu_2+\mu-1} d\xi \leqslant V(x_t, y_t)$$

$$\leqslant c_8\|x(t)\|^{\nu_1} + c_9\|y(t)\|^{\nu_2} + c_7\tau\|y(t)\|^{\nu_2-1} \int\limits_{t-\tau}^{t} \|y(\xi)\|^{\mu} d\xi$$

$$+ (\alpha_1 + \beta_1\tau) \int\limits_{t-\tau}^{t} \|x(\xi)\|^{\nu_1} d\xi + (\alpha_2 + \beta_2\tau) \int\limits_{t-\tau}^{t} \|y(\xi)\|^{\nu_2+\mu-1} d\xi,$$

$$\dot{V} \leqslant -m_{14}\|x(t)\|^{\nu_1} - m_{24}\|y(t)\|^{\nu_2+\mu-1} + c_1\|x(t)\|^{\nu_1-1}(\|x(t)\|^{\mu} + \|y(t)\|^{\mu})$$

$$+ c_2\|x(t)\|^{\nu_1-1} \int\limits_{t-\tau}^{t} (\|x(\xi)\|^{\mu} + \|y(\xi)\|^{\mu}) d\xi + c_3\|y(t)\|^{\nu_2-1}(\varepsilon_1\|y(t)\|^{\mu} + h_1\|x(t)\|^{\mu})$$

$$+ \tau c_{10}\|y(t)\|^{\nu_2-2} \int\limits_{t-\tau}^{t} \|y(\xi)\|^{\mu} d\xi \left(\|x(t)\|^{\mu} + \|y(t)\|^{\mu} + \int\limits_{t-\tau}^{t} (\|x(\xi)\|^{\mu} + \|y(\xi)\|^{\mu}) d\xi\right)$$

$$+ \varepsilon_2 c_4\|y(t)\|^{\nu_2-1} \int\limits_{t-\tau}^{t} \|y(\xi))\|^{\mu} d\xi + h_2 c_4\|y(t)\|^{\nu_2-1} \int\limits_{t-\tau}^{t} \|x(\xi)\|^{\mu} d\xi$$

$$- \beta_1 \int\limits_{t-\tau}^{t} \|x(\xi)\|^{\nu_1} d\xi - \beta_2 \int\limits_{t-\tau}^{t} \|y(\xi)\|^{\nu_2+\mu-1} d\xi$$

$$+ (\alpha_1 + \beta_1\tau)\|x(t)\|^{\nu_1} - \alpha_1\|x(t-\tau)\|^{\nu_1} + (\alpha_2 + \beta_2\tau)\|y(t)\|^{\nu_2+\mu-1}$$

$$- \alpha_2\|y(t-\tau)\|^{\nu_2+\mu-1}.$$

Here, $c_k > 0$, $k = 5, \ldots, 10$.

By Young's inequality [7], if the homogeneity orders of the functions $V_1(x)$ and $V_2(y)$ satisfy the condition $1 < (\nu_2 + \mu - 1)/\nu_1 < \mu$ and the values $\varepsilon_1, \varepsilon_2, \alpha_1, \beta_1, \alpha_2, \beta_2$, and δ are sufficiently small, we arrive at the relations

$$\frac{1}{2}\left(c_6\|y(t)\|^{\nu_2}+\alpha_2\int\limits_{t-\tau}^{t}\|y(\xi)\|^{\nu_2+\mu-1}d\xi\right)+c_5\|x(t)\|^{\nu_1}+\alpha_1\int\limits_{t-\tau}^{t}\|x(\xi)\|^{\nu_1}d\xi\leqslant V(x_t,y_t)$$

$$\leqslant c_8\|x(t)\|^{\nu_1}+(\alpha_1+\beta_1\tau)\int\limits_{t-\tau}^{t}\|x(\xi)\|^{\nu_1}d\xi+2\left(c_9\|y(t)\|^{\nu_2}+(\alpha_2+\beta_2\tau)\int\limits_{t-\tau}^{t}\|y(\xi)\|^{\nu_2+\mu-1}d\xi\right),$$

$$\dot V\leqslant-\frac{1}{2}\left(m_{14}\|x(t)\|^{\nu_1}+m_{24}\|y(t)\|^{\nu_2+\mu-1}+\beta_1\int\limits_{t-\tau}^{t}\|x(\xi)\|^{\nu_1}d\xi+\beta_2\int\limits_{t-\tau}^{t}\|y(\xi)\|^{\nu_2+\mu-1}d\xi\right)$$

holding for $\|x_t\|_\tau+\|y_t\|_\tau<\delta$.

The proof of Theorem 2 is complete.

Proof of Theorem 3. We choose the Lyapunov–Krasovskii functional

$$V(s_t,\omega_t)=\frac{1}{2}\lambda\omega^\top(t)\Theta\omega(t)+\frac{1}{2}\|s(t)-r\|^2+(s(t)\times r)^\top F^{-1}\Theta\omega(t)$$
$$+b(s(t)\times r)^\top F^{-1}\int\limits_{t-\tau}^{t}(\xi+\tau-t)\|s(\xi)-r\|^{\mu-1}s(\xi)\times r d\xi$$
$$+\int\limits_{t-\tau}^{t}(\alpha+\beta(\xi+\tau-t))\|s(\xi)-r\|^{\mu+1}d\xi,$$

where λ,α, and β are positive parameters.

This functional and its derivative along the trajectories of system (10), (12) admit the upper bounds

$$c_1\lambda\|\omega(t)\|^2+\frac{1}{2}\|s(t)-r\|^2-c_2\|s(t)-r\|\|\omega(t)\|-c_3|b|\tau\|s(t)-r\|\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu}d\xi$$
$$+\alpha\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu+1}d\xi\leqslant V(s_t,\omega_t)\leqslant c_4\lambda\|\omega(t)\|^2+\frac{1}{2}\|s(t)-r\|^2+c_2\|s(t)-r\|\|\omega(t)\|$$
$$+c_3|b|\tau\|s(t)-r\|\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu}d\xi+(\alpha+\beta\tau)\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu+1}d\xi,$$

$$\dot V\leqslant-(\lambda c_5-c_6)\|\omega(t)\|^2+\lambda a\|\omega(t)\|\|s(t)-r\|^{\mu}+b(\lambda+c_7\tau)\|\omega(t)\|\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu}d\xi$$
$$+c_8\|\omega(t)\|^2\|s(t)-r\|-(a-\tau b)c_9\|s(t)-r\|^{\mu-1}\|s(t)\times r\|^2$$
$$-\beta\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu+1}d\xi+(\alpha+\beta\tau)\|s(t)-r\|^{\mu+1}-\alpha\|s(t-\tau)-r\|^{\mu+1}.$$

Here, $c_k>0$, $k=1,\dots,9$.

By Young's inequality, if the number $\lambda > 0$ is sufficiently large and the positive numbers α, β, and δ are sufficiently small, then

$$\frac{1}{2}\left(c_1\lambda\|\omega(t)\|^2 + \frac{1}{2}\|s(t)-r\|^2 + \alpha\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu+1}d\xi\right) \leqslant V(s_t,\omega_t)$$

$$\leqslant 2\left(c_4\lambda\|\omega(t)\|^2 + \frac{1}{2}\|s(t)-r\|^2 + (\alpha+\beta\tau)\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu+1}d\xi\right),$$

$$\dot{V} \leqslant -\frac{1}{2}\left(\lambda c_5\|\omega(t)\|^2 + (a-\tau b)c_9\|s(t)-r\|^{\mu+1} + \beta\int\limits_{t-\tau}^{t}\|s(\xi)-r\|^{\mu+1}d\xi\right)$$

under $\|s_t - r\|_\tau + \|\omega(t)\| < \delta$.

Hence, according to [6, p. 22], the equilibrium (11) is asymptotically stable.

The proof of Theorem 3 is complete.

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